Optimization of flexible supports for seismic response reduction of arches and frames

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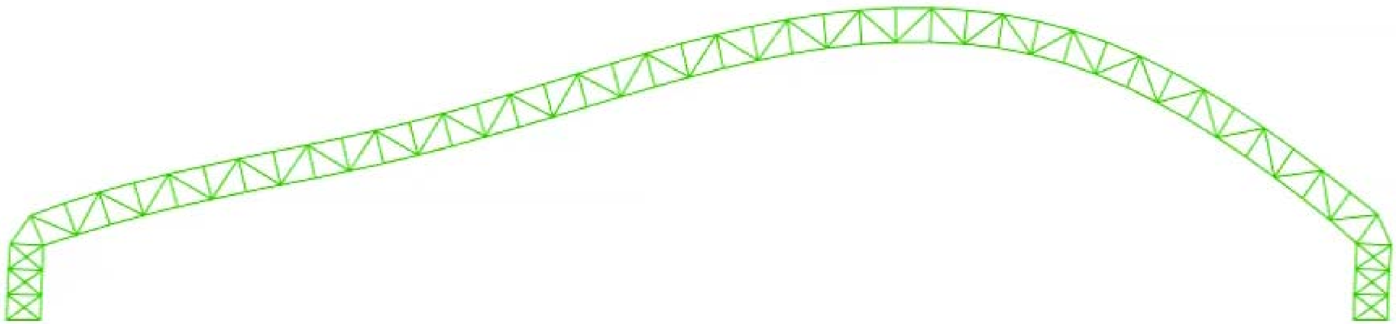
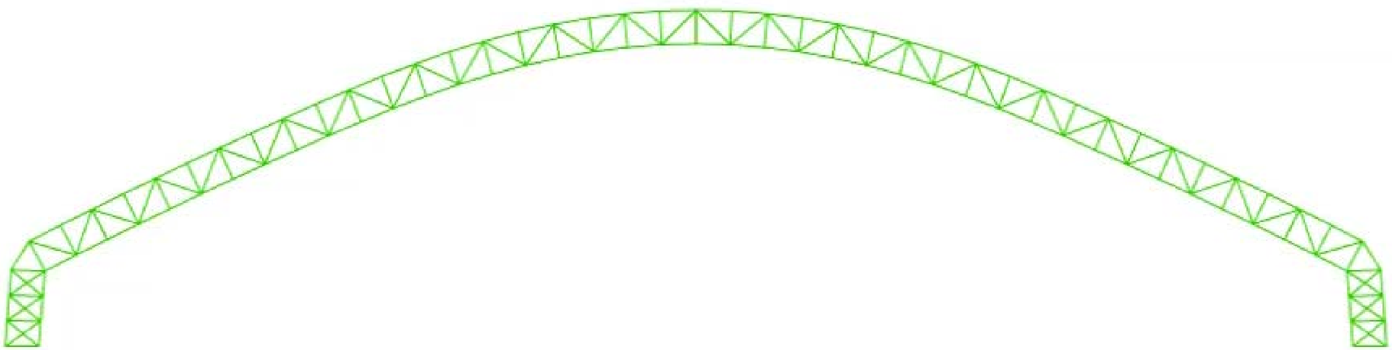
Background

- Complex dynamic property of long-span structure.
- Interaction between upper and supporting structure.
- Interaction of multiple modes.
- Dependence on flexibility of support.

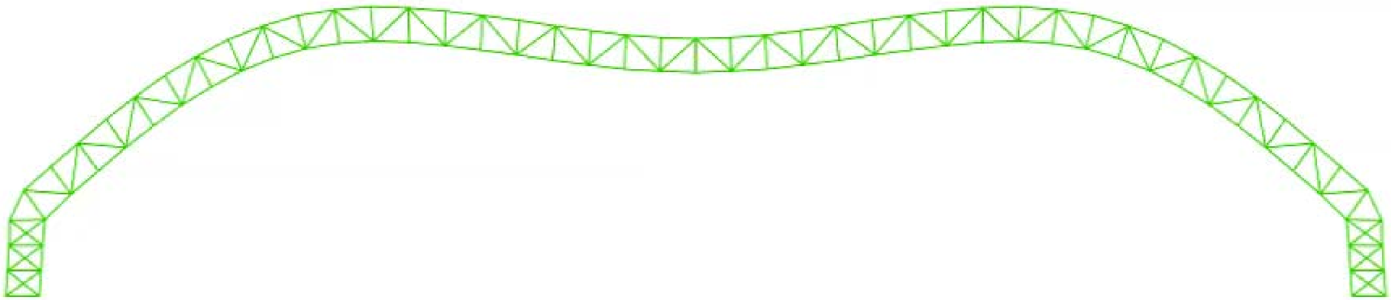
1st mode



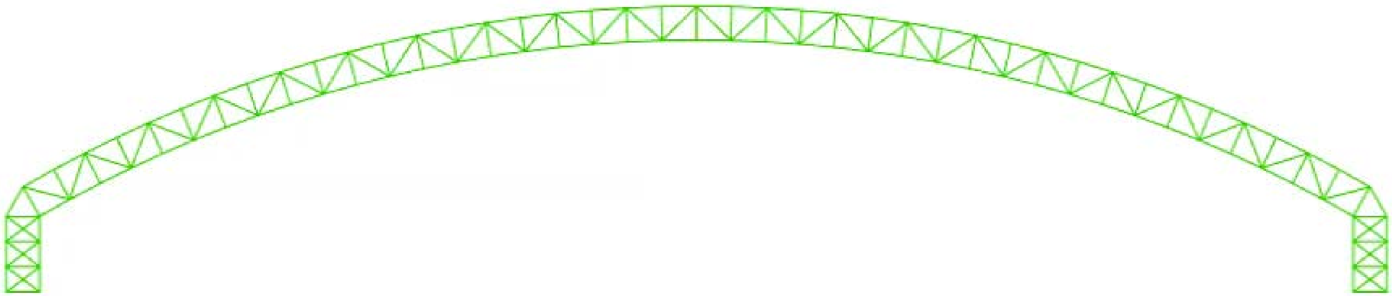
2nd mode



3rd mode



Seismic response



Purpose

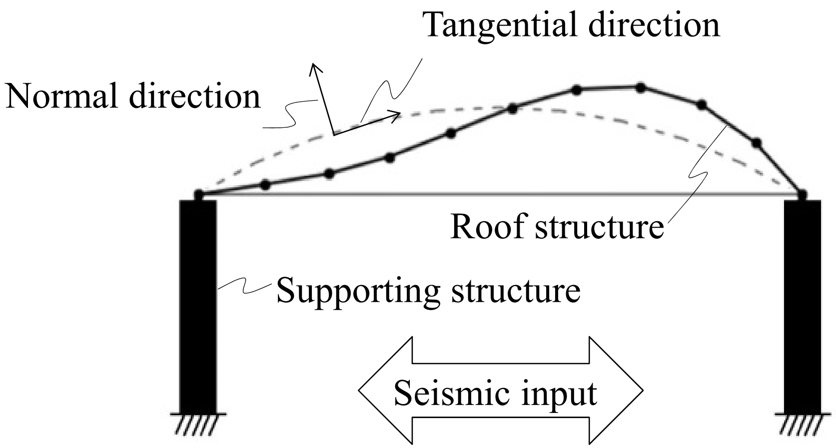
1. Optimization of supporting structure of arch subjected to seismic excitation.
2. Reduction of acceleration and deformation of upper structure.
3. Utilization of flexibility of support.

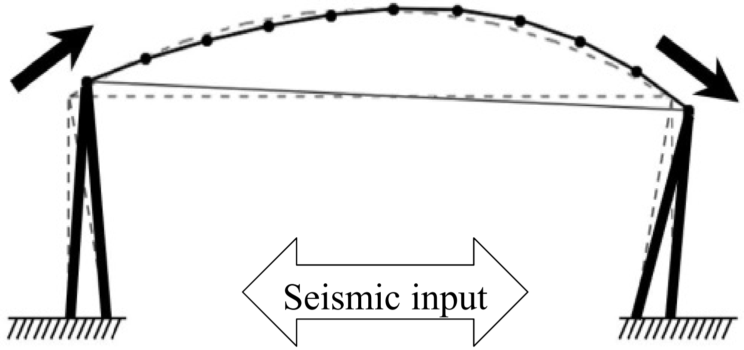
Three-step optimization of supporting structure

Step 1: Maximization of vertical/horizontal displacement ratio against static horizontal load.

Step 2: Minimization of structural volume.

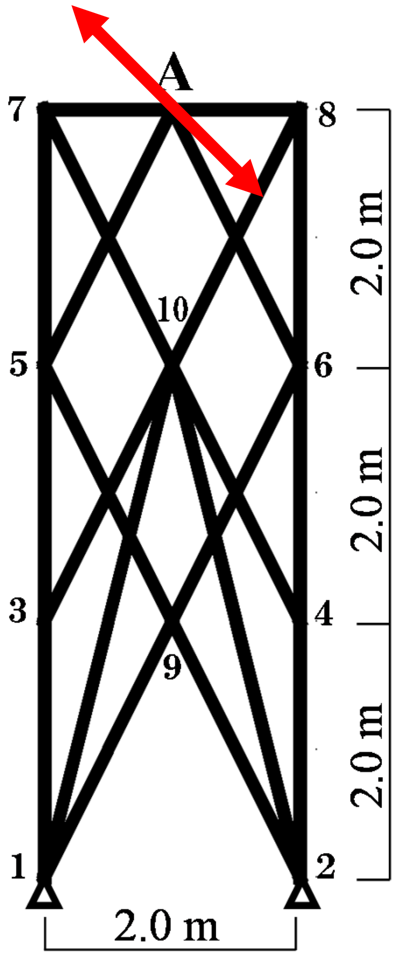
Step 3: Dynamic response reduction of upper structure.





Previous study (geometrically linear model)

Direction of displacement

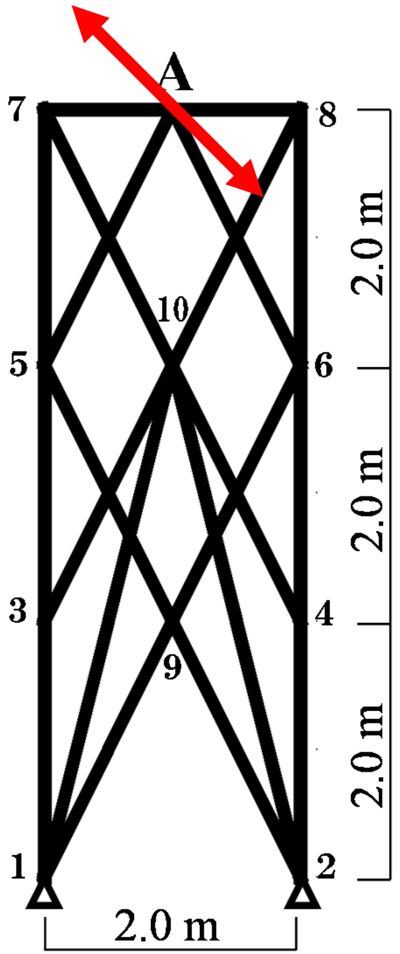


Ground structure for topology optimization

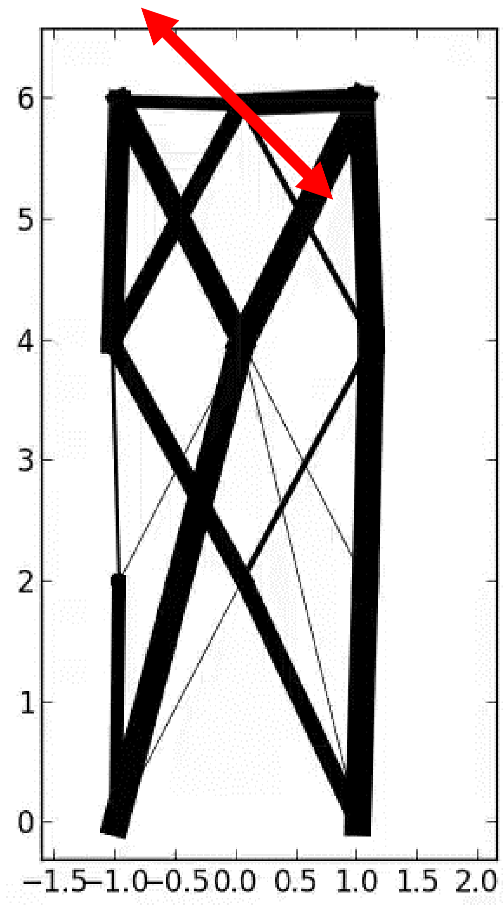
- Pin-jointed truss
- Variable: cross-sectional area
- Remove unnecessary members.

Previous study
(geometrically linear model)

Direction of displacement



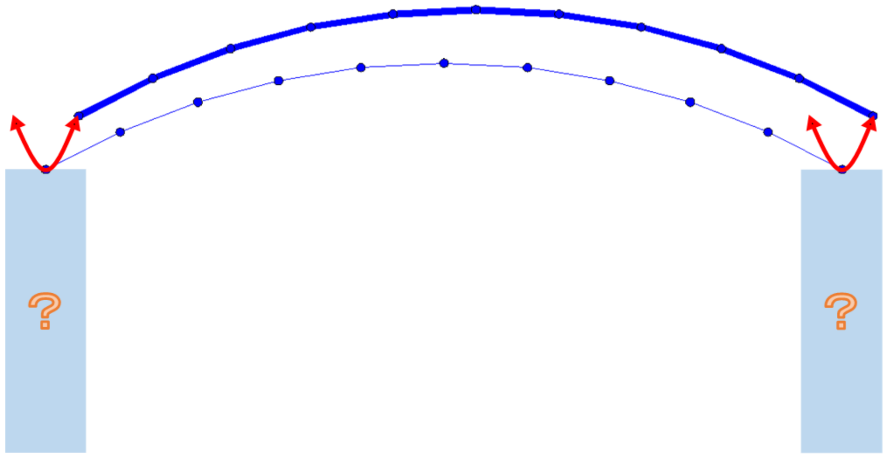
Ground
Structure
approach



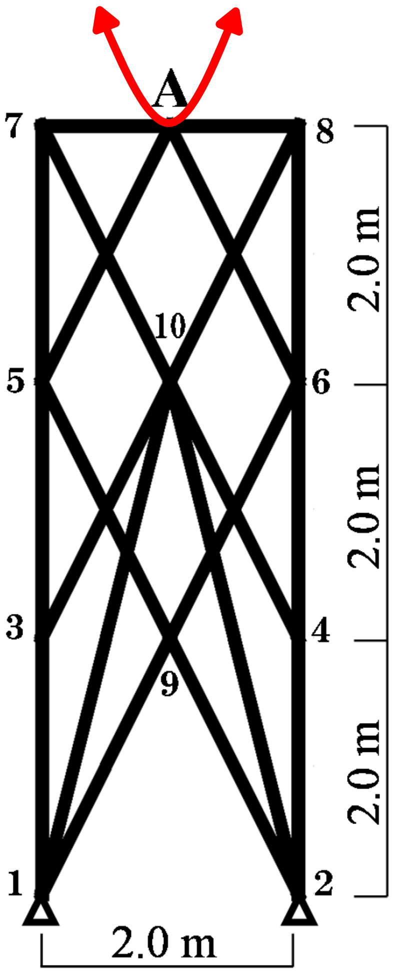
Previous study
(geometrically nonlinear model)

- Symmetric truss model considering geometrical nonlinearity.
- Optimization of cross-section and nodal location.

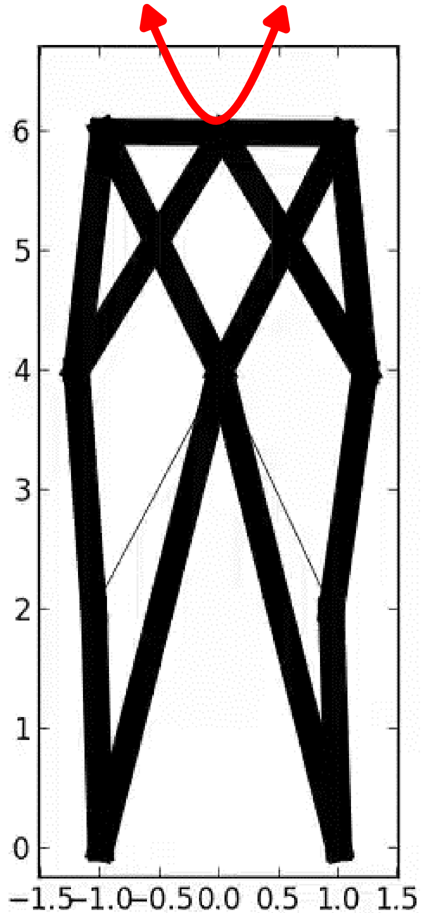
Deformation like reverse pendulum



Previous study (geometrically nonlinear model)

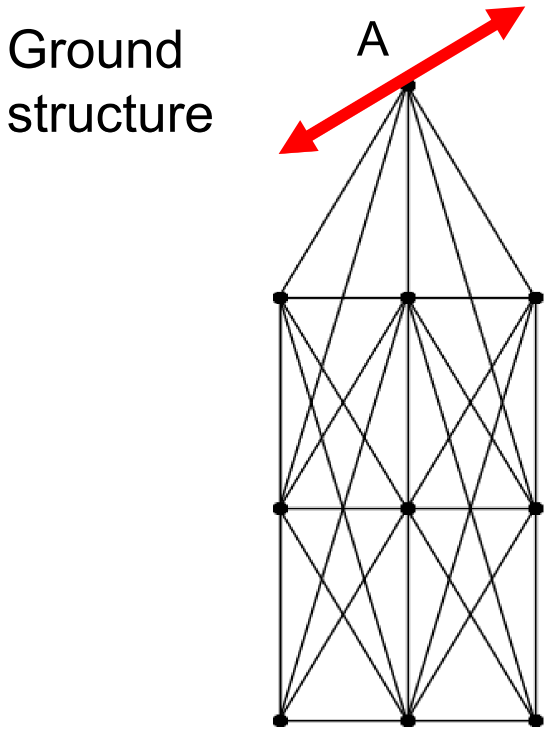


Maximize upward displacements for both right and left deformations.



Geometrically linear model

Direction of displacement



- Pin-jointed truss
- Young's modulus: 2.05×10^5 N/mm^2
- Mass at node A: 1800kg
- Mass at nodes 3～10: 600kg

- Variable: cross-sectional area
- Standard ground structure approach

Optimization problem (Step 1)

Maximize upward/horizontal disp. ratio due to horizontal forced disp.

$$\text{Maximize} \quad R(\boldsymbol{A}) = \frac{d_{hv}(\boldsymbol{A})}{d_{hh}(\boldsymbol{A})}$$ ← Disp. Ratio

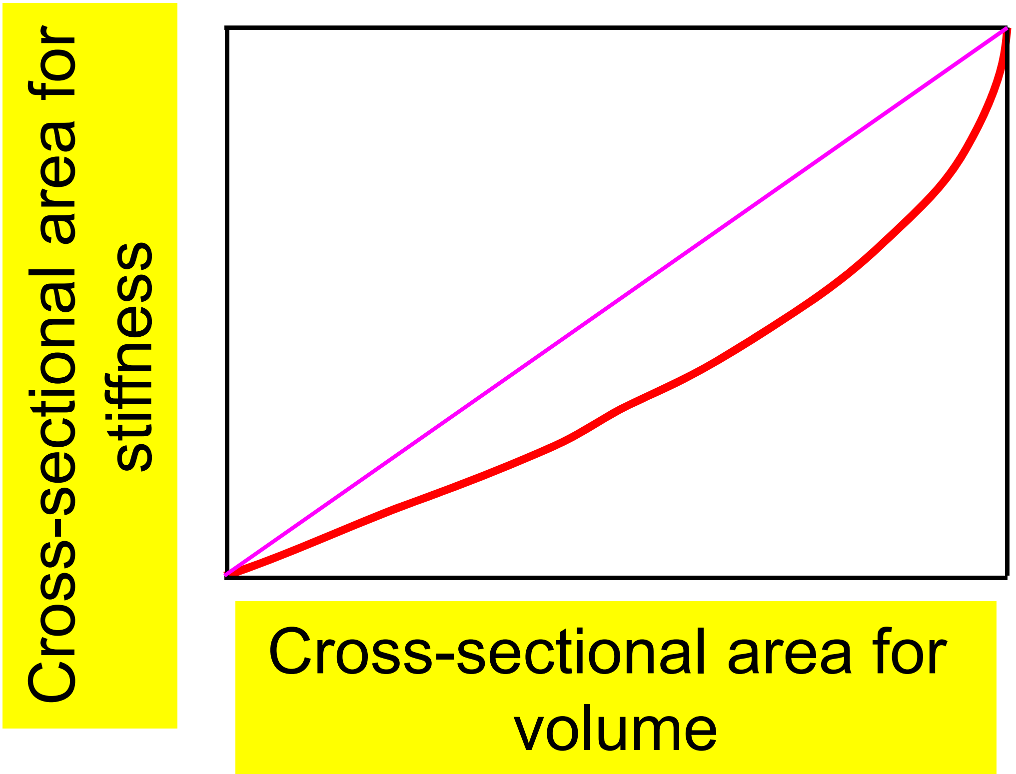
$$\text{subject to} \quad d_{gh}(\boldsymbol{A}) \geq d_{gh}^{L}$$ ← Stiffness for self-weight

$$d_{gv}(\boldsymbol{A}) \geq d_{gv}^{L}$$ ← Stiffness for self-weight

$$d_{hh}(\boldsymbol{A}) \leq d_{hh}^{U}$$ ← Stiffness for horizontal load

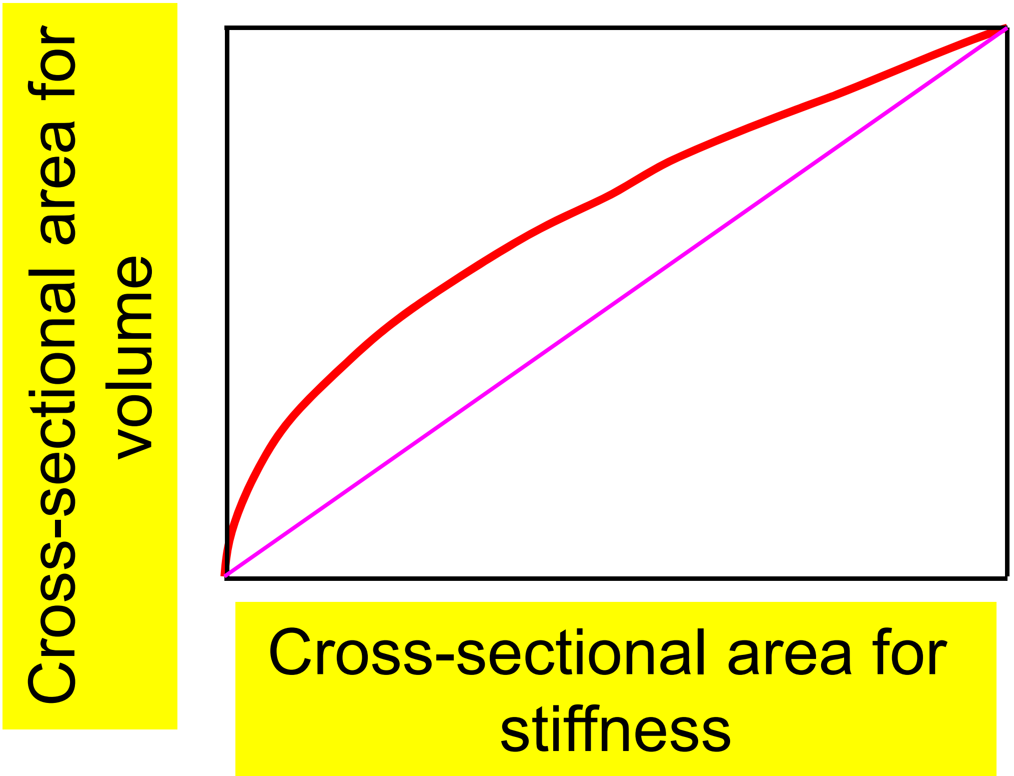
$$A_i^{L} \leq A_i \leq A_i^{U}$$

Penalization of intermediate cross-sectional area



Underestimate stiffness
→ Error in structural response

Penalization of intermediate cross-sectional area



Overestimate volume
→ No error in structural response

Optimization problem (Step 2)

Minimize volume under constraint on vertical/horizontal disp. ratio due to horizontal forced disp.

$$\text{Maximize} \quad V\left(\tilde{\boldsymbol{A}}(\boldsymbol{A})\right)$$ ← Structural volume

$$\text{subject to} \quad R\left(\boldsymbol{A}\right) \geq CR_{opt}$$ ← Disp. Ratio

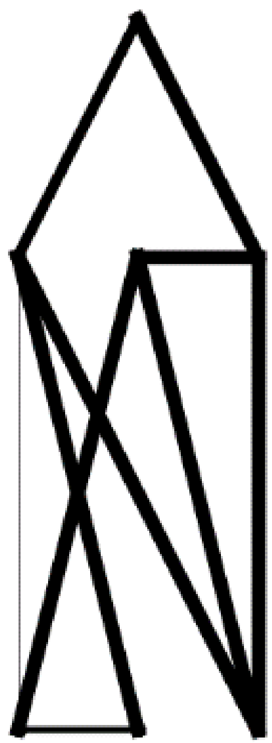
$$d_{gh}\left(\boldsymbol{A}\right) \geq d_{gh}^{L}$$ ← Stiffness for self-weight

$$d_{gv}\left(\boldsymbol{A}\right) \geq d_{gv}^{L}$$ ← Stiffness for self-weight

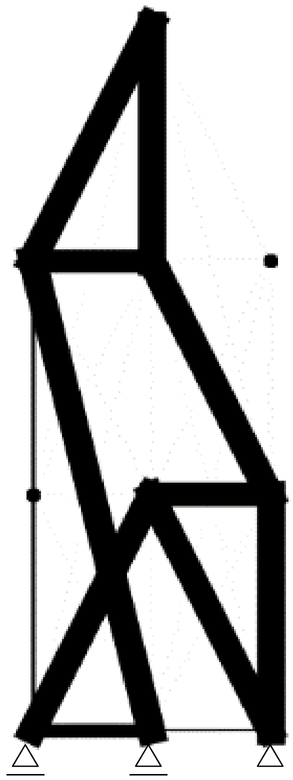
$$d_{hh}\left(\boldsymbol{A}\right) \leq d_{hh}^{U}$$ ← Stiffness for horizontal load

$$A_i^L \leq A_i \leq A_i^U$$

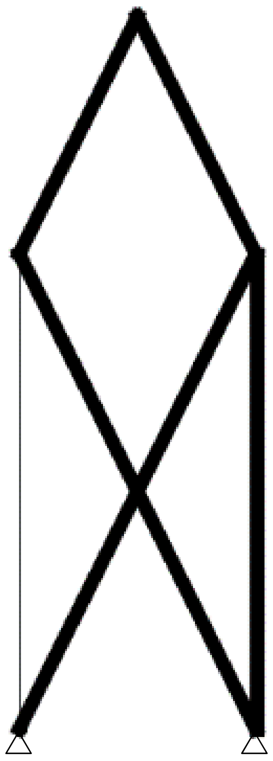
Optimal solution



Optimal solution

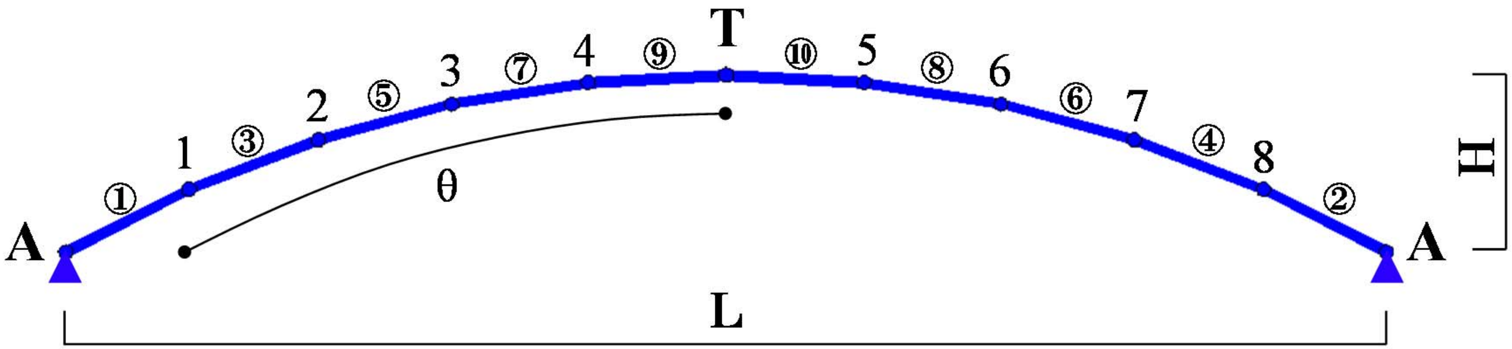


Solution for larger upper-bound area

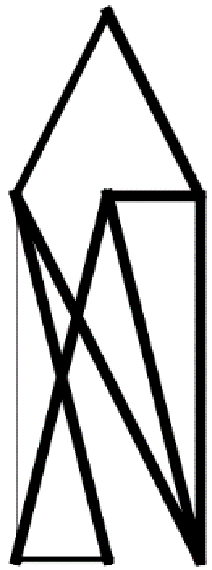


Simplified solution

Arch model



- Span L=19.5 m
- Rise H=2.613 m
- Open angle θ=60°
- Member section H-300×150×6.5×9
- Member length 2.04 m
- Young's modulus 2.05×10^5 N/mm^2
- Mass: nodes A, T 800 kg
  nodes 1～8 800 kg



Attach arch to opt 1, and carry out further optimization

Optimization problem (Step 3)

$$\text{Minimize} \quad F(A_{\mathrm{A}}) = \sqrt{\sum_{i=11}^{19} \left(\ddot{u}_i^n (A_{\mathrm{A}}) \right)^2}$$

←Norm of acc. In normal dir.

$$\text{subject to} \quad d_{gh}(\boldsymbol{A}) \geq d_{gh}^L$$

←Stiffness against self-weight

$$d_{gv}(\boldsymbol{A}) \geq d_{gv}^L$$

$$A_i^L \leq A_i \leq A_i^U$$

D_{A}^v : Vertical disp. at node A against self-weight

D_{A}^h : Horizontal disp. at node A against self-weight

Objective function:

Square norm of acceleration in normal direction.

Modal analysis: CQC method

Rayleigh damping with *h*=0.02 for 1st and 2nd modes.

CQC method
(complete quadratic combination)

Max. acceleration of node ***i***: $\left|\alpha_i^N\right|$

$$\left|\alpha_i^N\right| = \sqrt{\sum_{s=1}^{N}\sum_{r=1}^{N}\left(\beta_s\,{}^{\mathrm{N}}\phi_s^i S_s\left(T_s, h_s\right)\right)\rho_{sr}\left(\beta_r\,{}^{\mathrm{N}}\phi_r^i S_r\left(T_r, h_r\right)\right)}$$

β_s : participation factor T_s : natural period h_s : damping factor

S_s : acceleration response spectrum ω_s : natural circular frequency

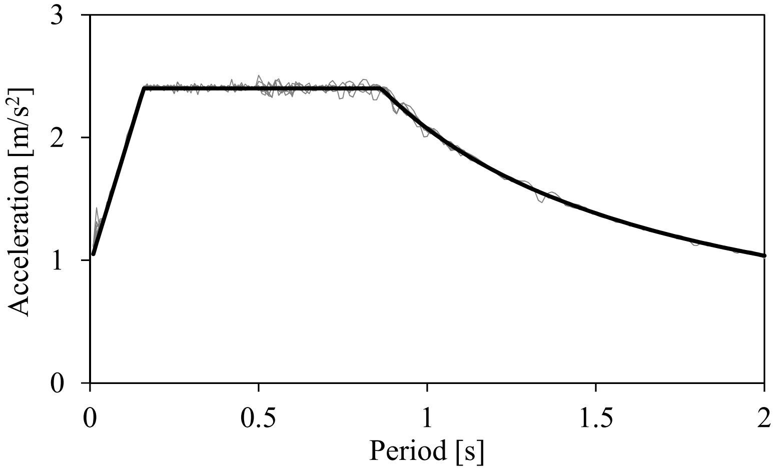
${}^{\mathrm{N}}\phi_s^i$: normal displacement component at node ***i***

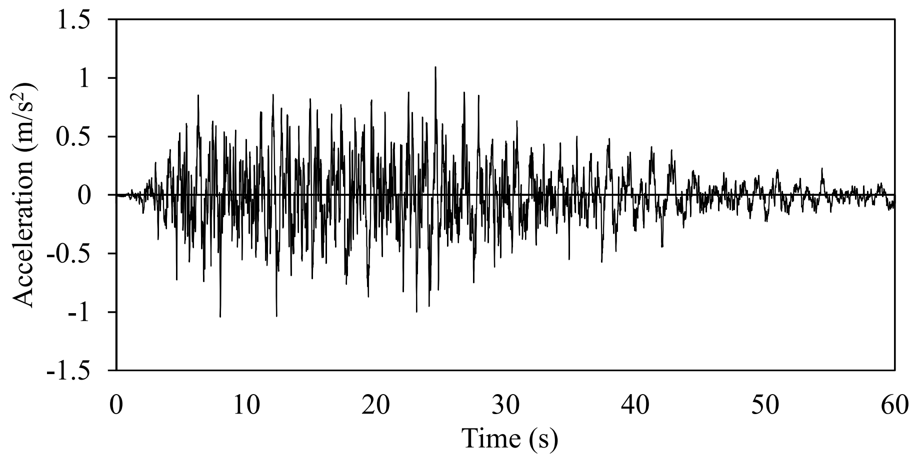
ρ_{sr} : modal correlation coefficient

$$\rho_{sr} = \frac{8\sqrt{h_s h_r}\left[h_r + \chi^3 h_s + 4\chi h_s h_r\left(h_r + \chi h_s\right)\right]\sqrt{\chi}}{\sqrt{\left(1+4h_s^{\ 2}\right)\left(1+4h_r^{\ 2}\right)}\alpha}$$

$$\alpha = \left(1-\chi^2\right)^2 + 4\chi h_s h_r\left(1+\chi^2\right) + 4\left(h_s^{\ 2} + h_r^{\ 2}\right)\chi^2 \qquad \chi = \omega_r / \omega_s$$

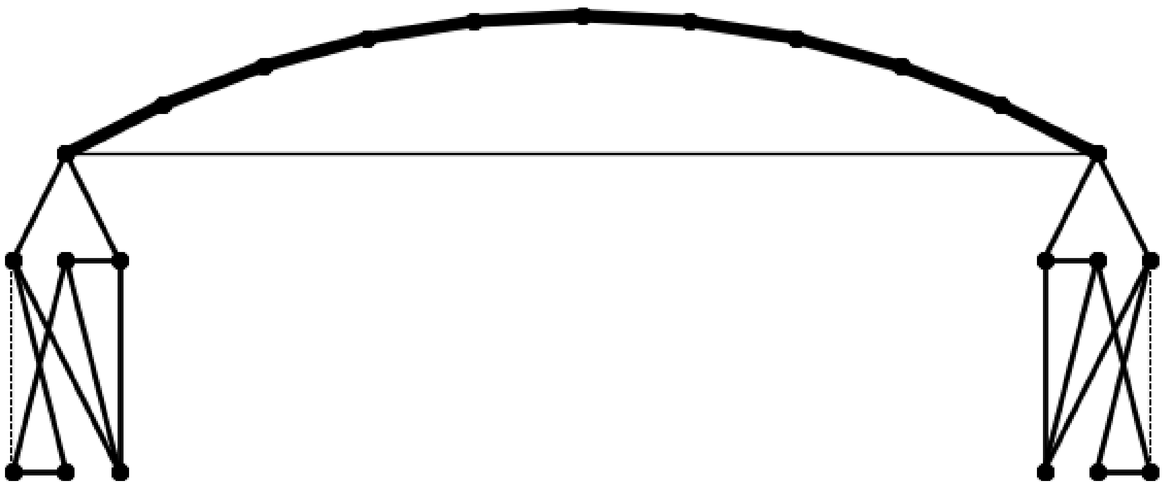
Response spectrum





$$S_a(T_s, h_s) = \frac{1.5}{1+10h_s}\begin{cases} 0.96 + 9.0T_s & \text{for } T_s \leq 0.16 \\ 2.4 & \text{for } 0.16 \leq T_s \leq 0.864 \\ 2.074 / T_s & \text{for } 0.864 \leq T_s \end{cases}$$

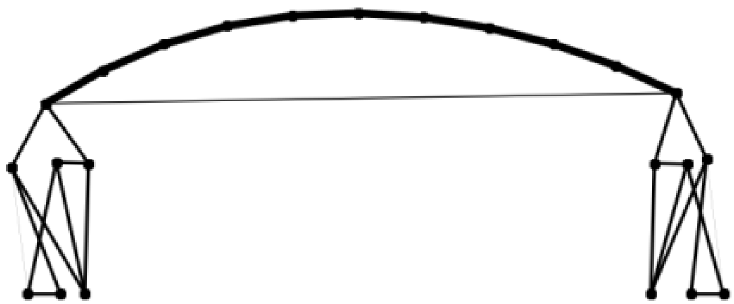
Optimization result



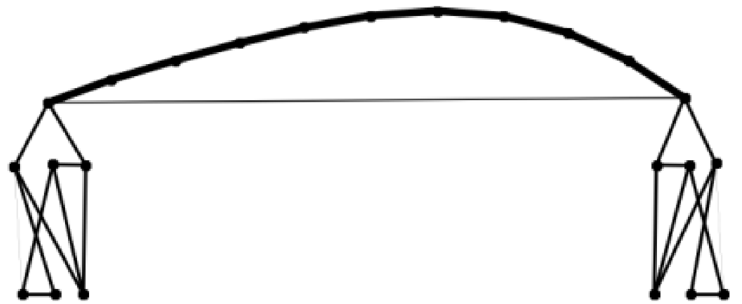
Vibration properties of optimal solution

Mode	Period T_s [s]	Damping factor h_s	Effective mass ratio in X-dir $\bar{M}_s^X$ [%]	Effective mass ratio in Y-dir $\bar{M}_s^Y$ [%]
1	0.4488	0.0200	49.194	0.000
2	0.3593	0.0200	1.235	0.000
3	0.2878	0.0210	0.000	50.771
4	0.1637	0.0284	0.000	2.395

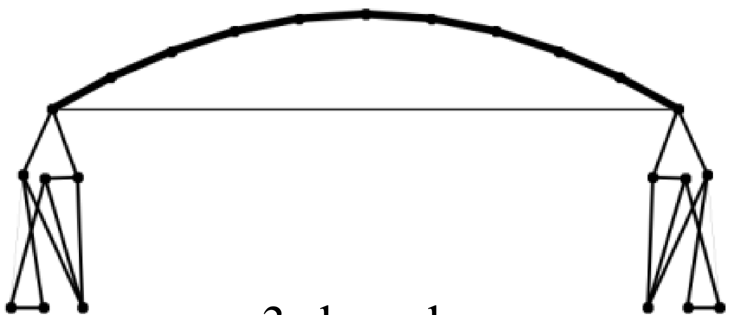
Vibration modes of optimal solution



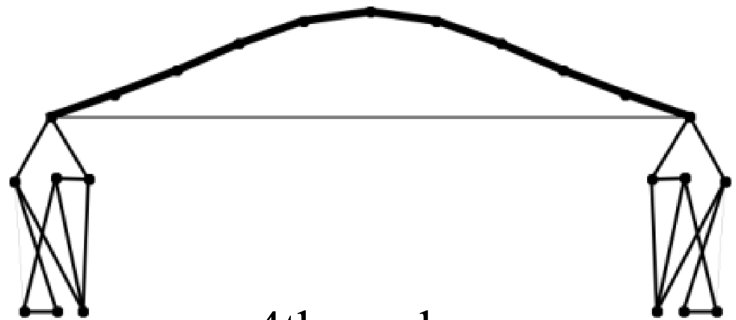
1st mode



2nd mode

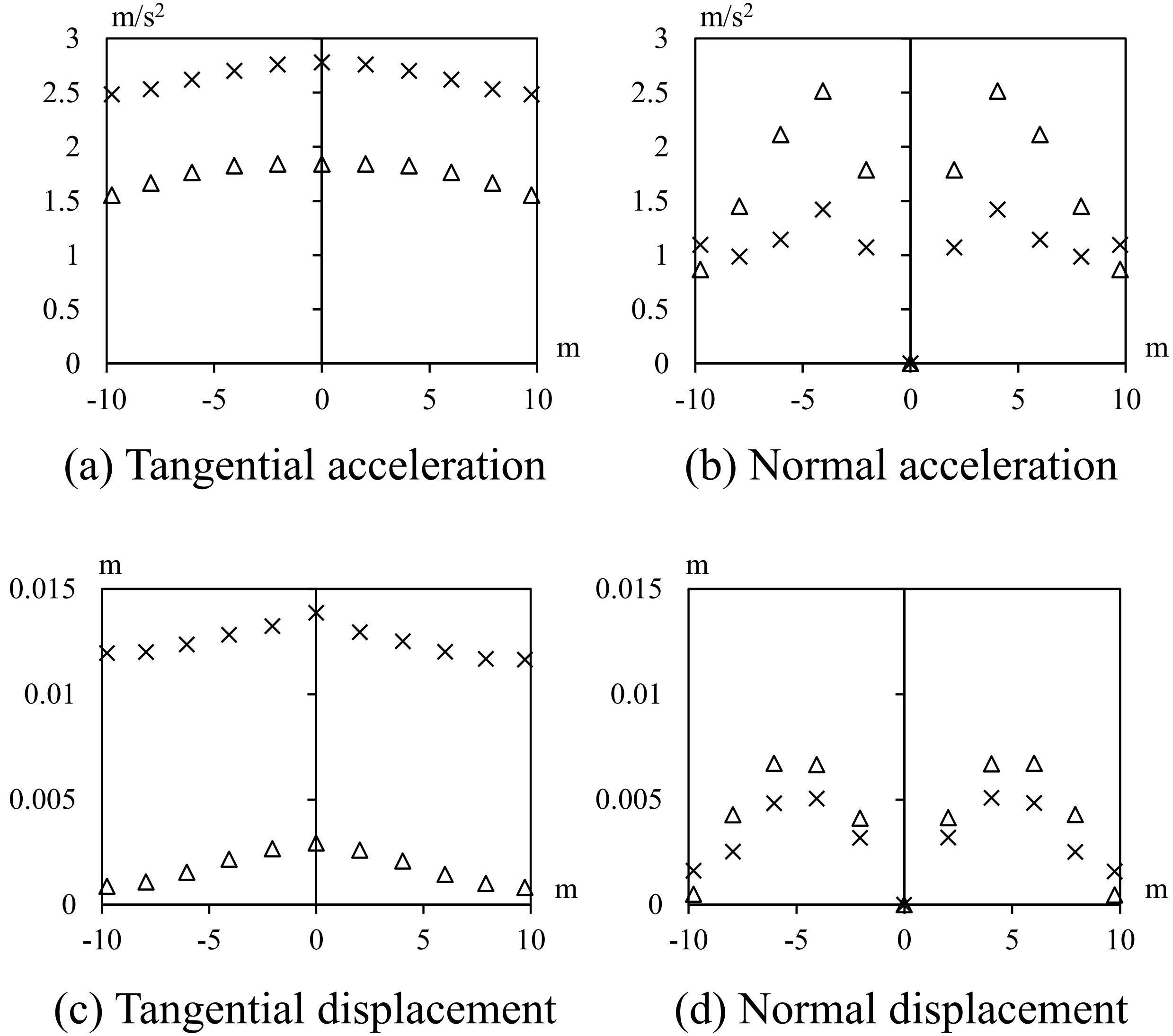


3rd mode



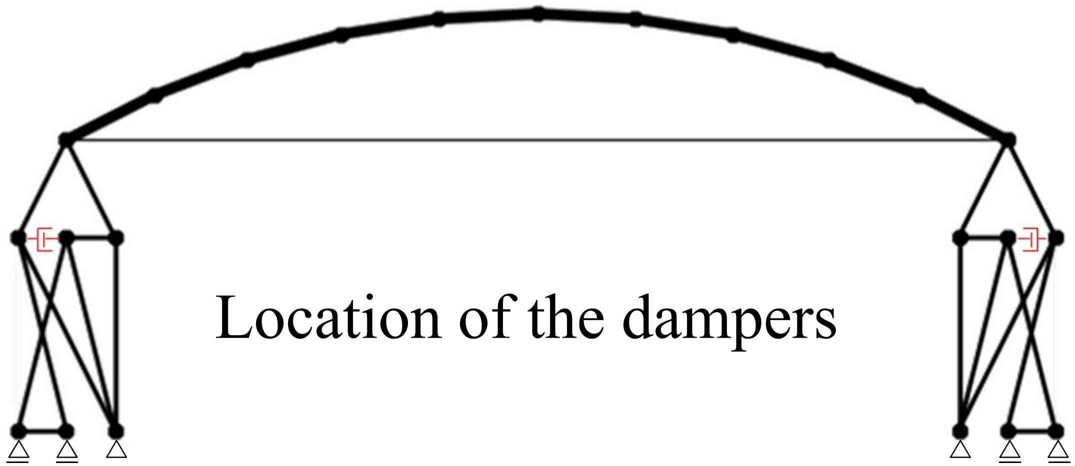
4th mode

Mean-maximum responses of acceleration and displacement

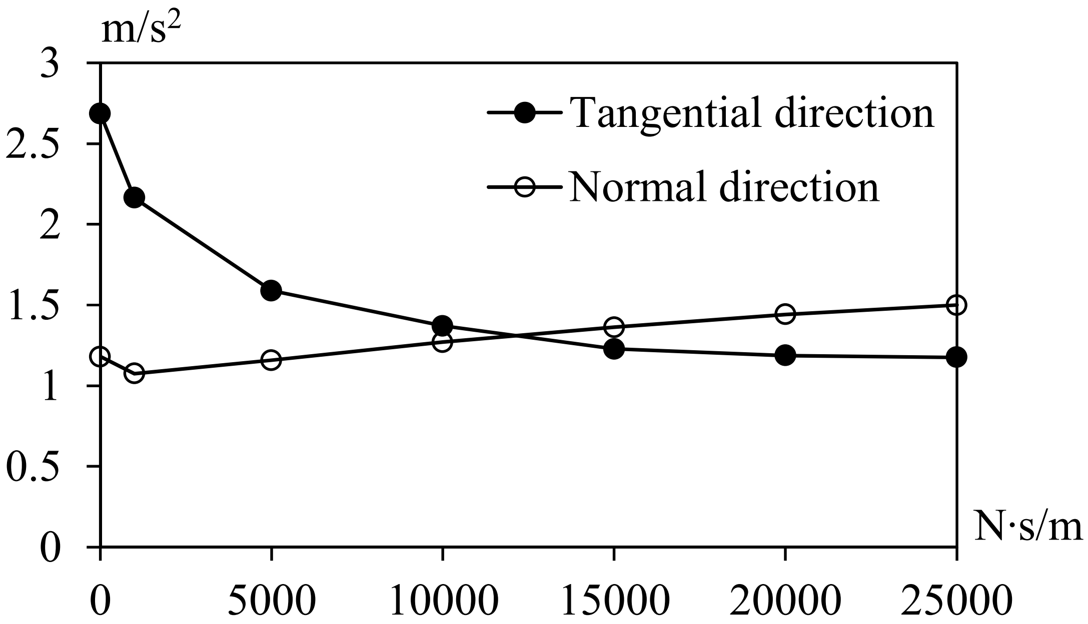


△: Stiff-model, ×:Flexible-model

Attachment of viscous damper

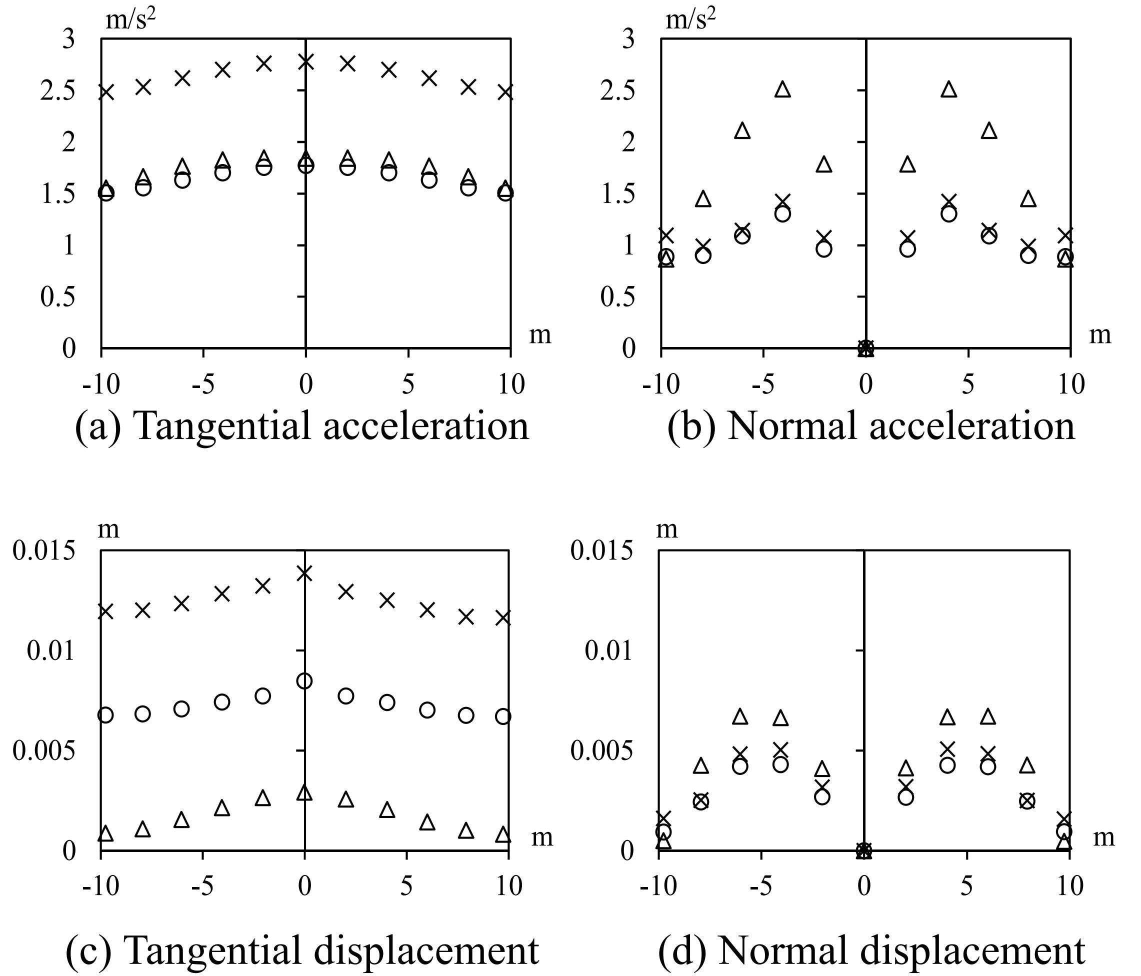


Location of the dampers



Relation between damping coefficient and acceleration response

Attachment of viscous damper

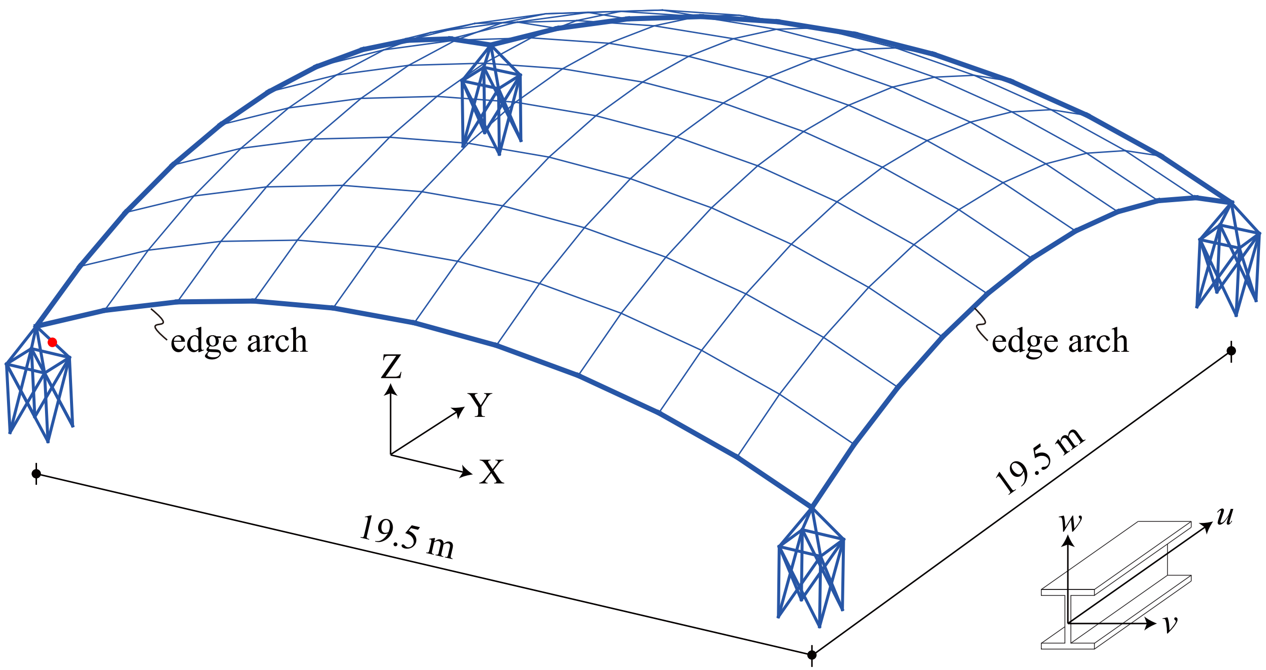


○: Flexible-model with dampers

×: Flexible-model without dampers

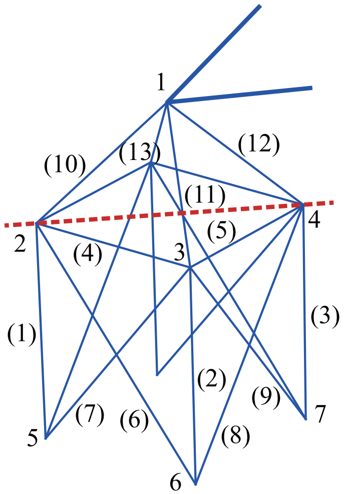
△:Stiff-model

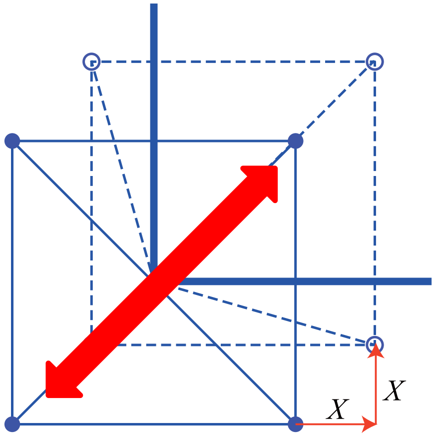
Extension to single-layer grid



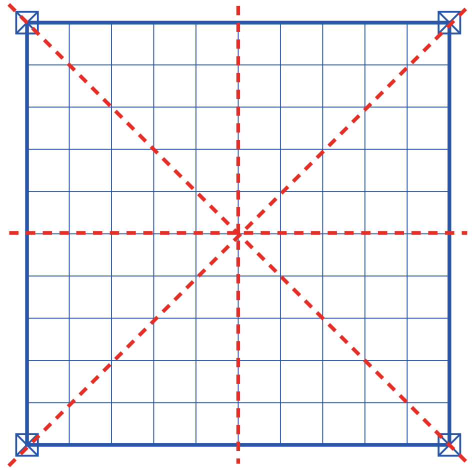
Minimize interaction force between roof and supporting structure

Geometry of supporting structure



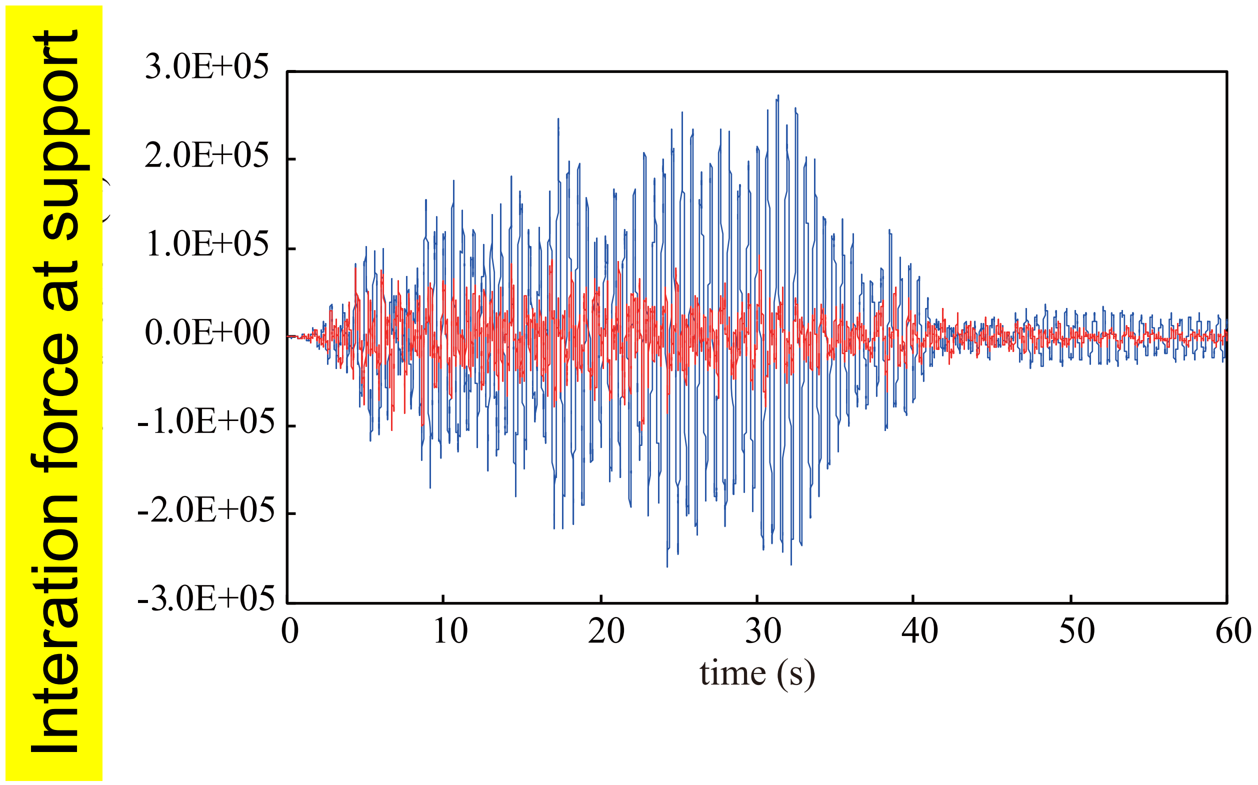


Diagonal location of top node of support



Optimal objective function value:
about 78 % of stiff-model.

Attachment of viscous damper



Red: with damper
Blue: without damper

Conclusions

- Flexibility of supports can be effectively utilized for reduction of seismic responses of structures.
- Three-step procedure:
  - 1st step: static optimization
    maximization of vertical displacement
  - 2nd step: static optimization
    minimization of structural volume:
  - 3rd step: dynamic optimization
    seismic response reduction

Optimization of flexible base for reduction of seismic response of buildings

Rocking mechanism

Dissipate seismic energy using rocking of frame and plastic dissipation at column base

+

Compliant bar-joint structure

Isolate structure using a flexible support

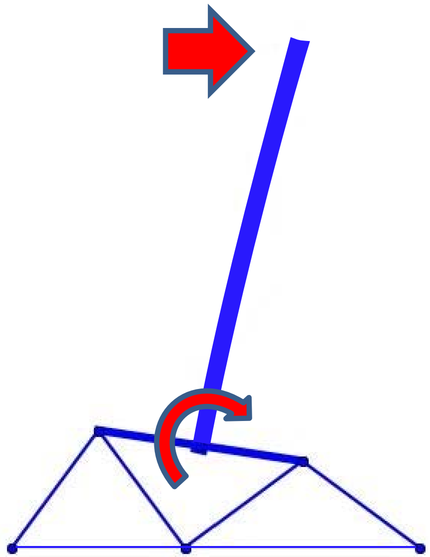
↓

Reduce seismic response combining rocking mechanism and compliant mechanism

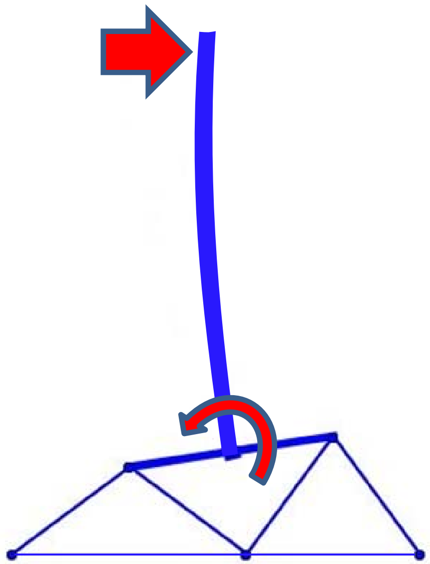
Purpose

Optimize flexible base to control mode shape
and reduce response displacement of building frame

Rotate opposite direction against horizontal input

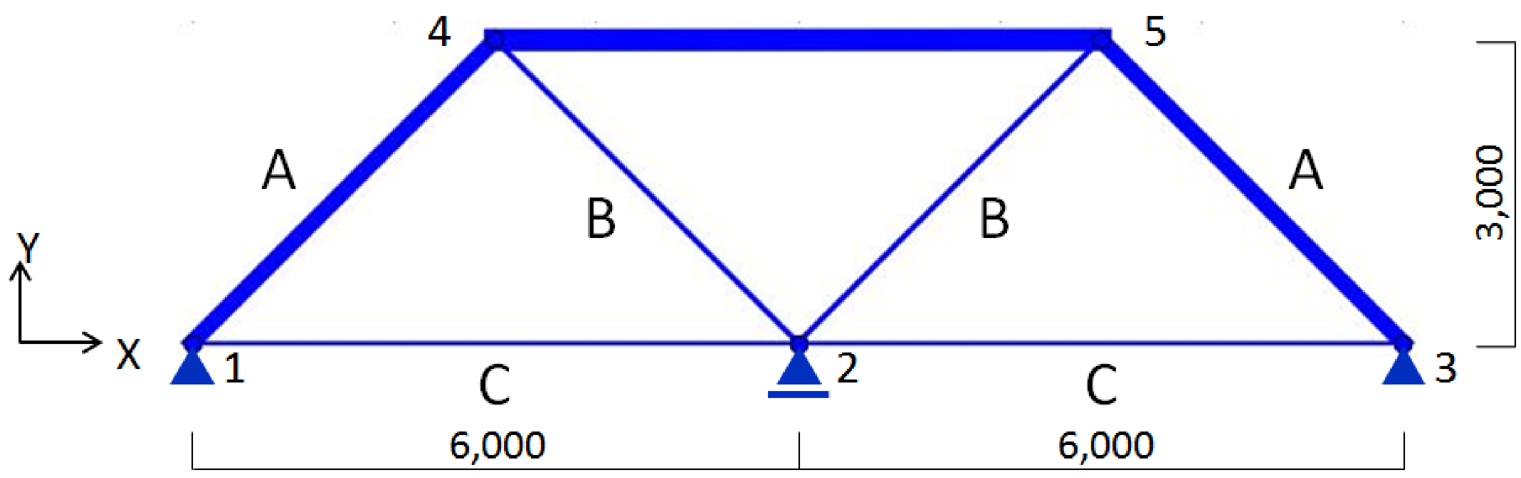


Standard base



Flexible base

Details of flexible base



Truss members (cm^2)

A	(members 1-4,3-5)	200.0
B	(members 2-4,2-5)	50.0
C	(members 1-2,2-3)	1.0

Material: steel

Manufacture member C using a spring

Optimization problem

Design variables: nodal coordinate $\boldsymbol{X}$

cross-sectional area $\boldsymbol{A}$

Objective function : roof displacement $|y|_{\max}$

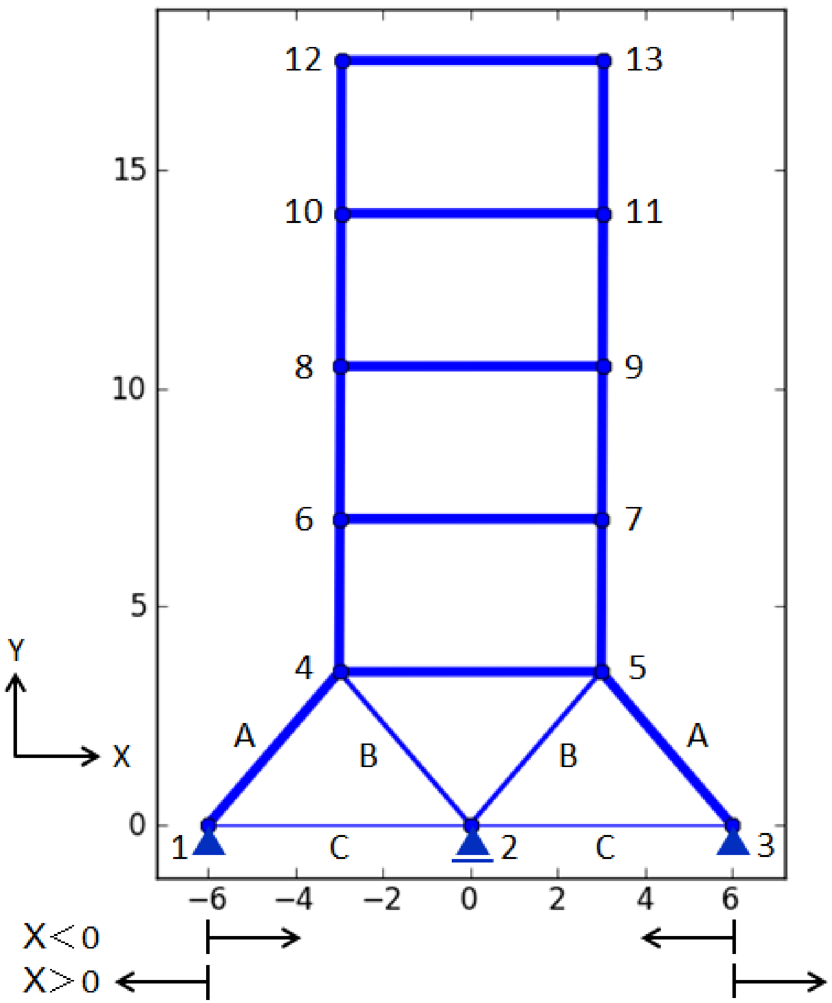
➡ minimize

Response spectrum approach (SRSS rule)

$$|y|_{\max} = \sqrt{\sum_{i=1}^{3} \left|\beta_i \cdot u_i \cdot S_{D_i}\right|^2}$$

Constraint : lowest natural period $T_1 \leq 1.0$

Optimization result



- Damping factor 2%
- Young's modulus：200kN/mm^2
- Story mass: 8000kg

	Beam section (SN400B)
2〜R	H－400×200×8×13
	Column section (BCR295)
1〜4	□－350×350×16

- Base beam (points 4,5) 45000kg

Design variables:

location of pin support (1, 3) X(m)

$$-3.0 \leq X \leq 3.0$$

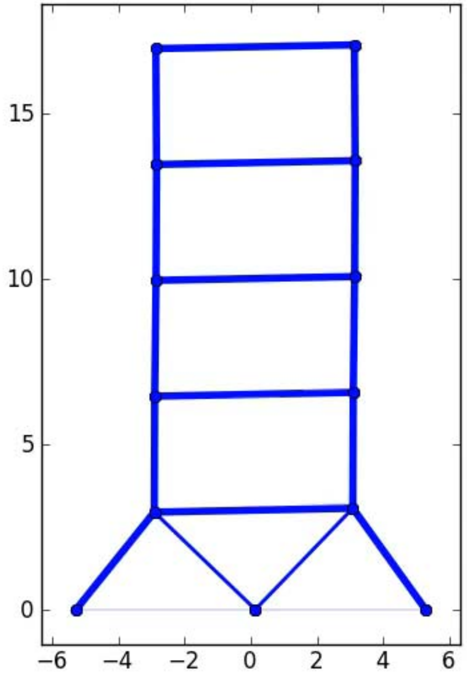
Cross-sectional area of horizontal member (C)   A(cm^2)

$$0.1 \leq A \leq 10$$

Optimization result

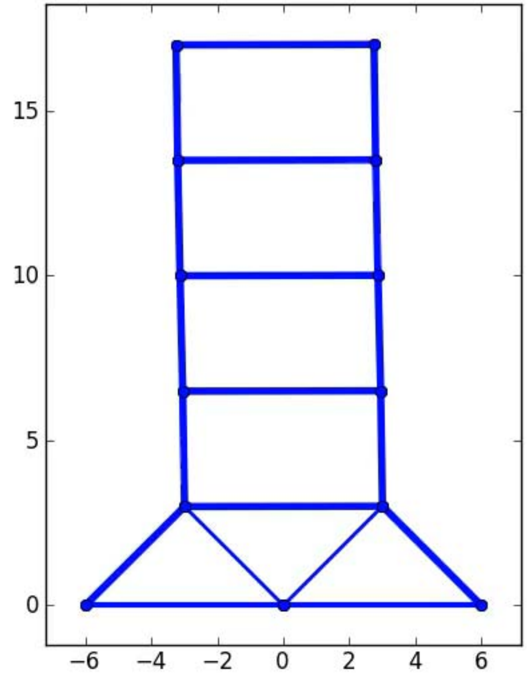
Objective function (m)	X (m)	A (cm^2)
1.078×10^{-2}	-0.719	1.191

Optimal model



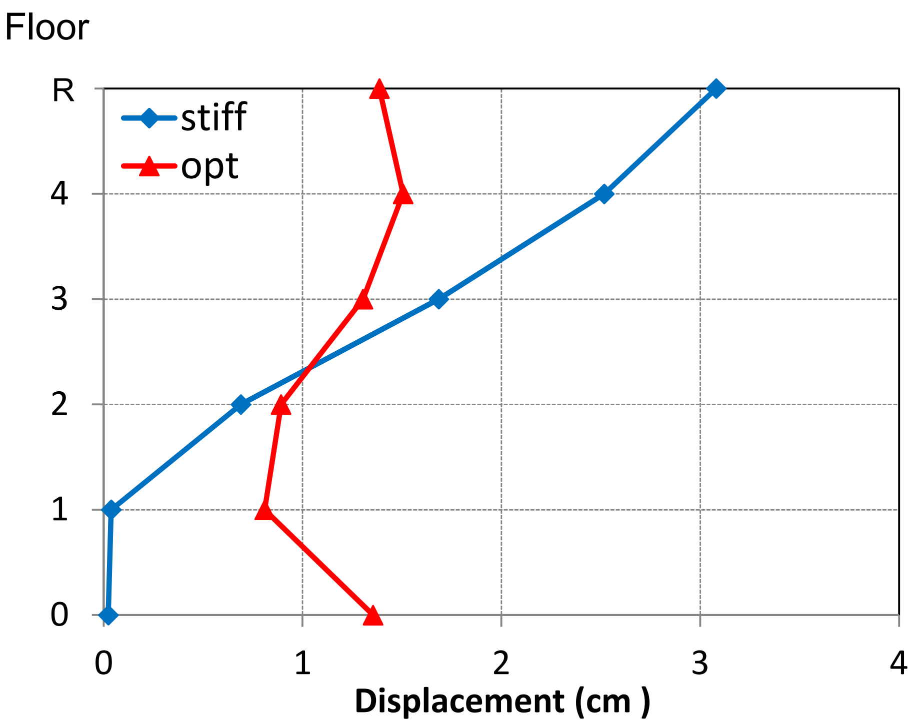
X=−0.72 A=1.19

Stiff model

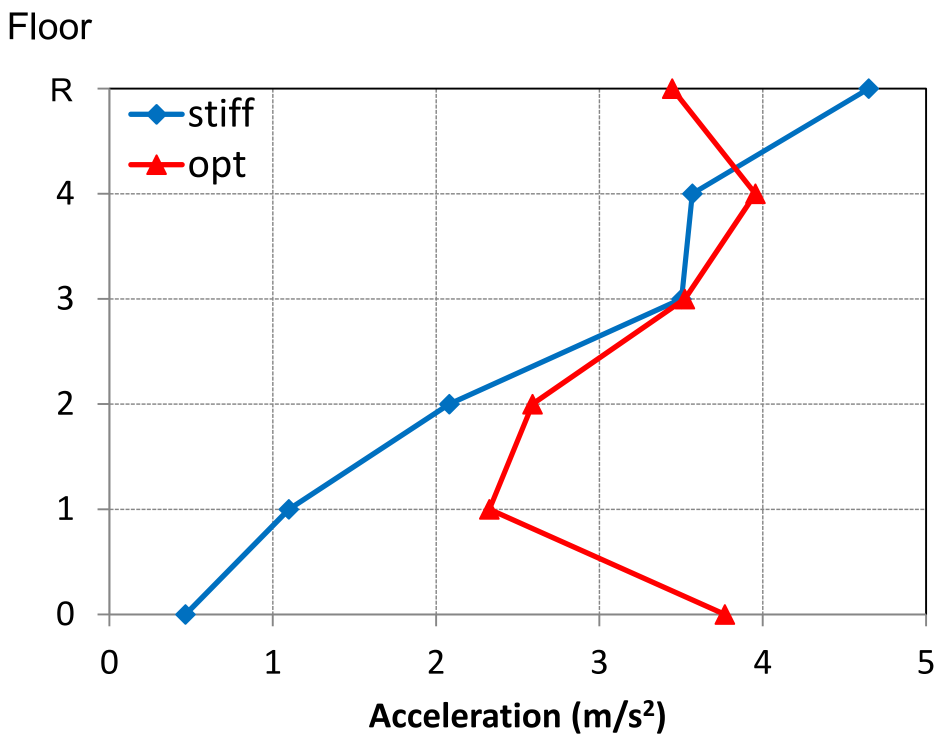


X=0.0 A=100.0

Maximum responses

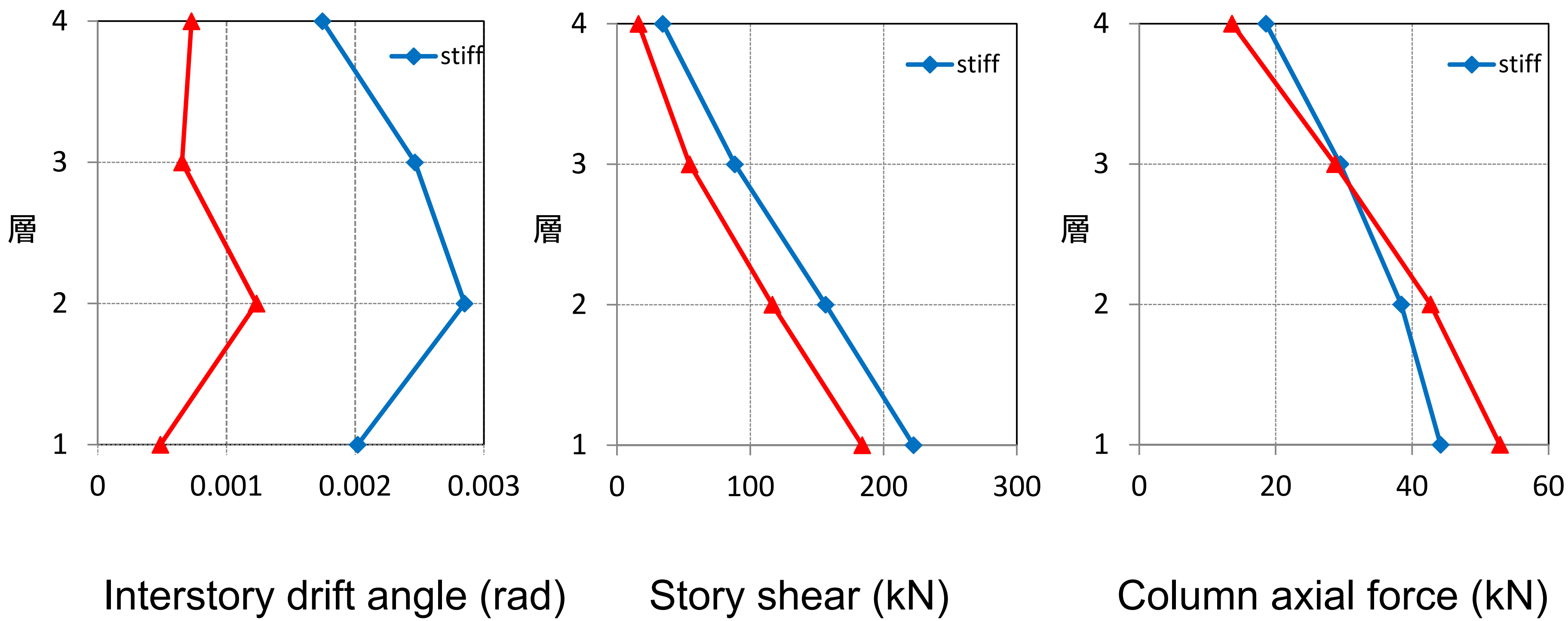


Displacement



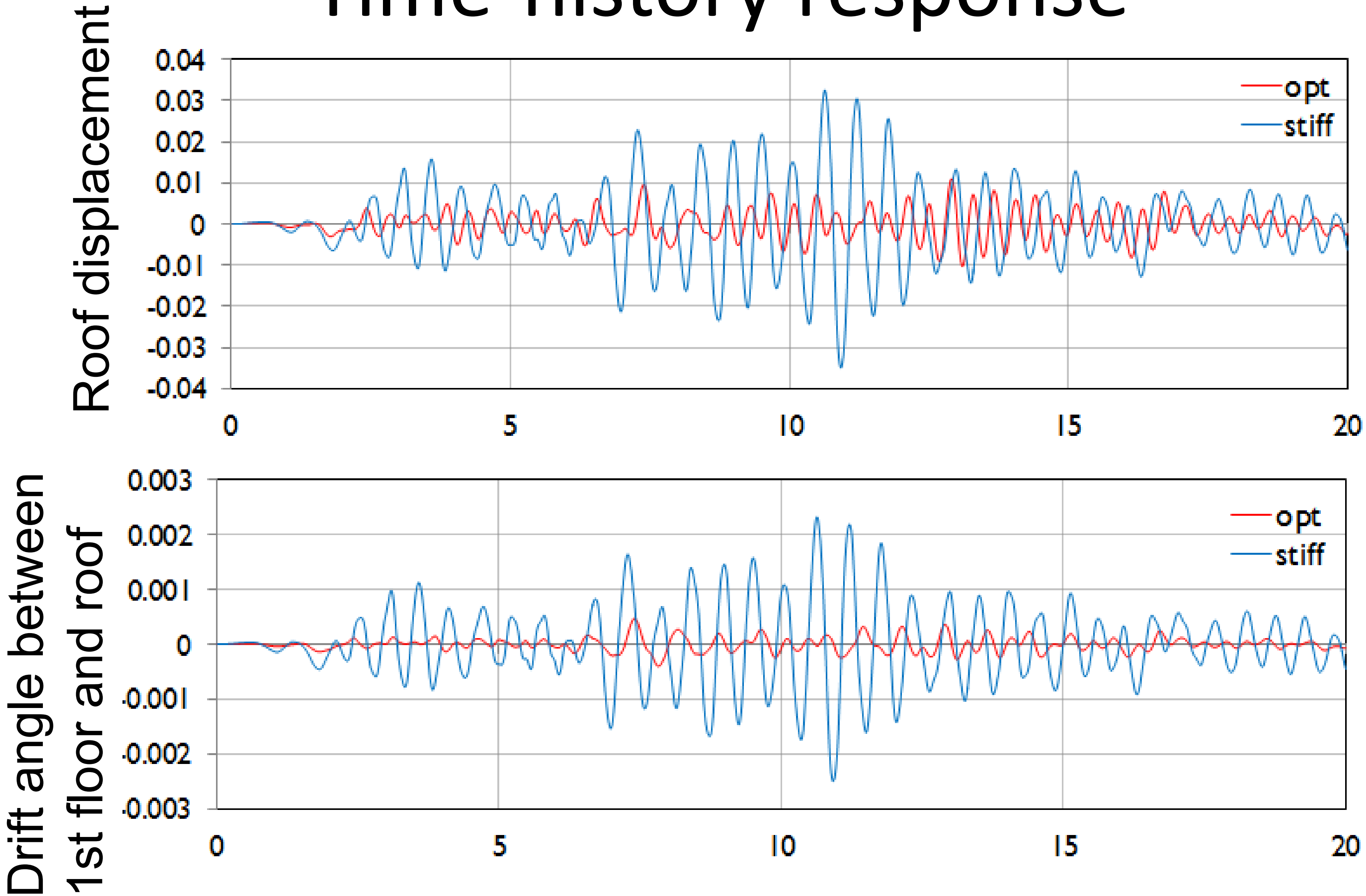
Acceleration

Maximum responses

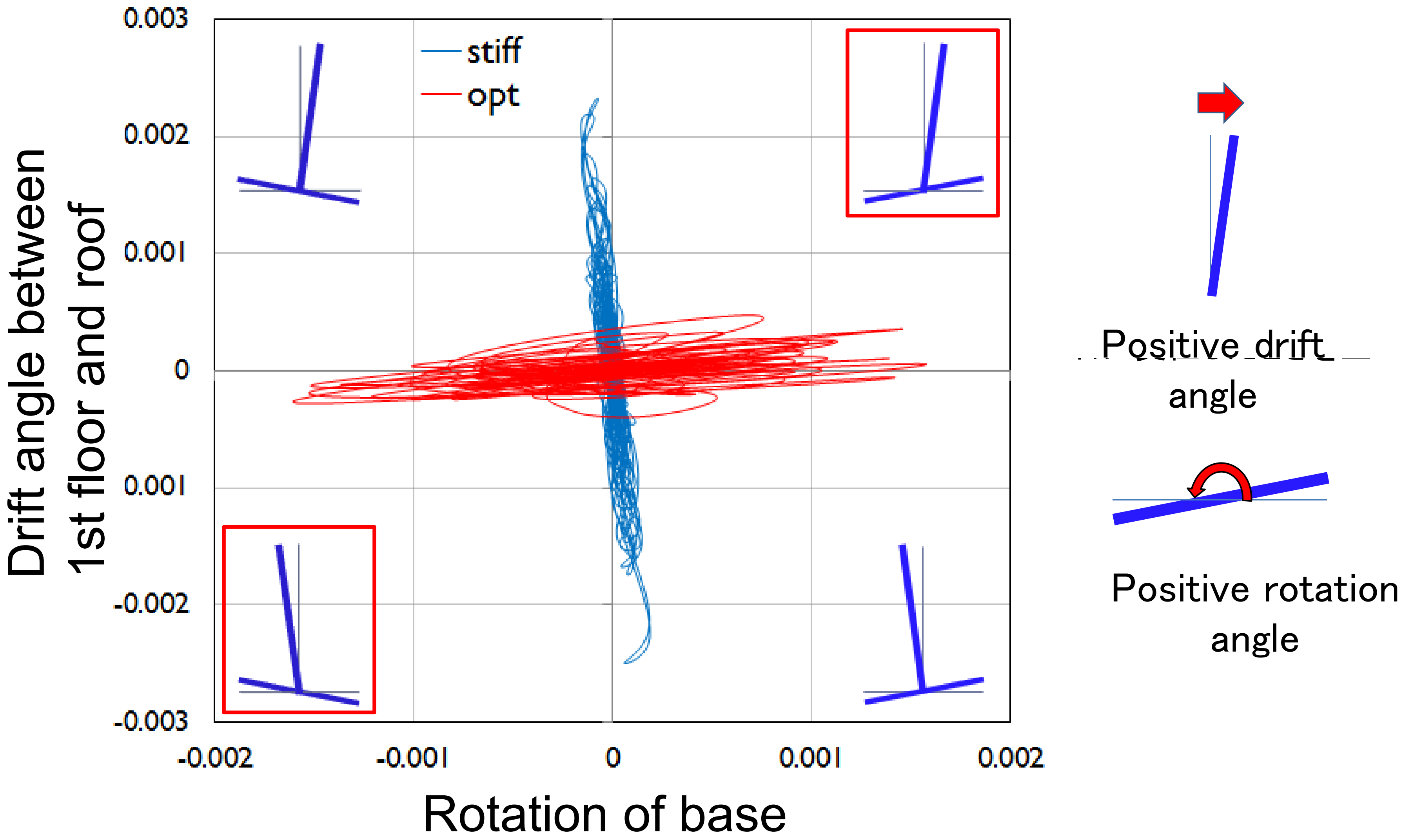


➡ Rocking response is enhanced

Time-history response



Trajectory of drift angle and rotation of base

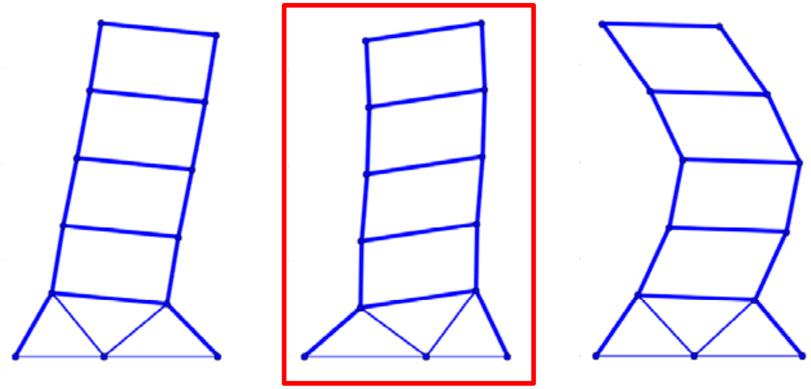


Eigenvalue analysis

Optimal model

	Period	Participation factor	Effective mass ratio
	T(s)	β	X-dir. (%)
1st	0.712	16.53	0.22
2nd	0.379	287.77	67.88
3rd	0.155	13.75	0.15

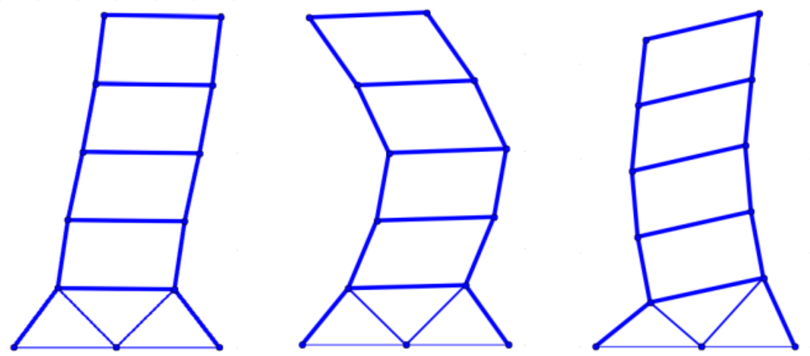
Eigenmode



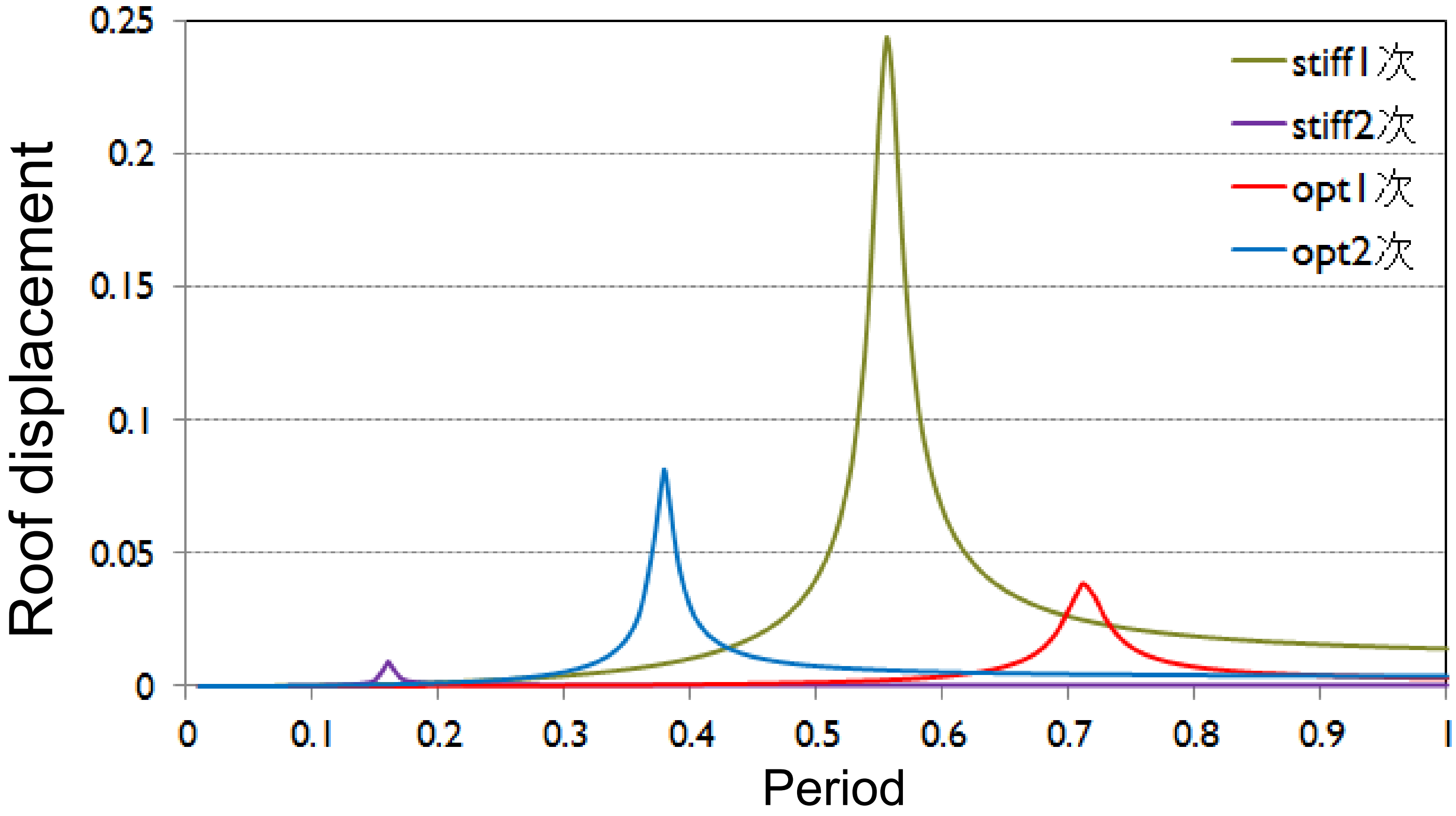
Stiff model

	Period	Participati on factor	Effective mass ratio
	T(s)	β	X-dir. (%)
1st	0.556	157.30	20.28
2nd	0.160	90.66	6.74
3rd	0.107	186.00	28.36

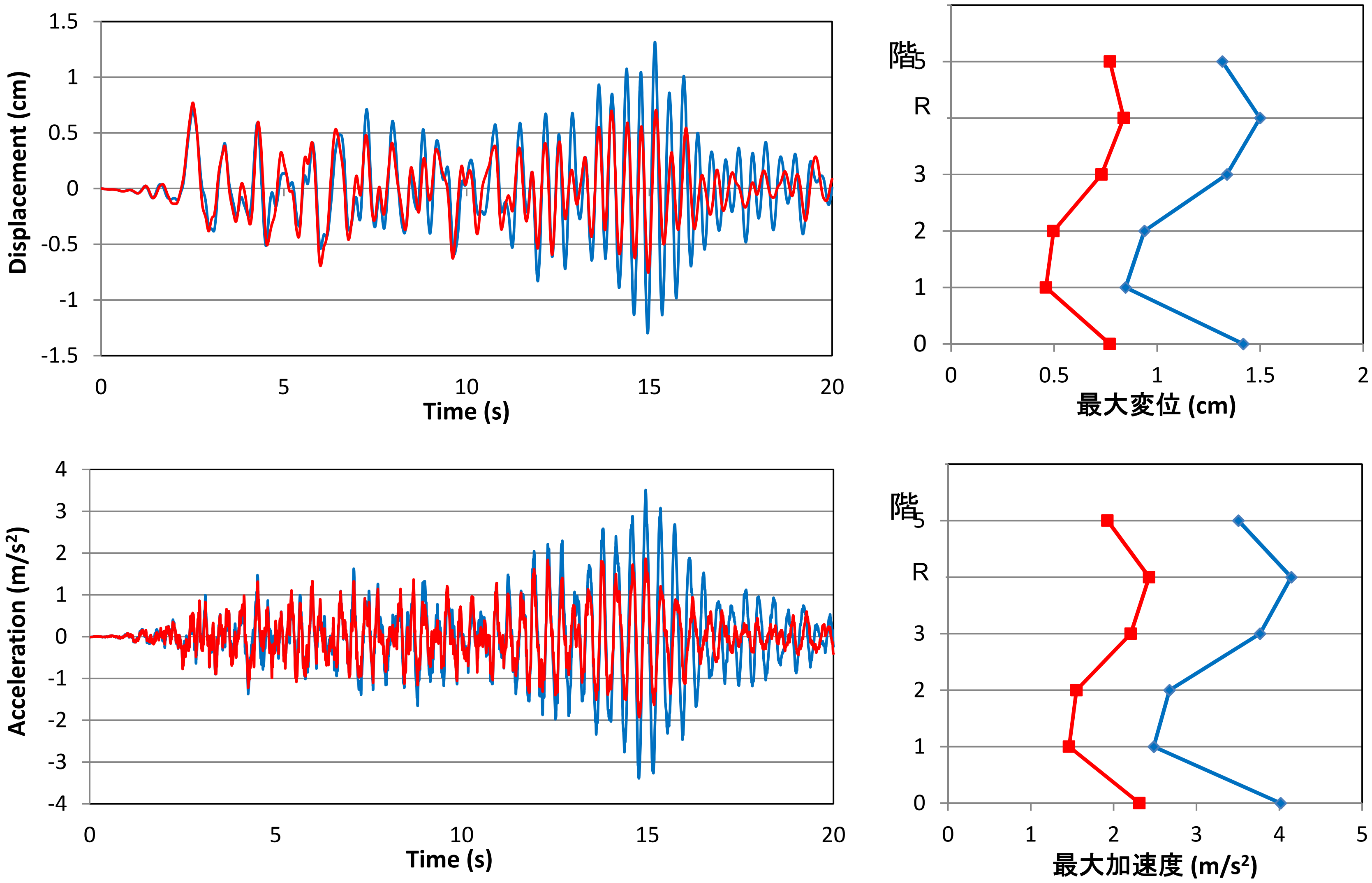
Eigenmode



Frequency response



Base with viscous damper



Red: with damper, Blue: without damper

Conclusions

- Flexibility of supports can be effectively utilized for reduction of seismic responses of structures.
- Two-stage procedure:
  - 1st stage: static optimization
    maximization of vertical displacement:
    minimization of structural volume:
- 2nd stage: dynamic optimization
  seismic response reduction
  variable: cross-sectional area