Optimization of flexible supports for seismic response reduction of arches and frames

Makoto Ohsaki (Kyoto Univ., Japan)
Seita Tsuda (Okayama Pref. Univ.)
Yuji Miyazu (Hiroshima Univ.)
Background

• Complex dynamic property of long-span structure.
• Interaction between upper and supporting structure.
• Interaction of multiple modes.
• Dependence on flexibility of support.
1st mode

2nd mode
3rd mode

Seismic response
Purpose

1. Optimization of supporting structure of arch subjected to seismic excitation.
2. Reduction of acceleration and deformation of upper structure.
3. Utilization of flexibility of support.
Three-step optimization of supporting structure

Step 1: Maximization of vertical/horizontal displacement ratio against static horizontal load.
Step 2: Minimization of structural volume.
Step 3: Dynamic response reduction of upper structure.
Previous study (geometrically linear model)

Direction of displacement

Ground structure for topology optimization

- Pin-jointed truss
- Variable: cross-sectional area
- Remove unnecessary members.
Previous study
(geometrically linear model)

Direction of displacement

Ground Structure approach
Previous study (geometrically nonlinear model)

- Symmetric truss model considering geometrical nonlinearity.
- Optimization of cross-section and nodal location.

Deformation like reverse pendulum
Previous study
(geometrically nonlinear model)

Maximize upward displacements for both right and left deformations.
Geometrically linear model

- Pin-jointed truss
- Young’s modulus: $2.05 \times 10^5$ N/mm$^2$
- Mass at node A: 1800kg
- Mass at nodes 3~10: 600kg

Variable: cross-sectional area
Standard ground structure approach
Optimization problem (Step 1)

Maximize upward/horizontal disp. ratio due to horizontal forced disp.

Maximize \[ R(A) = \frac{d_{hv}(A)}{d_{hh}(A)} \] ← Disp. Ratio

subject to \[ d_{gh}(A) \geq d_{gh}^{L} \] ← Stiffness for self-weight
\[ d_{gv}(A) \geq d_{gv}^{L} \] ← Stiffness for self-weight
\[ d_{hh}(A) \leq d_{hh}^{U} \] ← Stiffness for horizontal load
\[ A_{L}^{i} \leq A_{i} \leq A_{U}^{i} \]
Penalization of intermediate cross-sectional area

Cross-sectional area for
stiffness

Cross-sectional area for
volume

Underestimate stiffness
→ Error in structural response
Penalization of intermediate cross-sectional area

Cross-sectional area for volume

Cross-sectional area for stiffness

Overestimate volume → No error in structural response
Maximize \[ V\left(\tilde{A}(A)\right) \]
subject to \[ R(A) \geq CR_{opt} \]
\[ d_{gh}(A) \geq d_{gh}^L \]
\[ d_{gv}(A) \geq d_{gv}^L \]
\[ d_{hh}(A) \leq d_{hh}^U \]
\[ A_i^L \leq A_i \leq A_i^U \]

Minimize volume under constraint on vertical/horizontal disp. ratio due to horizontal forced disp.

Optimization problem (Step 2)

← Structural volume
← Disp. Ratio
← Stiffness for self-weight
← Stiffness for self-weight
← Stiffness for horizontal load
Optimal solution

Optimal solution
Solution for larger upper-bound area
Simplified solution
- Span $L = 19.5$ m
- Rise $H = 2.613$ m
- Open angle $\theta = 60^\circ$
- Member section $H-300 \times 150 \times 6.5 \times 9$
- Member length $2.04$ m
- Young's modulus $2.05 \times 10^5$ N/mm$^2$
- Mass: nodes A, T 800 kg
  nodes 1~8 800 kg

Attach arch to opt 1, and carry out further optimization
Optimization problem (Step 3)

Minimize \( F(A_A) = \sqrt{\sum_{i=1}^{19} (\ddot{u}_i(A_A))^2} \)

Subject to \( d_{gh}(A) \geq d_{gh}^L \)
\( d_{gv}(A) \geq d_{gv}^L \)
\( A_i^L \leq A_i \leq A_i^U \)

\( D_A^v \): Vertical disp. at node A against self-weight
\( D_A^h \): Horizontal disp. at node A against self-weight

Objective function:
Square norm of acceleration in normal direction.
Modal analysis: CQC method
Rayleigh damping with \( h=0.02 \) for 1st and 2nd modes.
CQC method
(complete quadratic combination)

Max. acceleration of node $i$: $|\alpha_i^N|$

$$|\alpha_i^N| = \sqrt{\sum_{s=1}^{N} \sum_{r=1}^{N} (\beta_s^N \phi_s^i S_s(T_s, h_s)) \rho_{sr} (\beta_r^N \phi_r^i S_r(T_r, h_r))}$$

$\beta_s$: participation factor  \hspace{1cm} T_s: natural period  \hspace{1cm} h_s: damping factor

$S_s$: acceleration response spectrum  \hspace{1cm} \omega_s: natural circular frequency

$\phi_s^i$: normal displacement component at node $i$

$\rho_{sr}$: modal correlation coefficient

$$\rho_{sr} = \frac{8h_s h_r [h_r + \chi^2 h_s + 4\chi h_s h_r (h_r + \chi h_s)]}{\sqrt{(1+4h_s^2)(1+4h_r^2)}}$$

$$\alpha = (1 - \chi^2)^2 + 4\chi h_s h_r (1 + \chi^2) + 4(h_s^2 + h_r^2)$$

$$\chi = \omega_r / \omega_s$$
Response spectrum

\[
S_a(T_s, h_s) = \frac{1.5}{1 + 10h_s} \begin{cases} 
0.96 + 9.0T_s & \text{for } T_s \leq 0.16 \\
2.4 & \text{for } 0.16 \leq T_s \leq 0.864 \\
2.074 / T_s & \text{for } 0.864 \leq T_s
\end{cases}
\]
Optimization result
## Vibration properties of optimal solution

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period $T_s$ [s]</th>
<th>Damping factor $h_s$</th>
<th>Effective mass ratio in X-dir $\bar{M}_S^X$ [%]</th>
<th>Effective mass ratio in Y-dir $\bar{M}_S^Y$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4488</td>
<td>0.0200</td>
<td>49.194</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.3593</td>
<td>0.0200</td>
<td>1.235</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.2878</td>
<td>0.0210</td>
<td>0.000</td>
<td>50.771</td>
</tr>
<tr>
<td>4</td>
<td>0.1637</td>
<td>0.0284</td>
<td>0.000</td>
<td>2.395</td>
</tr>
</tbody>
</table>
Vibration modes of optimal solution

1st mode

2nd mode

3rd mode

4th mode
Mean-maximum responses of acceleration and displacement

(a) Tangential acceleration
(b) Normal acceleration

(c) Tangential displacement
(d) Normal displacement

△: Stiff-model, ×:Flexible-model
Attachment of viscous damper

Location of the dampers

Relation between damping coefficient and acceleration response
Attachment of viscous damper

(a) Tangential acceleration
(b) Normal acceleration
(c) Tangential displacement
(d) Normal displacement

○: Flexible-model with dampers
×: Flexible-model without dampers
△: Stiff-model
Extension to single-layer grid

Minimize interaction force between roof and supporting structure
Geometry of supporting structure

Optimal objective function value:
about 78 % of stiff-model.
Attachment of viscous damper

Red: with damper
Blue: without damper
Conclusions

• Flexibility of supports can be effectively utilized for reduction of seismic responses of structures.

• Three-step procedure:
  – 1st step: static optimization
    maximization of vertical displacement
  – 2nd step: static optimization
    minimization of structural volume:
  – 3rd step: dynamic optimization
    seismic response reduction
Optimization of flexible base for reduction of seismic response of buildings

Rocking mechanism
Dissipate seismic energy using rocking of frame and plastic dissipation at column base

+ 

Compliant bar-joint structure
Isolate structure using a flexible support

Reduce seismic response combining rocking mechanism and compliant mechanism
Purpose

Optimize flexible base to control mode shape and reduce response displacement of building frame.

Rotate opposite direction against horizontal input.

Standard base

Flexible base
Details of flexible base

Truss members (cm²)  | Material: steel
A (members 1-4,3-5)  | 200.0
B (members 2-4,2-5)  | 50.0
C (members 1-2,2-3)  | 1.0

Manufacture member C using a spring
Optimization problem

Design variables: nodal coordinate $X$
cross-sectional area $A$

Objective function: roof displacement $|y|_{\text{max}}$

Response spectrum approach (SRSS rule)

$$|y|_{\text{max}} = \sqrt{\sum_{i=1}^{3} |\beta_i \cdot u_i \cdot S_{D_i}|^2}$$

Constraint: lowest natural period $T_1 \leq 1.0$
Optimization result

- Damping factor: 2%
- Young’s modulus: 200kN/mm²
- Story mass: 8000kg

<table>
<thead>
<tr>
<th></th>
<th>Beam section (SN400B)</th>
<th>Column section (BCR295)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2～R</td>
<td>H−400 × 200 × 8 × 13</td>
<td>□−350 × 350 × 16</td>
</tr>
</tbody>
</table>

- Base beam (points 4,5) 45000kg

Design variables:
- location of pin support (1, 3) X(m)
  -3.0 ≤ X ≤ 3.0
- Cross-sectional area of horizontal member (C) A(cm²)
  0.1 ≤ A ≤ 10
Optimization result

<table>
<thead>
<tr>
<th>Objective function (m)</th>
<th>$X$ (m)</th>
<th>$A$ (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.078 \times 10^{-2}$</td>
<td>-0.719</td>
<td>1.191</td>
</tr>
</tbody>
</table>

Optimal model

X = -0.72  A = 1.19

Stiff model

X = 0.0  A = 100.0
Maximum responses

Displacement

Accelerator
Maximum responses

Interstory drift angle (rad)  Story shear (kN)  Column axial force (kN)

Rocking response is enhanced
Time-history response

Roof displacement

Drift angle between 1st floor and roof
Trajectory of drift angle and rotation of base

Drift angle between 1st floor and roof

Drifting angle

Rotation of base

Positive rotation angle

Positive drift angle
# Eigenvalue analysis

## Optimal model

<table>
<thead>
<tr>
<th>Period</th>
<th>Participation factor</th>
<th>Effective mass ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(s)</td>
<td>β</td>
<td>X-dir. (%)</td>
</tr>
<tr>
<td>1st</td>
<td>0.712</td>
<td>16.53</td>
</tr>
<tr>
<td>2nd</td>
<td>0.379</td>
<td>287.77</td>
</tr>
<tr>
<td>3rd</td>
<td>0.155</td>
<td>13.75</td>
</tr>
</tbody>
</table>

![Eigenmode](image1)

## Stiff model

<table>
<thead>
<tr>
<th>Period</th>
<th>Participation factor</th>
<th>Effective mass ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(s)</td>
<td>β</td>
<td>X-dir. (%)</td>
</tr>
<tr>
<td>1st</td>
<td>0.556</td>
<td>157.30</td>
</tr>
<tr>
<td>2nd</td>
<td>0.160</td>
<td>90.66</td>
</tr>
<tr>
<td>3rd</td>
<td>0.107</td>
<td>186.00</td>
</tr>
</tbody>
</table>

![Eigenmode](image2)
Frequency response

![Graph showing frequency response with different color lines for roof displacement vs period]
Base with viscous damper

Red: with damper, Blue: without damper
Conclusions

• Flexibility of supports can be effectively utilized for reduction of seismic responses of structures.

• Two-stage procedure:
  – 1st stage: static optimization
    maximization of vertical displacement:
    minimization of structural volume:
  – 2nd stage: dynamic optimization
    seismic response reduction
    variable: cross-sectional area