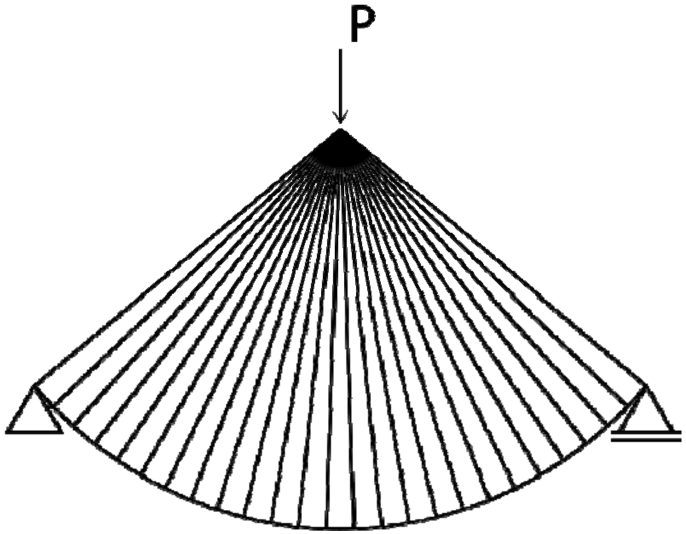
Linear mixed integer programming for topology optimization of trusses and plates

Makoto Ohsaki and Ryo Watada

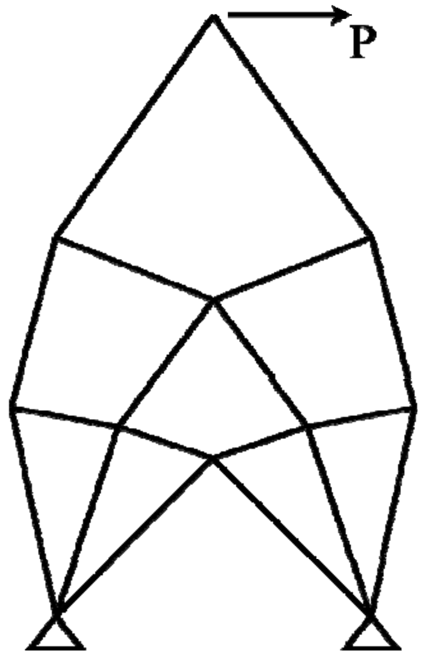
Kyoto University, JAPAN

Michell truss



A.G.M. Michell, The limits of economy in frame structures, 1904.

Infinite number of members in principal stress line

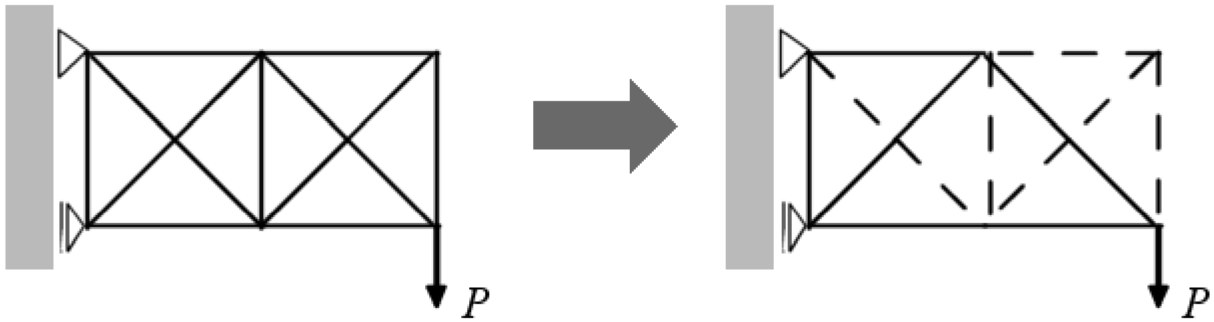


W. Prager, A note on discretized Michell structure,1974

Finite number of members
Consider nodal cost

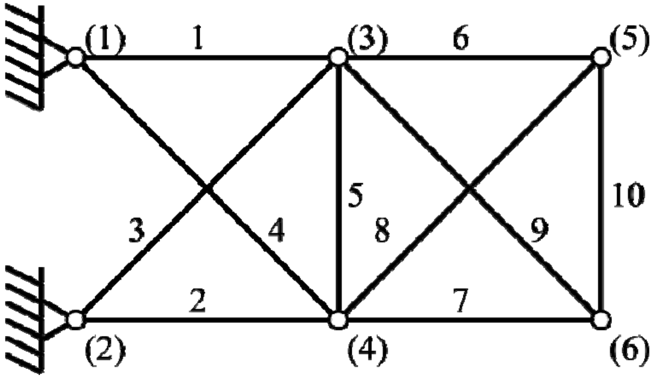
Ground structure approach to topology optimization

W. Dorn et al., Automatic design of optimal structures, 1964

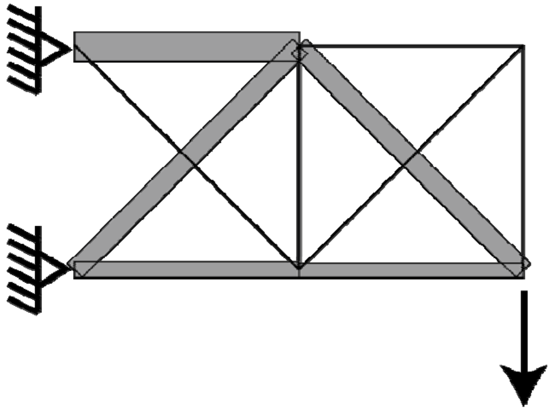


1. Assign all the nodes and members.
2. Optimize cross-sectional areas as continuous variables.
3. Remove unnecessary members and nodes.

Optimization under stress constraints

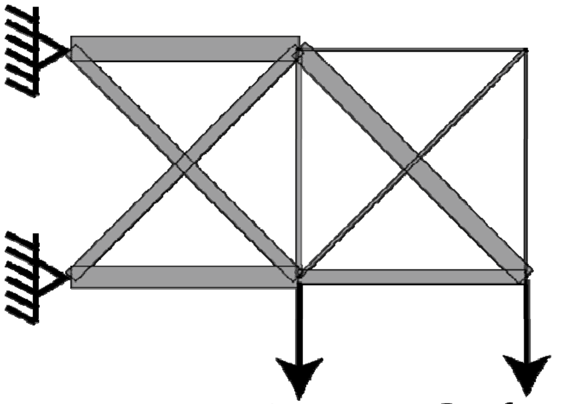


Single loading condition



Statically determinate
fully stressed

Multiple loading condition



Statically indeterminate
generally not fully stressed

Formulation of topology optimization under stress constraints

Minimize total structural volume

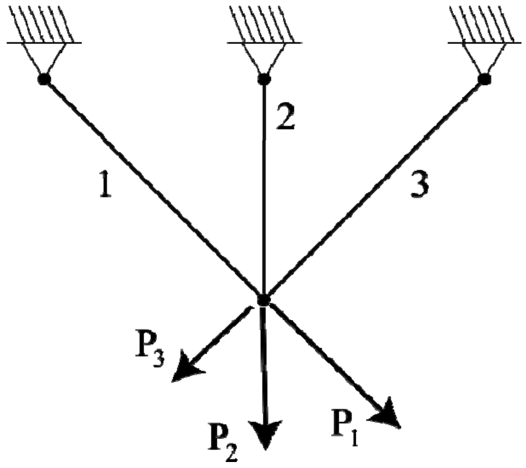
s. t. $\sigma_i^L \le \sigma_i^k(\mathbf{A}) \le \sigma_i^U$ for $A_i \ge 0$

Stress constraints are given only for existing members

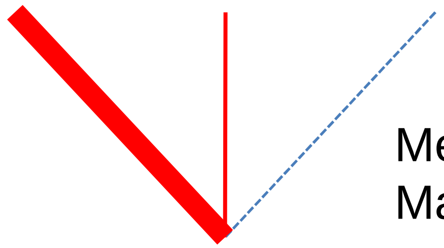
A_i : cross-sectional area of member i

$\sigma_i^k(\mathbf{A})$: stress of member i for load k

Multiple loading conditions



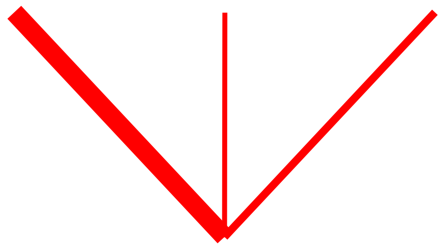
Optimal topology



Member 3 does not exist
Max. stress ratio = 4.03

V = 12.812

Fully stressed design

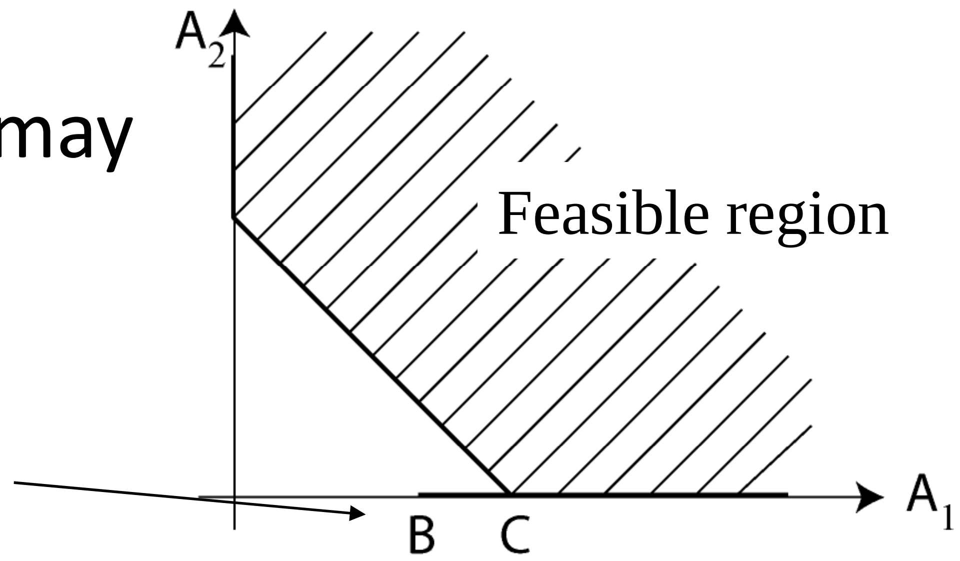


V = 15.986

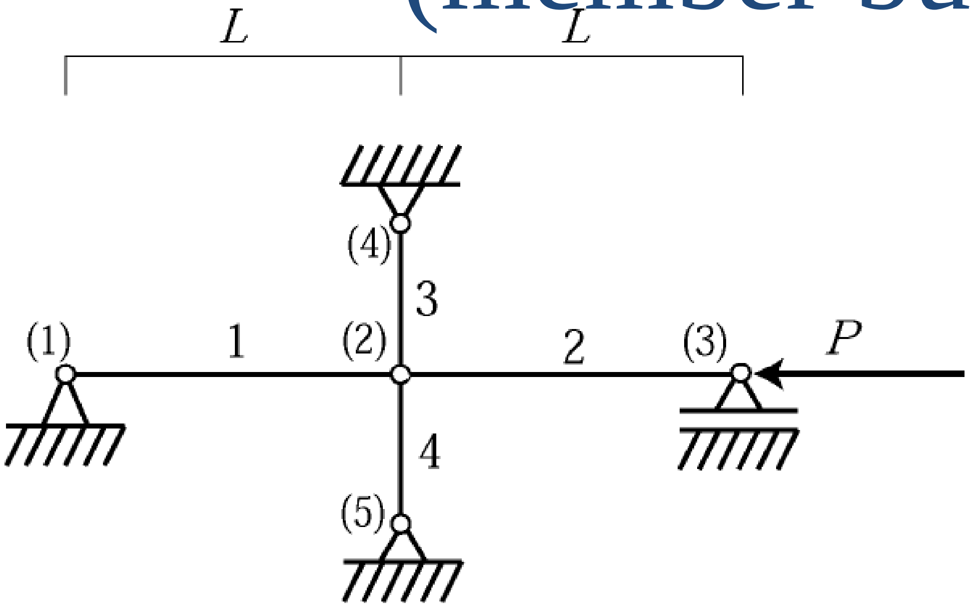
Optimal topology under multiple loading conditions cannot be obtained by conventional ground structure approach

Difficulties in topology optimization of trusses

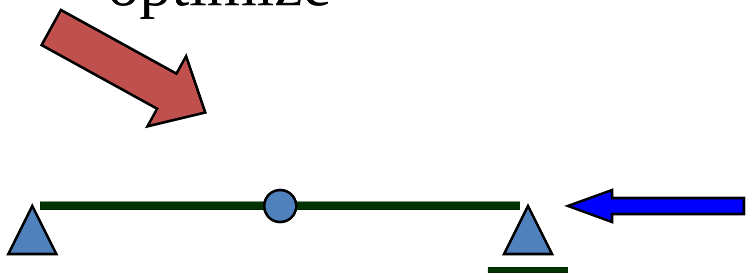
- No stress constraint for non-existing member.
- Discontinuity in problem formulation.
  - Stress constraint suddenly disappear as area approach 0.
- Optimal solution may exist at a singular point (cusp).



Unstable optimal solution (member buckling)



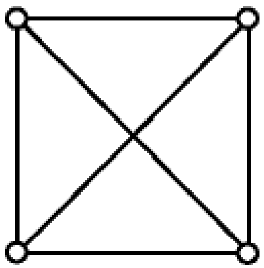
optimize



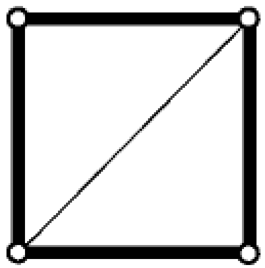
Fix node 2 → different slenderness ratio

→ different stress bound

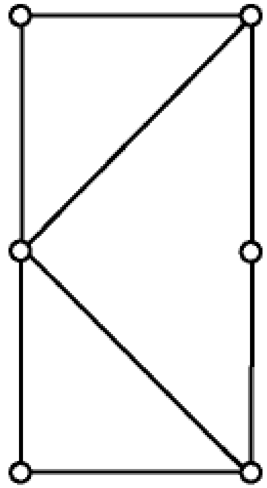
Infeasible optimal solutions



Intersection of members



Very thin member



Unstable node

Mixed integer programming for truss topology optimization with discrete variables

Topology optimization problem

minimize $V = \sum_j a_j l_j$ Structural volume

subject to $\mathbf{Bs} = \mathbf{f}$ equilibrium

Stress constraint

$$a_j \sigma_j^{\min} \le s_j \le a_j \sigma_j^{\max}$$

Force constraint

$$s_j = a_j \frac{E_j}{l_j} \mathbf{b}_j^T \mathbf{u}$$ Nonlinear relation

$$a_j \ge 0$$

Variables: **s**, **u**, a_j

Select a_j from list of available sections

Select from available list

$$a_j \in \mathbf{A}_j = \{0, a_{j1}, \cdots, a_{jN_j}\}$$

- 0-1variable

$$x_{jk} = \begin{cases} 1 & \text{if } a_j = a_{jk} \\ 0 & \text{otherwise} \end{cases}$$

- Express using x_{jk}

$$a_j = \sum_{k=1}^{N_j} x_{jk} a_{jk}, \quad \sum_{k=1}^{N_j} x_{jk} = x_j \leq 1$$

$$s_{jk} = x_{jk} a_{jk} \frac{E_j}{l_j} \mathbf{b}_j{}^T \mathbf{u}, \quad s_j = \sum_{k=1}^{N_j} s_{jk}$$

Stress-displ. relation

Linearization of stress-disp. relation

- Linearization of nonlinear constraints (Stolpe and Svanberg)

$$s_{ik} = x_{ik} a_{ik} \frac{E_i}{l_i} \mathbf{b}_i{}^T \mathbf{u}, \quad x_{ik} = 0 \; or \; 1$$

⇔

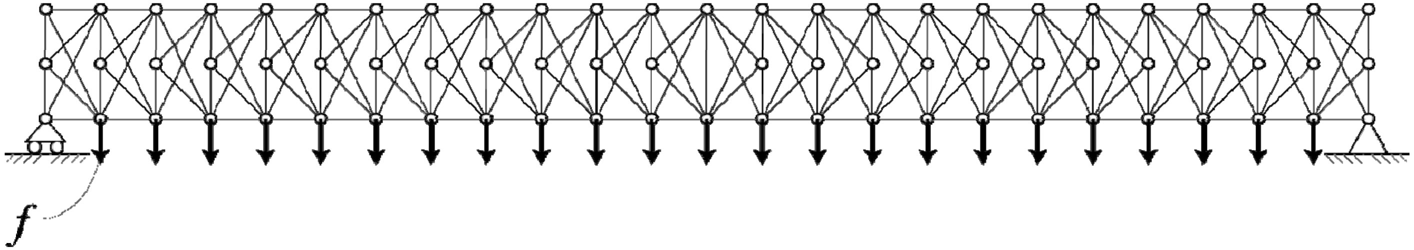
$$x_{ik} c^{\min} \le s_{ik} \le x_{ik} c^{\max}$$

$$(1 - x_{ik}) c^{\min} \le a_{ik} \frac{E}{l_i} \mathbf{b}_j^{\mathrm{T}} \mathbf{u} - s_{ik} \le (1 - x_{ik}) c^{\max}$$

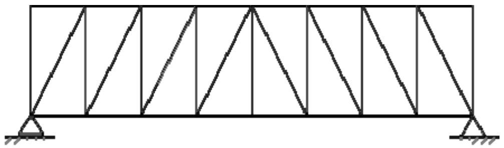
C^{max}, C^{min}: sufficiently large/small value

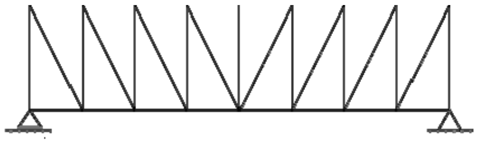
⇒ Mixed integer linear programming (MILP)

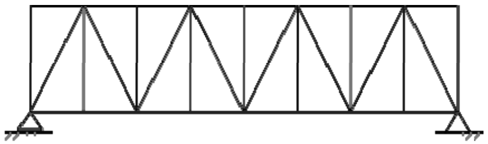
Optimization of bridge truss from traditional configurations

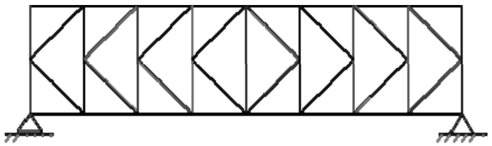


- Traditional configurations

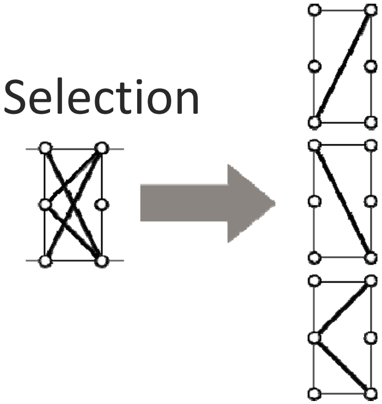

Howe truss


Platt truss


Warren truss


K-truss

- Selection

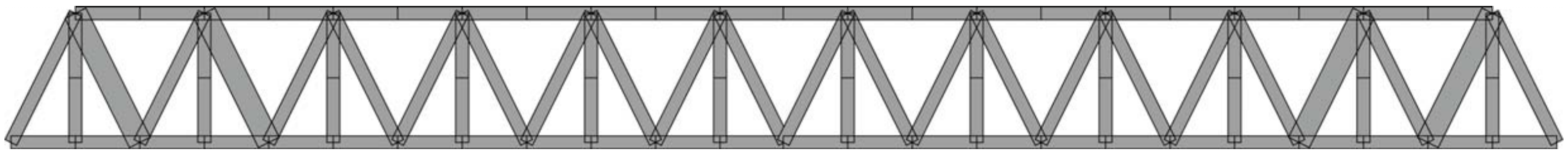


Select from three types

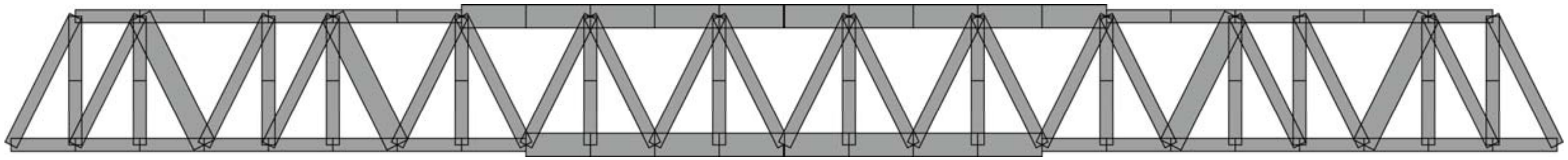
⇒ combination of traditional configurations.

Optimization results by CPLEX

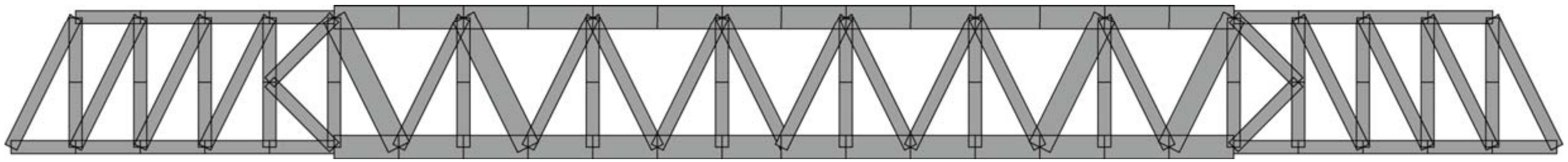
- Case 1: $f = 0.7 \times 10^{-4}$ V=65.410 CPU = 67900(sec)



- Case 2: $f = 1.0 \times 10^{-4}$ V=74.610 CPU = 14600(sec)



- Case 3: $f = 1.4 \times 10^{-4}$ V=81.203 CPU = 10600(sec)

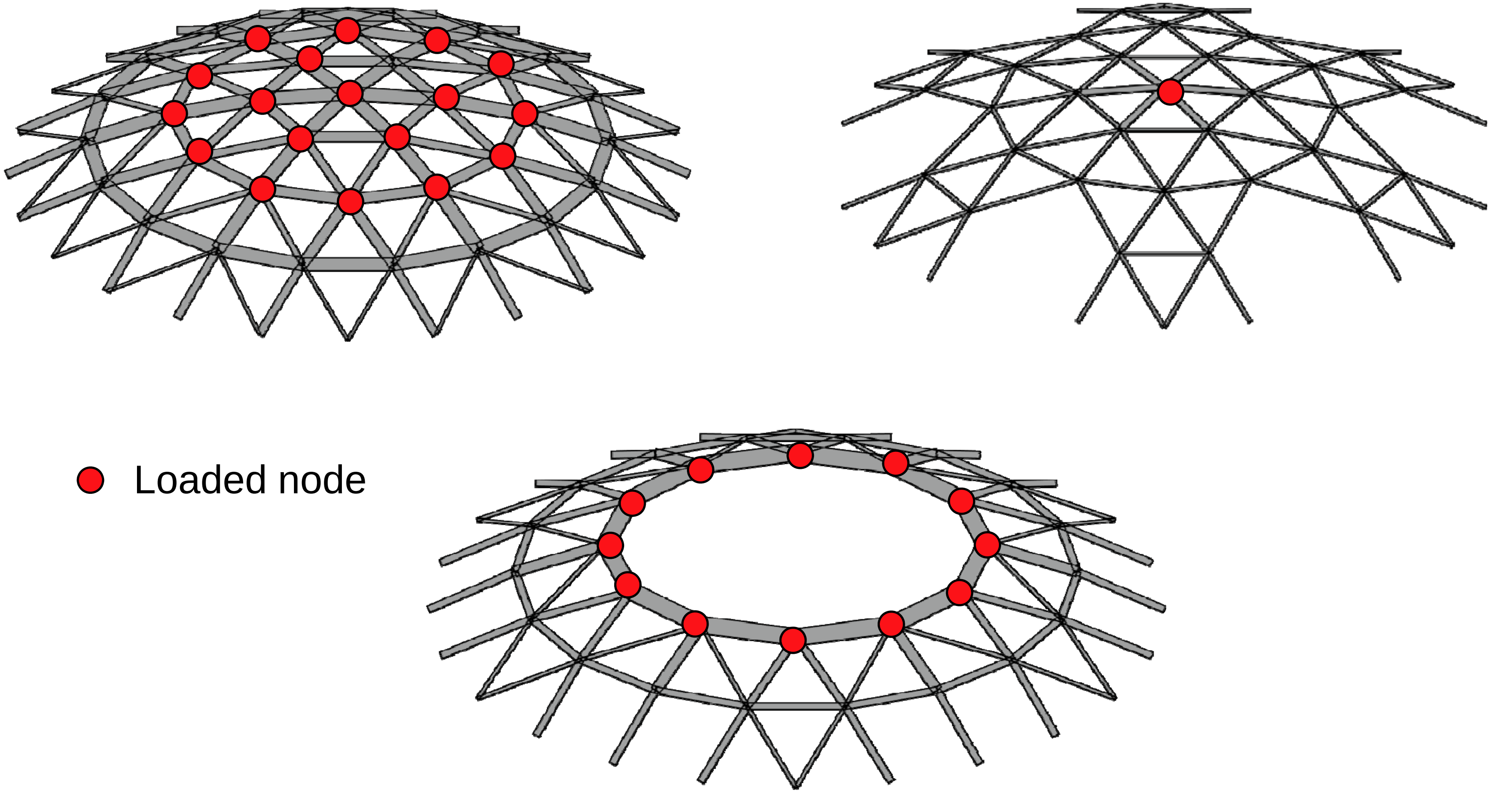


- Warren truss for small loads.

  ⇒ Howe truss near support for large loads.
- CPU time strongly depends on load magnitude.

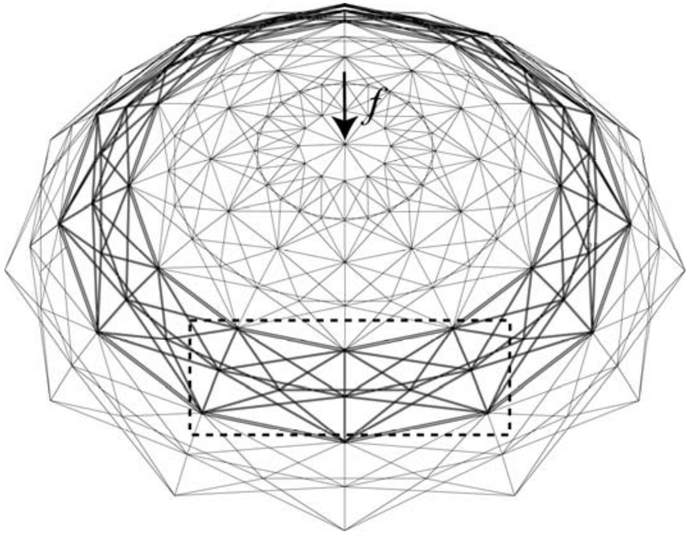
$(\mathbf{A} = \{0.0, 1.0, 1.8\})$

Optimization of space truss

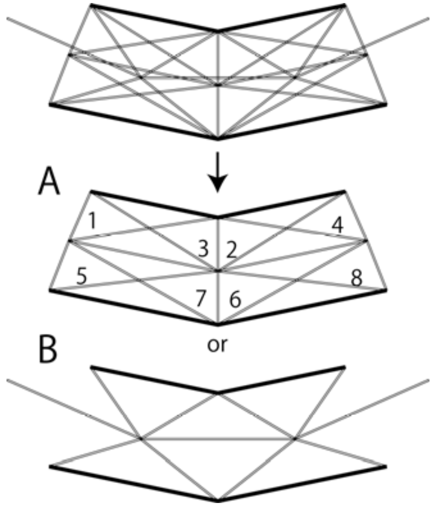


Optimization of space truss

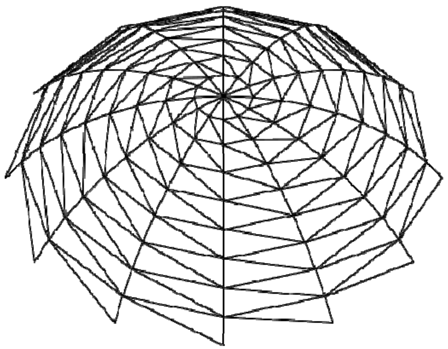
- Ground structure



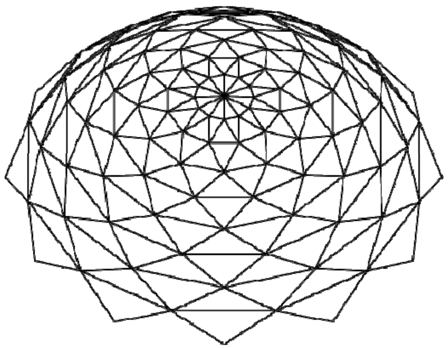
- Selection pattern



- Traditional configurations



Schuwedler dome

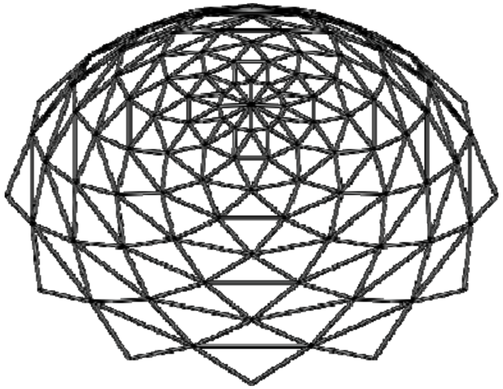


Lamella dome

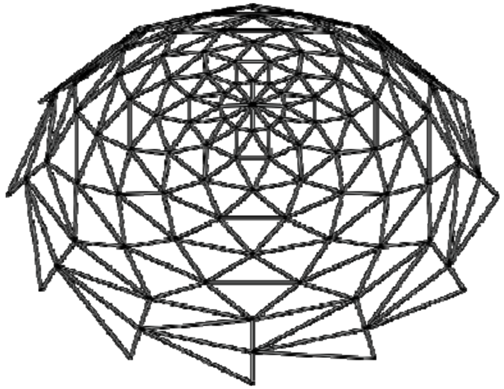
A: Schuwedler dome
B: Lamella dome

Optimization results

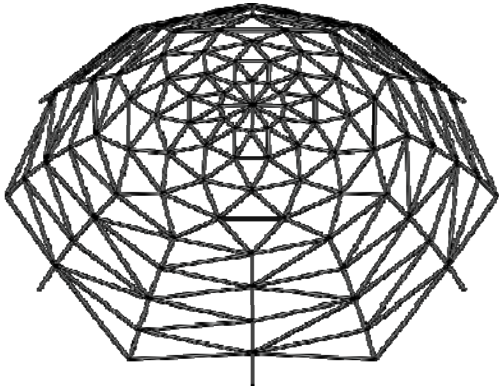
- Case 1: $f = 1.0 \times 10^{-4}$ V=1404.3



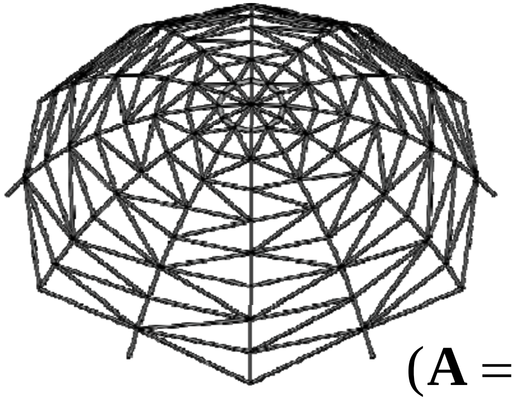
- Case 2: $f = 3.0 \times 10^{-4}$ V=1441.1



- Case 3: $f = 1.0 \times 10^{-4}$ V=1404.3



- Case 4: $f = 10.0 \times 10^{-4}$ V=1527.2



$(\mathbf{A} = \{0.0, 1.0, 2.0\})$

- Lamella dome for small loads.

  ⇒ Schuwedler dome in perimeter region for large loads.