Optimization of tensegrity lattice with truncated octahedral units

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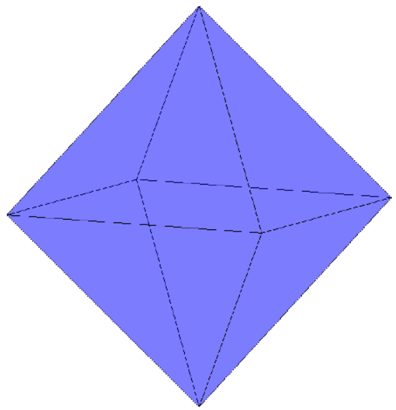
Background

- Stability of tensegrity structures
  - Global buckling: Super-stability
  - Local buckling: Euler buckling of bars
- Design of flexible support of vehicle or structure
  - Utilize buckling of bars (struts)
  - Reduce maximum reaction under impact force
  - Reduce tangent stiffness at large deformation
- Properties of tensegrity lattice
  - Stiffness/flexibility
  - Wave propagation

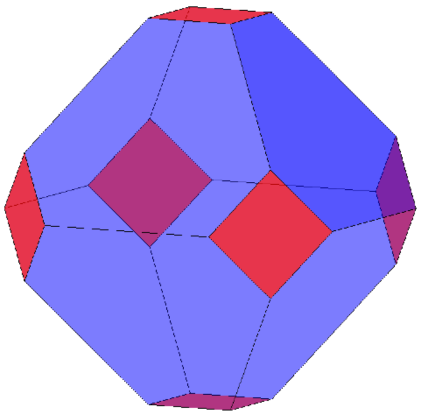
Objective of Study

- Present optimization method for a tensegrity lattice
  - Eight truncated octahedral units with threefold symmetry
- Maximize strain energy under specified forced vertical displacement
- Obtain stiff structure with degrading tangent stiffness for vertical deformation
- Add horizontal bars to stabilize the structure for horizontal deformation.

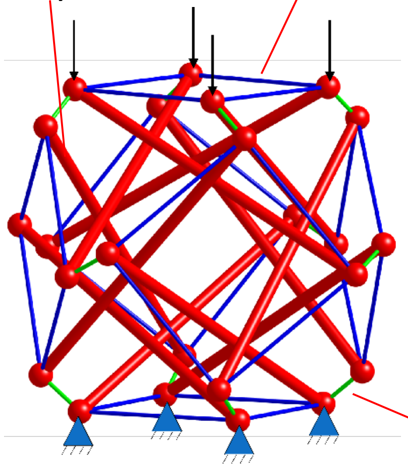
Truncated octahedral tensegrity



Regular octahedron



Truncated octahedron

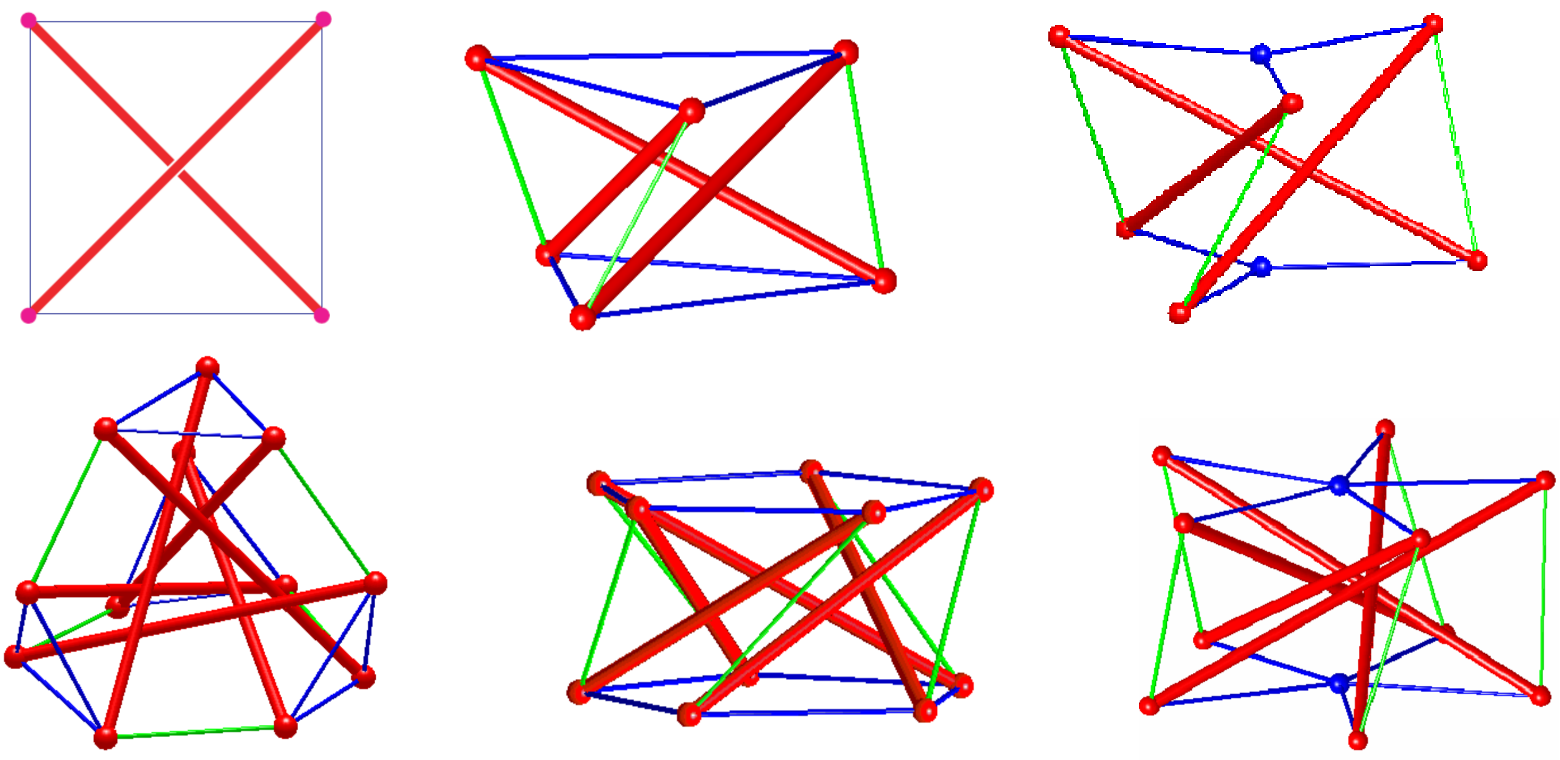


Truncated octahedral tensegrity

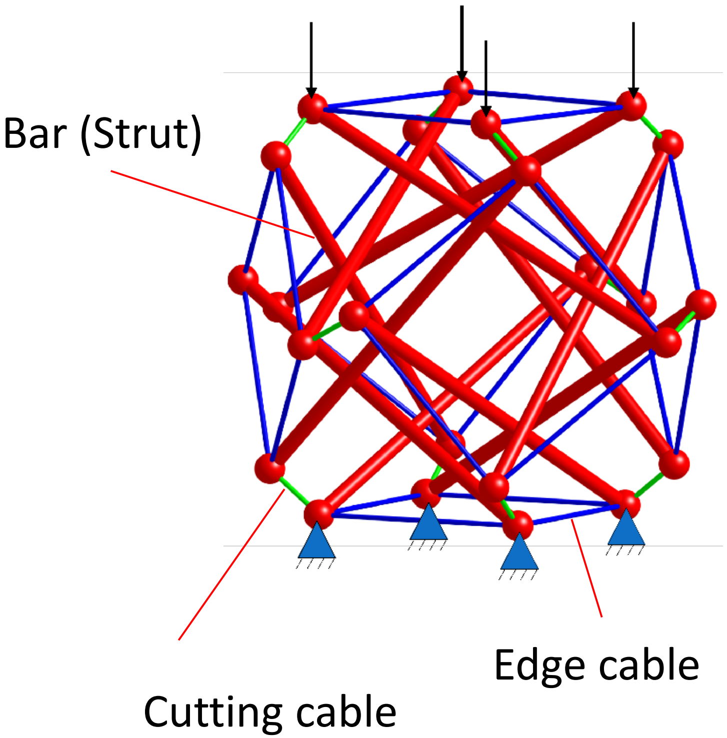
Super-stability of tensegrity

Stable for any level of prestresses
Return back from large deformation

→ Super-stable structure:
Only ratios of forces should be considered for shape design



Force density vector



Symmetry condition

Force densities
(Force / member length)

$$q_{\mathrm{c}} = \frac{b^2 - 5a^2 + \sqrt{a^4 + 14a^2b^2 + b^4}}{4a}$$

$$a = \frac{q_{\mathrm{e}} + q_{\mathrm{b}}}{2}, \ \ b = \frac{q_{\mathrm{e}} - q_{\mathrm{b}}}{2}$$

$q_{\mathrm{e}} = 1:$ normalization

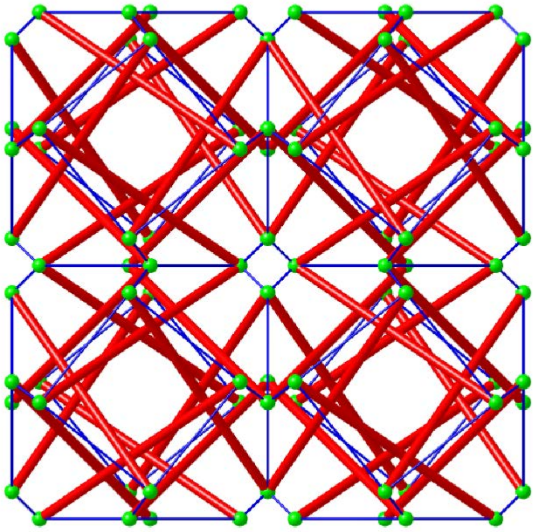
$q_{\mathrm{b}}:$ parameter $(-1 < q_{\mathrm{b}} < 0)$

Super-stability condition

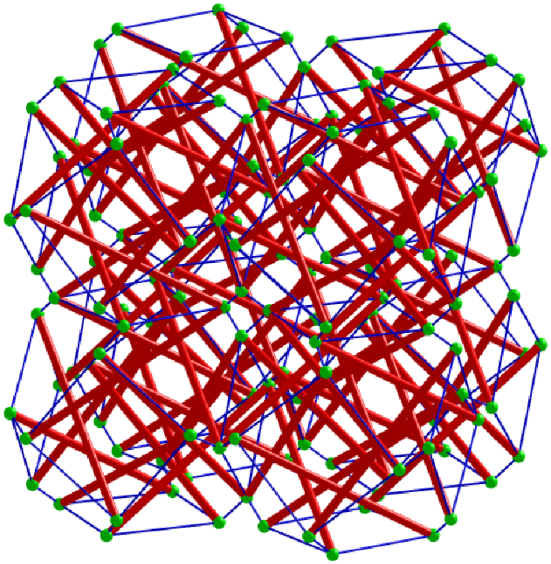
$\mathbf{q} = (q_{\mathrm{e}}, q_{\mathrm{c}}, q_{\mathrm{b}}):$ normalization

$\mathbf{q} \Rightarrow \beta\mathbf{q}:$ scaling

Eight-unit tensegrity lattice



Plan/side view

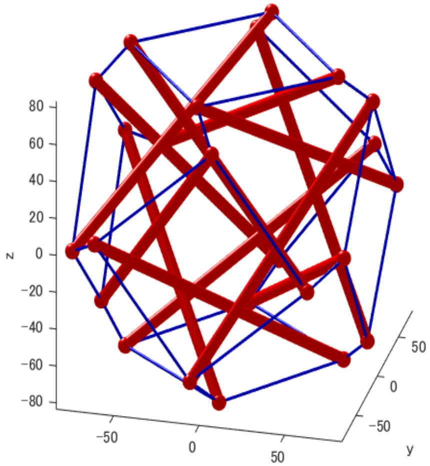


Diagonal view

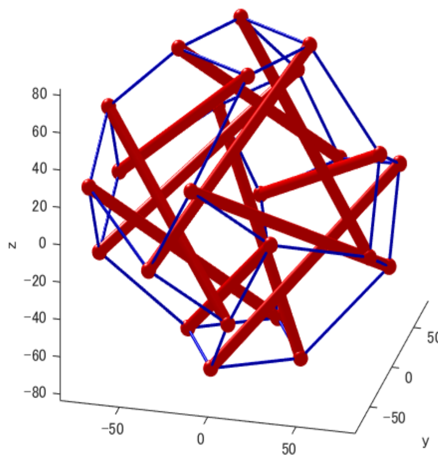
- Connect mirror images of units
- Remove one of the duplicate edge cables at the connection
- 96 nodes, 336 members

Equilibrium shape

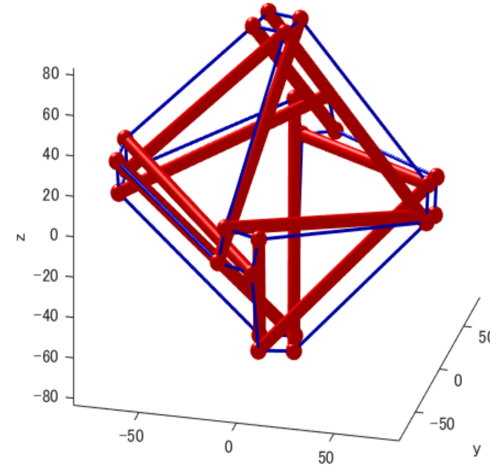
- Equilibrium shape depends on q_b only (q_e = 1).



q_b = –0.2



q_b = –0.5



q_b = –0.8

Cutting cables become shorter when q_b is reduced.

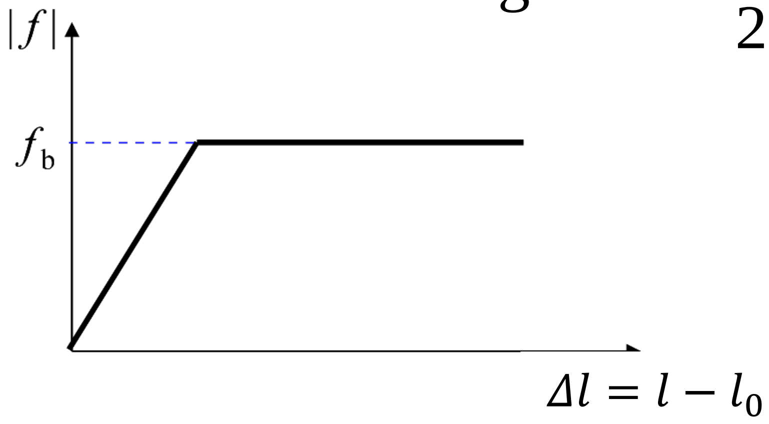
Material property

- Steel material for cables and bars
- Bilinear elastic model for buckling of bars

Strain energy:

before buckling $$S = \frac{EA}{2l_0}(l - l_0)^2$$

after buckling $$S = \frac{f_b^{\,2} l_0}{2EA} + f_b \left| l - l_0 - \frac{f_b l_0}{EA} \right|$$



E : Young's modulus

A : Cross-sectional area

l_0 : Initial length

l : Length after deformation

f_b : Buckling force

Optimization problem

- Maximize total strain energy S^* at specified vertical displacement
- Design variables: Cross-sectional areas of bars A_b and cables A_c; prestress level β
- Constraints: Material volume

$$\text{Maximize} \quad S^*(A_b, A_c, \beta)$$
$$\text{sunject to} \quad A_b L_b + A_c L_c = V_0$$
$$A_b^L \le A_b \le A_b^U$$
$$A_c^L \le A_c \le A_c^U$$
$$\beta^L \le \beta \le \beta^U$$

A_b, A_c : Cross-sectional areas of bars and cables

β : Prestress level

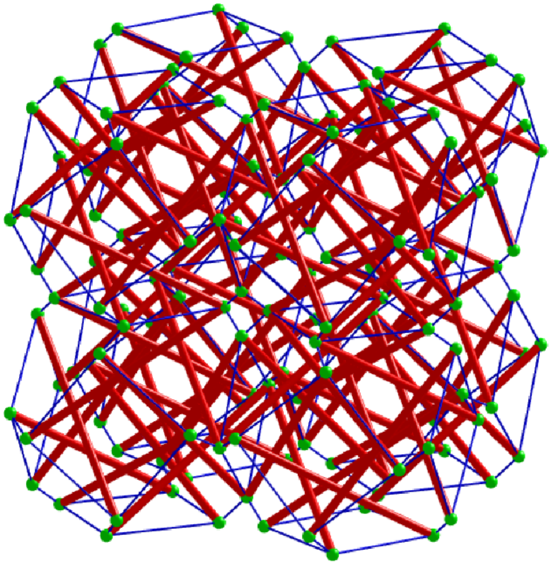
L_b, L_c : Total lengths of bars and cables

V_0 : Specified material volume

$(\ )^U$, $(\ )^L$: Upper and lower bounds

Optimization results

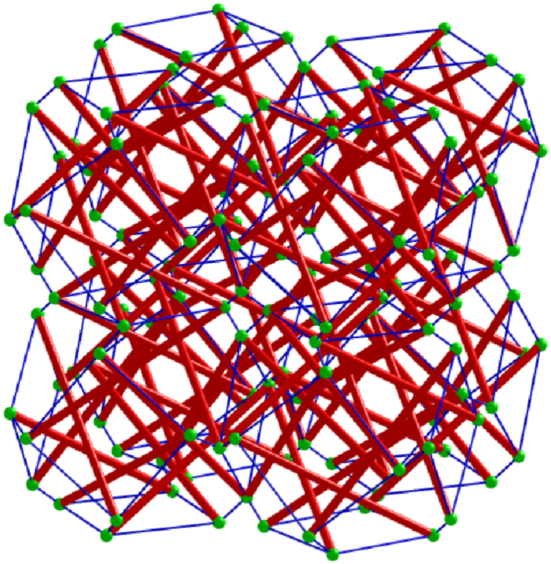
- Young's modulus: $E = 2.05\times10^5$ N/mm^2
- Volume of surrounding rectangular parallelepiped: $8\times0.004 = 0.032$ m^3 (unit size = 0.2 x 0.2 x 0.1)
- Radius of gyration of rectangular bar section: 1.2 mm (slenderness ratio is about 140) Width of rectangular bar is proportional to cross-sectional area
- Forced vertical displacement: 80 mm (Height is about 200 mm)
- V_0 = material volume of initial solution



q_b

Optimization results

- Boundary condition:
  Bottom and top planes can expand horizontally constraining rigid-body displacements
- Displacement increment method with Newton iteration at each step
- Small axial stiffness $EA_b/100$ after buckling to stabilize computation
- Optimize for q_b between -0.3 and -0.7
- Check yielding and slackening of cable after optimization



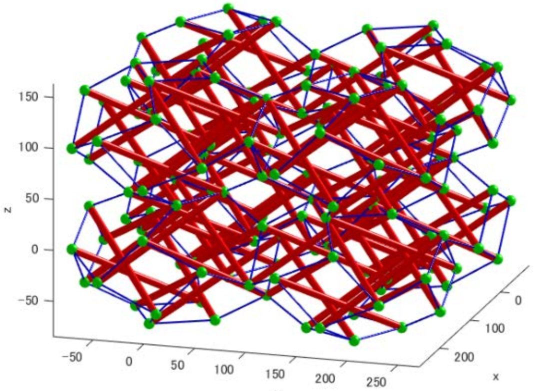
q_b

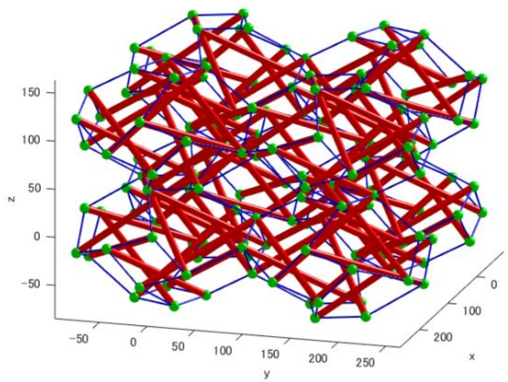
Optimal values

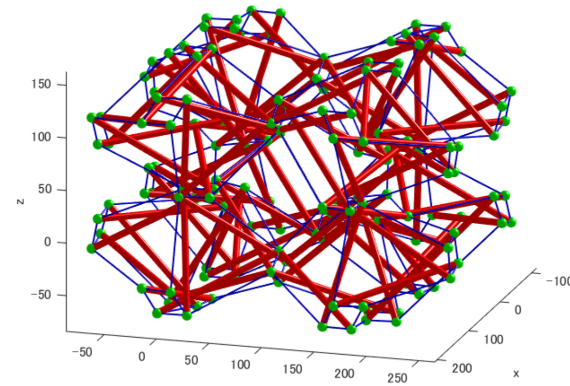
q_b	A_b (mm^2)	A_c (mm^2)	β	S^* (Nm)
–0.3	0.9179	0.1000	1.6536	9601
–0.4	0.9806	0.1000	1.7617	10130
–0.5	1.0456	0.1000	1.9449	14185
–0.6	1.1106	0.1000	1.9451	23137
–0.7	1.1742	0.1000	1.8395	37734

- Cable cross-sectional area A_c is equal to lower-bound for all cases
- Bar cross-sectional area A_b and strain energy S^* increase as q_b is decreased
- Stress level β mostly increases as b_q is decreased

Optimal shape after deformation





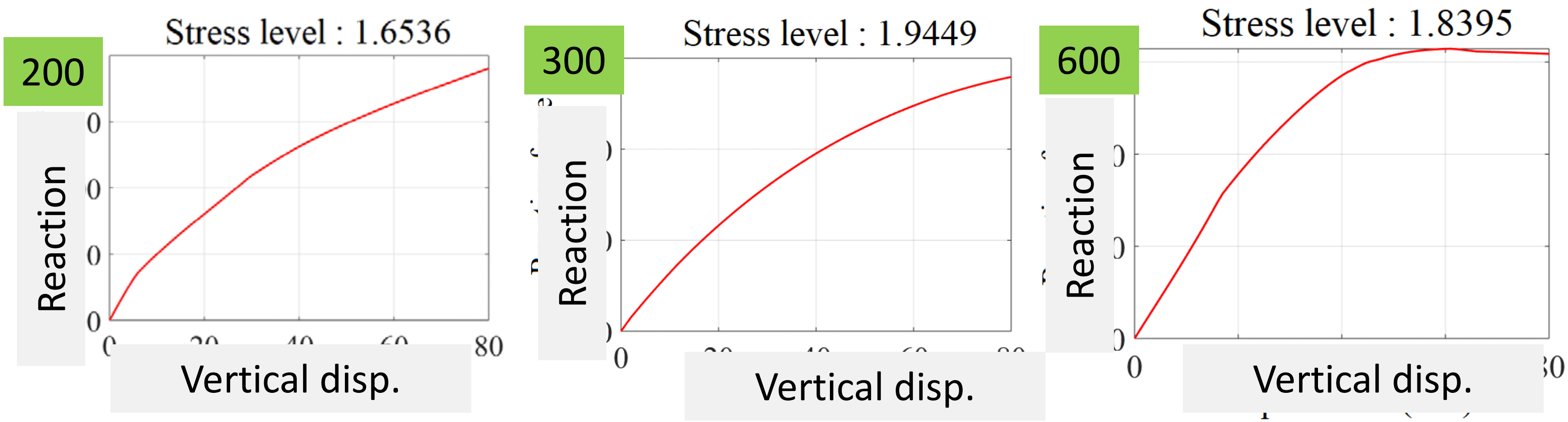


$q_b = -0.2$

$q_b = -0.5$

$q_b = -0.7$

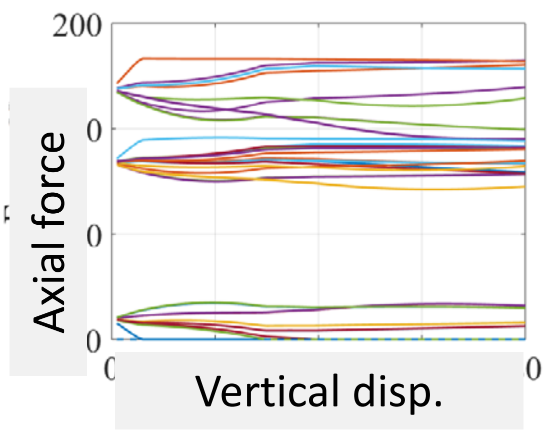
Reaction-displacement relation

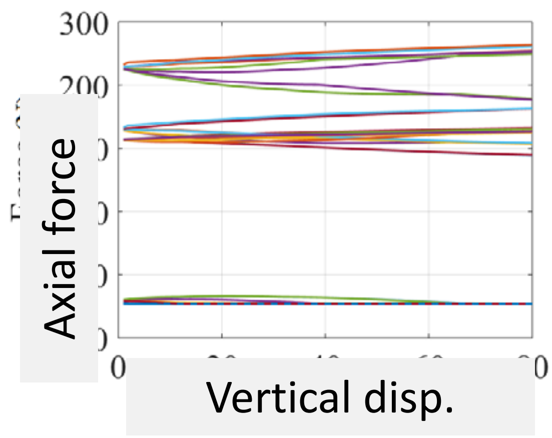


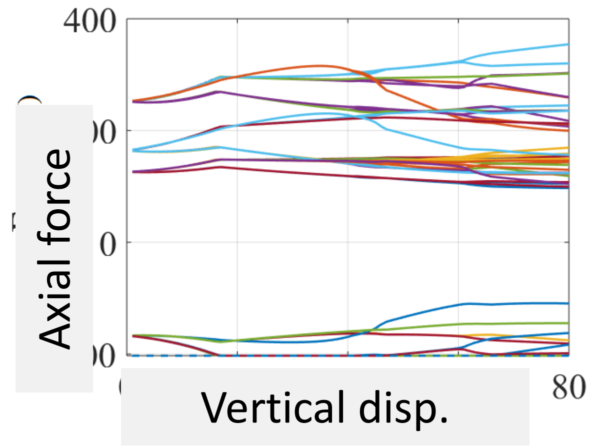
$q_b = -0.3$ $q_b = -0.5$ $q_b = -0.7$

- Vertical stiffness decreases as the downward displacement is increased
- Maximum reaction force increases as q_b is decreased
- Structure has a limit point instability for $q_b = -0.7$

Displacement-member force relation





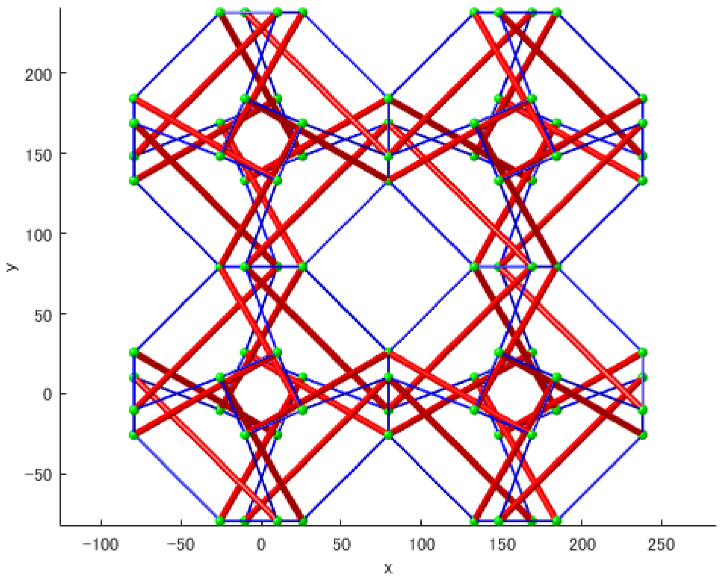


$q_b = -0.3$ $q_b = -0.5$ $q_b = -0.7$

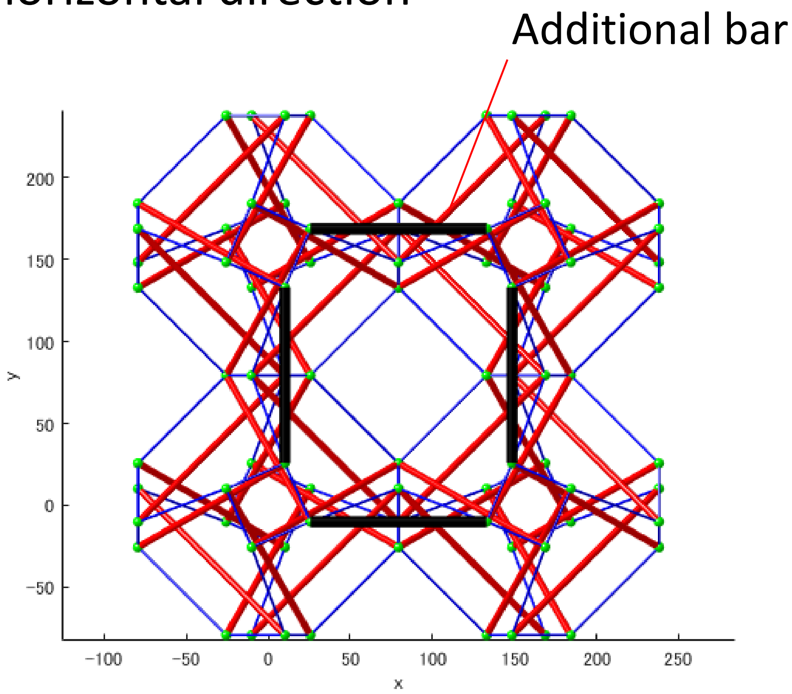
- Axial force has positive value in tensile state
- Buckling of bars
  -> Forces of linear elastic cable have non-smooth distributions
- Number of buckled members increases as vertical displacement is increased

Additional bars

- Vertical stiffness has been reduced successfully
- Horizontal stiffness may be lost due to shear deformation of upper and lower units
- Add bars to prevent instability in horizontal direction



Without additional bars



With additional bars

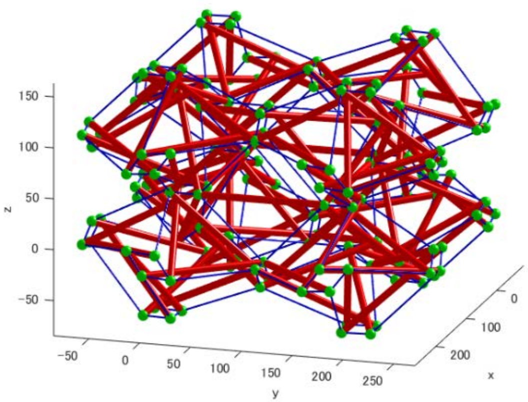
Optimization results with additional bars

Layers for adding bars	q_b	A_s (mm^2)	A_c (mm^2)	β	S^* (Nm)
Top	–0.7	1.1294	0.1000	1.8137	38338
	–0.8	1.1825	0.1000	1.8103	38549
Top and middle	–0.7	1.0879	0.1000	1.5261	44891
	–0.8	1.1322	0.1000	1.5060	72591
Top and bottom	–0.7	1.0879	0.1000	1.7596	37265
	–0.8	1.1322	0.1000	1.6076	63305
Top, middle, and bottom	–0.7	1.0493	0.1000	1.3264	62863
Middle	–0.7	1.1294	0.1000	1.8230	38653

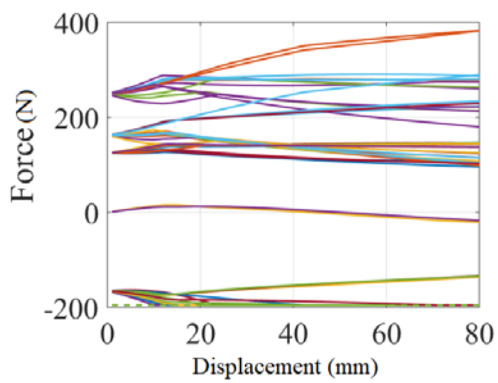
Almost similar property as the case without additional bars

Property under vertical loading of optimal solution

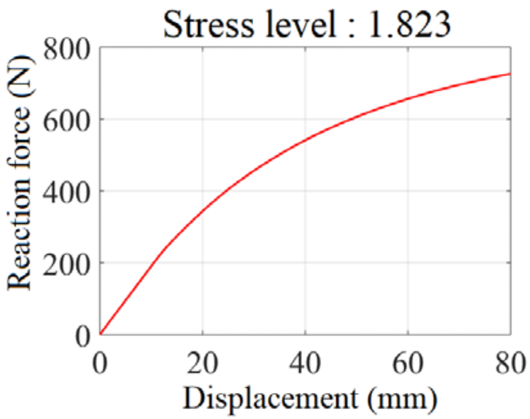
q_b = –0.7, bars on top and middle layers



Deformed shape



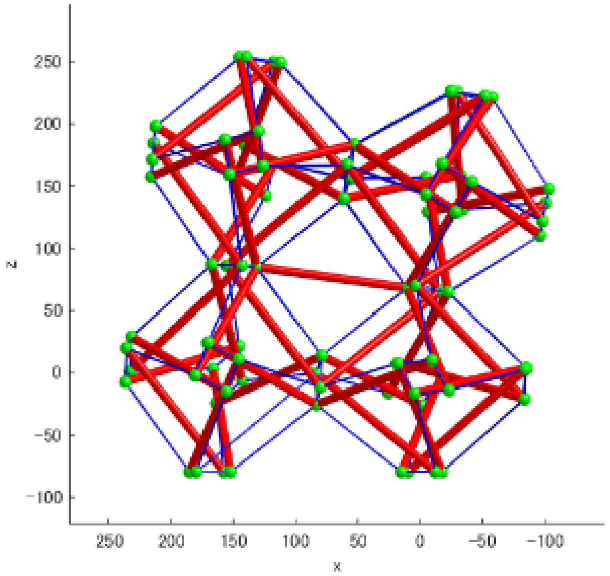
Axial force

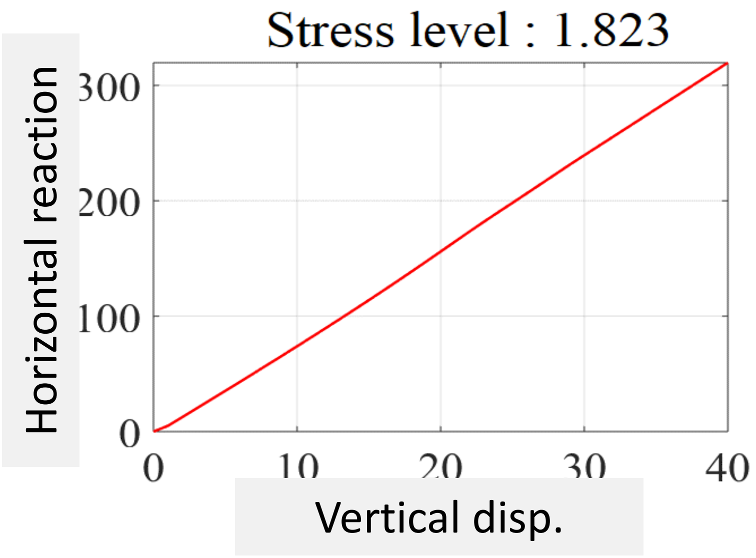


Reaction

Property under horizontal loading of optimal solution

$q_b = -0.7$, bars on top and middle layers





Deformed shape under horizontal displacement

Positive horizontal stiffness

Conclusions

- Optimization method for tensegrity lattices composed of eight truncated octahedral units.
- Maximize stored strain energy under specified forced vertical displacement and constraints on structural volume.
- A flexible structure with degrading vertical stiffness is obtained as a result of optimization.
- Although structures generally retain compression stiffness even after large deformations, shear stiffness may be lost.
  -> Add four bars in some of the layers of the lattice.
- Tensegrity lattice with flexibility in vertical direction and adequate shear stiffness for isolation for vertical motion.