Approximate method for cutting-pattern optimization of membrane structures

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Form-finding and cutting-pattern design

Form finding

- Specify boundary shape

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- Assume uniform stress distribution
- Obtain equilibrium shape that may not be generated from plane sheets

Cutting-pattern design

- Divide surface into several parts using geodesic lines

↓

- Remove initial stress and obtain cutting patterns

Inevitable error to generate curved surface from plane sheets

Existing optimization approach

1. Assume shape of plane cutting sheets.
2. Divide sheets into triangular finite elements.
3. Carry out forced deformation analysis to connect sheets to boundary.
4. Correct shape of cutting sheets to achieve uniform stress distribution and reduce error from target shape, and go to step 2.

Finite element analysis should be carried out many times.

Purpose of study

1. Propose approximate method for generating cutting patterns from curved equilibrium shape.

   ⬇

   Iterative formula for updating target ideal stress.
   Reduce number of forced deformation analysis for approximate optimization of cutting patterns.

2. Analysis of equilibrium shape including pneumatic membrane based on energy minimization.

3. Modeling of ETFE as isotropic bilinear elastoplastic material.

Algorithm

1. Assign initial curved surface and its triangulation.
2. Assign ideal stress.
3. Project surface on a plane.
4. Compute edge lengths after reducing stress.
5. Optimize cutting patter by minimization of edge-length error.

Cutting pattern generation

6. Find equilibrium shape by energy minimization.
7. Compute stress at equilibrium and go to Step 2.

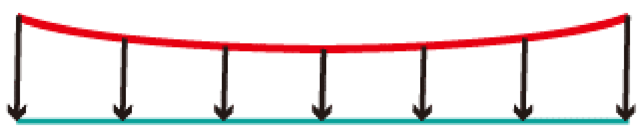
Equilibrium shape analysis

Projection to plane

Prevent distortion of element shape due to projection

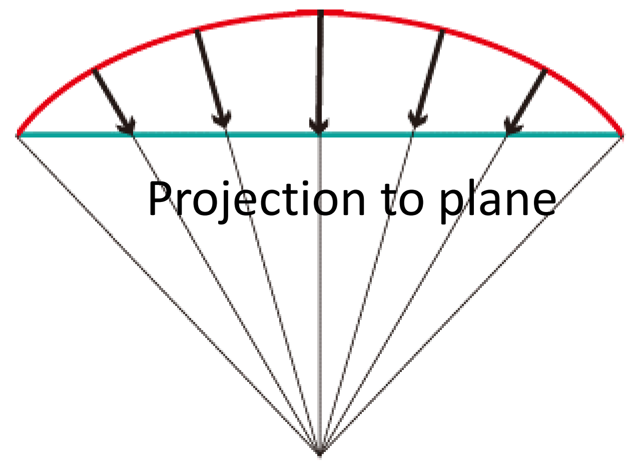
Different method depending on curvature of surface

(a) frame-supported membrane



Projection to plane

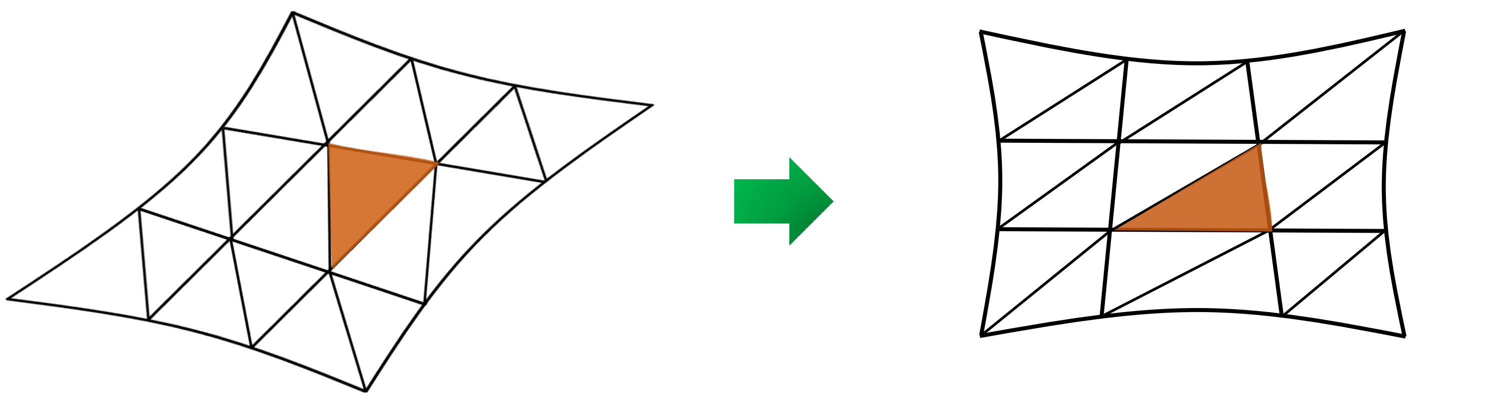
(b) Pneumatic structure



Center of curvature

Approximate generation of cutting pattern

1. L_{k1}, L_{k2}, L_{k3} : Edge lengths
   Specify stresses σ_{kx}, σ_{ky} in principal directions
2. Compute length $L_{k1}^{0}, L_{k2}^{0}, L_{k3}^{0}$ after removing stress
3. Triangulate cutting sheets with same topology as surface



Triangulation of surface

Triangulation of cutting sheet

Approximate generation of cutting pattern

3. Find nodal locations on cutting sheets to minimize errors between L_{k1}, L_{k2}, L_{k3} and $L^0_{k1}, L^0_{k2}, L^0_{k3}$

Objective function

$$\sum_k \left\{ \frac{(L_{k1} - L^0_{k1})^2}{L^0_{k1}} + \frac{(L_{k2} - L^0_{k2})^2}{L^0_{k2}} + \frac{(L_{k3} - L^0_{k3})^2}{L^0_{k3}} \right\}$$

Modification of target stress

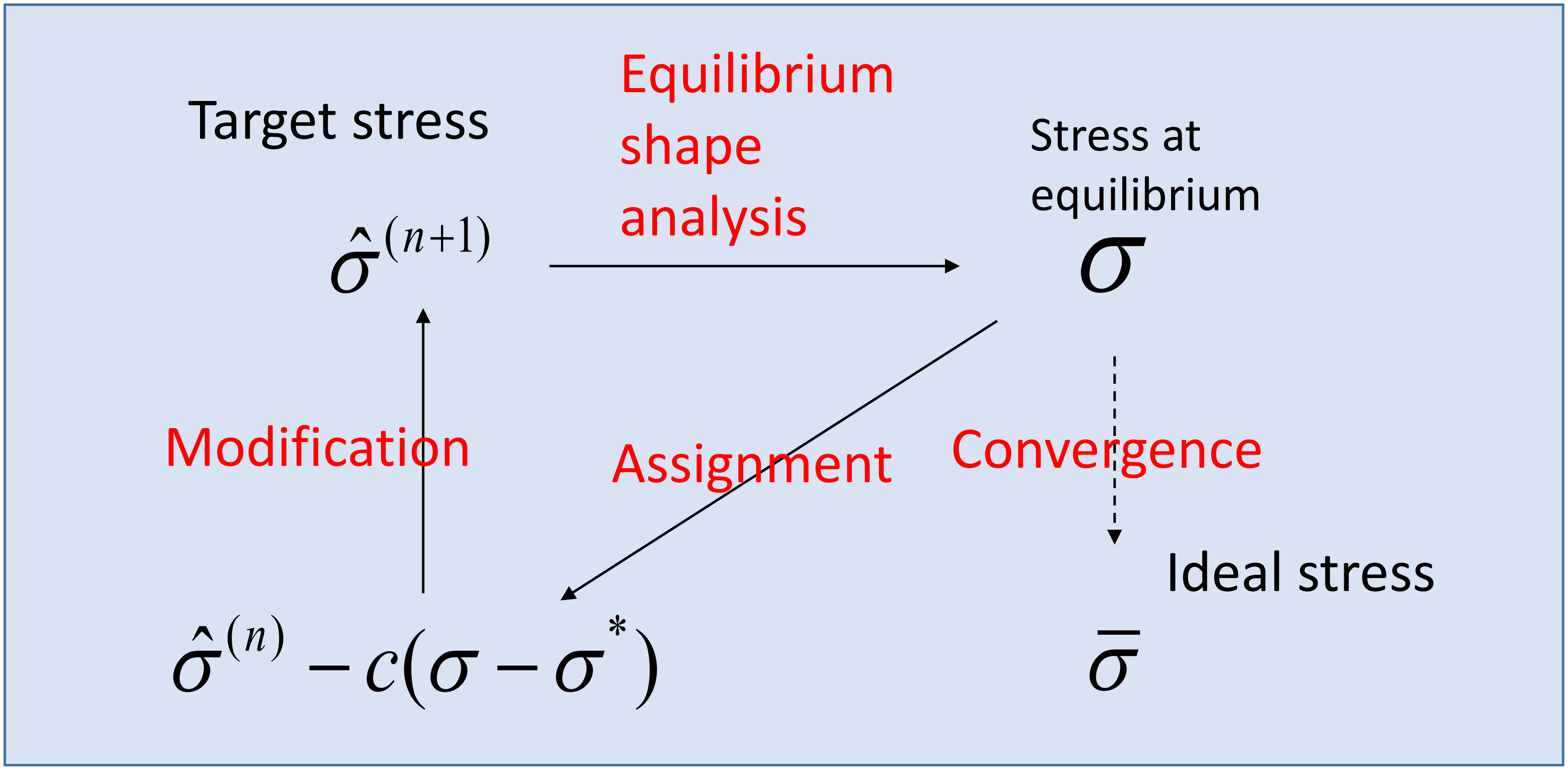
Correct the target stress iteratively to obtain uniform distribution of stresses.

$$\hat{\sigma}_i^{(n+1)} = \hat{\sigma}_i^{(n)} - c\left(\sigma_i^{(n)} - \bar{\sigma}\right)$$

- $\bar{\sigma}$: Ideal stress
- $\hat{\sigma}_i^{(n)}$: Target stress at *n*th step
- $\hat{\sigma}_i^{(n+1)}$: Target stress at (n+1)th step
- $\sigma_i^{(n)}$: Stress at equilibrium at *n*th step
- c : Convergence parameter

Modification of target stress

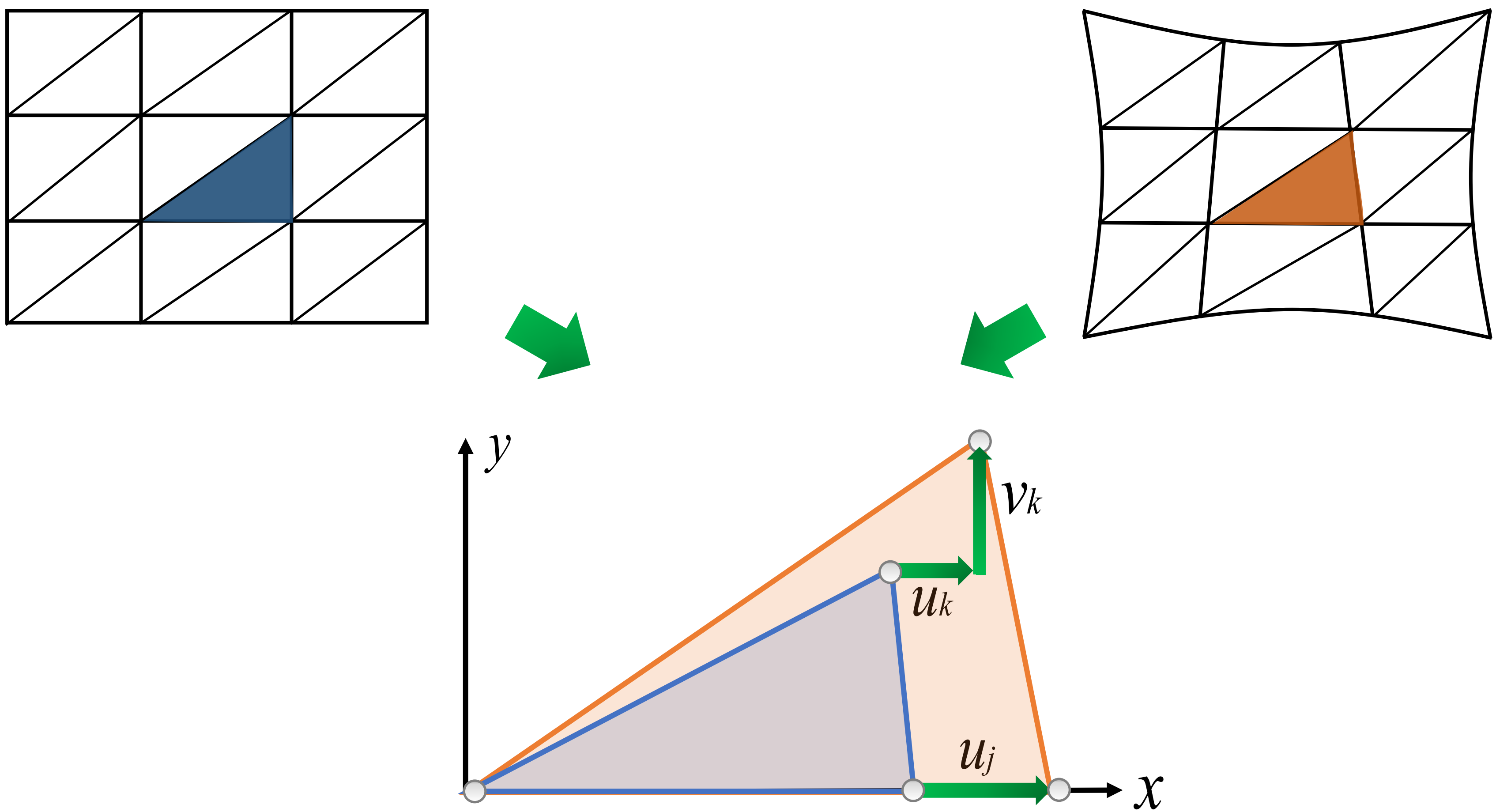
$$\hat{\sigma}_i^{(n+1)} = \hat{\sigma}_i^{(n)} - c\left(\sigma_i^{(n)} - \sigma^*\right)$$



Energy minimization for equilibrium analysis

Undeformed shape on plane

Deformed shape on surface



Energy minimization for equilibrium analysis

Strain-disp. relation

$$\boldsymbol{\varepsilon} = \mathbf{C}^0 \mathbf{u}$$

$$\mathbf{C}^0 = \frac{1}{2A^0} \begin{pmatrix} y_k^0 & 0 & 0 \\ 0 & 0 & x_j^0 \\ -x_k^0 & x_j^0 & 0 \end{pmatrix}$$

- $\mathbf{u} = \left(u_j \quad u_k \quad v_k\right)^{\mathbf{T}}$: Disp. vector
- $\boldsymbol{\varepsilon} = \left(\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}\right)^{\mathbf{T}}$: Local strains
- A^0 : Initial area of element

Stress-strain relation (orthotropic material)

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$$

$$\kappa = G/E_y \qquad \gamma = E_x/E_y$$

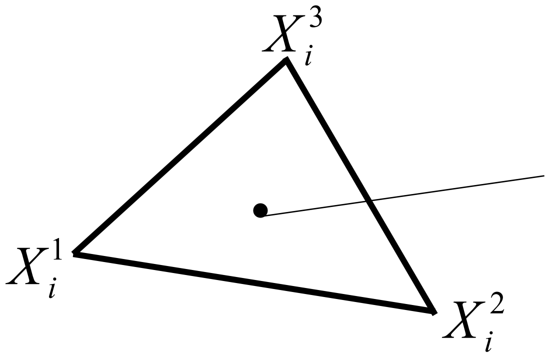
$$\mathbf{D} = \frac{E_y}{1-\gamma\nu^2} \begin{pmatrix} \gamma & \gamma\nu & 0 \\ \gamma\nu & 1 & 0 \\ 0 & 0 & \kappa\left(1-\gamma\nu^2\right) \end{pmatrix}$$

- E : Young's modulus
- G : Shear modulus
- ν : Poisson's ratio

Objective function to be minimized

Strain energy of membrane $S(\mathbf{X}) = \frac{1}{2} A_0 \boldsymbol{\varepsilon}^{\mathrm{T}} \mathbf{D} \boldsymbol{\varepsilon}$

Pneumatic membrane



$$\frac{X_i^1 + X_i^2 + X_i^3}{3} = X_i^0$$: center of gravity

- α : air pressure
- A_i : element area
- n_i : normal vector
- $V(X)$: volume

$$\begin{aligned}\Pi(\boldsymbol{X}) &= S(\boldsymbol{X}) - W(\boldsymbol{X}) \\ &= S(\boldsymbol{X}) - \alpha V(\boldsymbol{X}) \\ &= S(\boldsymbol{X}) - \sum_{i=1}^{m} \alpha A_i(\boldsymbol{X})[n_i(\boldsymbol{X})]^T \frac{X_i^1 + X_i^2 + X_i^3}{3}\end{aligned}$$

Objective function to be minimized

Pneumatic membrane

Stationary condition → Equilibrium equation :

$$\frac{\partial \Pi(\boldsymbol{X})}{\partial \boldsymbol{X}} = \frac{\partial S(\boldsymbol{X})}{\partial \boldsymbol{X}}$$

Internal force by tension

$$-\sum_{i=1}^{m} \alpha A_i(\boldsymbol{X})[\boldsymbol{n}_i(\boldsymbol{X})]^T \frac{\partial \boldsymbol{X}_i^0}{\partial \boldsymbol{X}}$$

External force by pressure

$$-\sum_{i=1}^{m} \alpha \left[\frac{\partial \left(A_i(\boldsymbol{X}) \boldsymbol{n}_i(\boldsymbol{X}) \right)}{\partial \boldsymbol{X}} \right]^T \boldsymbol{X}_i^0$$

Redundant term ≈ 0

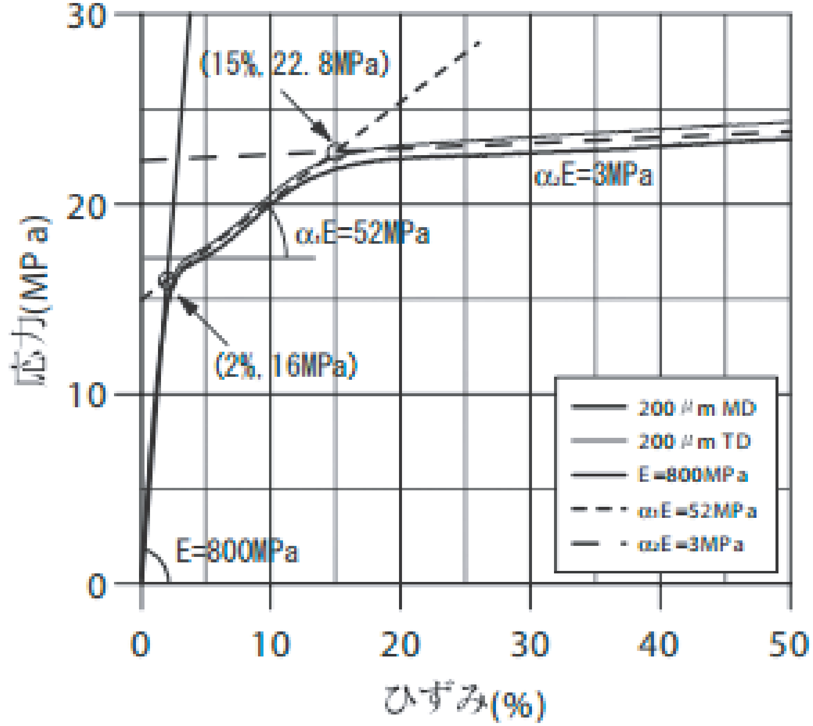
$$= 0$$

Different from the formula by Bouzidi and Le van

Application to ETFE film

- Transparent polymeric material
  Ithotropic elastoplastic material.
- Von Mises yield condition by
  Yoshino and Kato
- Tri-linear hardening property.

Stress strain relation



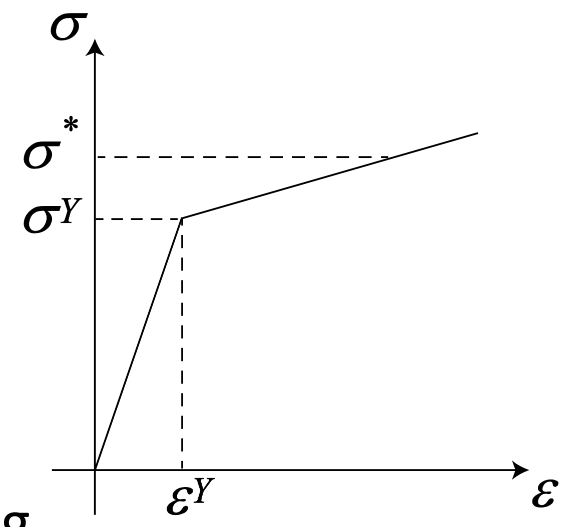
Equivalent stress: $$\overline{\sigma} = \sqrt{\sigma_x{}^2 - \sigma_x \sigma_y + \sigma_y{}^2 + 3\tau_{xy}{}^2}$$

Equivalent strain: $$\overline{\varepsilon} = \frac{2}{\sqrt{3}} \sqrt{(\varepsilon_x)^2 + \varepsilon_x \varepsilon_y + (\varepsilon_y)^2 + \frac{1}{4}(\gamma_{xy})}$$

Application to ETFE film

Monotonic loading

$\longrightarrow$ Bilinear elastic material



- $\mathbf{D_1}$: Elastic stress-strain matrix
- $\mathbf{D_2}$: Plastic stress-strain matrix

Stress, strain, and strain energy after yielding

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \boldsymbol{D}_2(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}')$$

$$S = \frac{1}{2}A\boldsymbol{\varepsilon}'\boldsymbol{\sigma}' + \frac{1}{2}A(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}')(\boldsymbol{\sigma} + \boldsymbol{\sigma}')$$

$$\boxed{\boldsymbol{\sigma}' = [\sigma_Y \quad \sigma_Y \quad 0]^T}$$

$$\boxed{\boldsymbol{\varepsilon}' = [\frac{1}{2}\varepsilon_Y \quad \frac{1}{2}\varepsilon_Y \quad 0]^T}$$

Assume properties of uniform stress distribution

$$\sigma_x = \sigma_y, \quad \tau_{xy} = 0$$

$$\varepsilon_x = \varepsilon_y, \quad \gamma_{xy} = 0$$

Example of pneumatic membrane

＜ETFE＞

Elastic modulus	160 [kN/m]
Hardening coefficient	10.4 [kN/m]
Yield stress, strain	3.2 [kN/m], 2%
Poisson's ratio	0.45

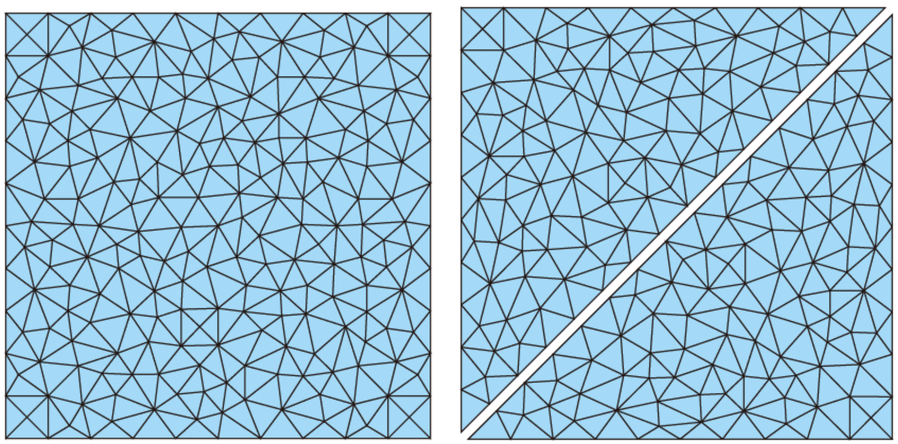
Pneumatic membrane：

Pressure *P*, stress *T*, and curvature 1/ρ satisfy

$$\frac{T_1}{\rho_1} + \frac{T_2}{\rho_2} = P$$

Example of pneumatic membrane

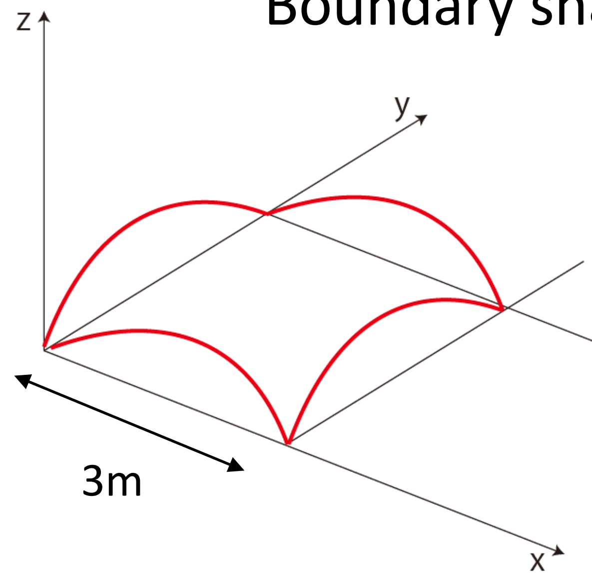
Initial cutting pattern



225 nodes
408 elements

224 nodes
374 members

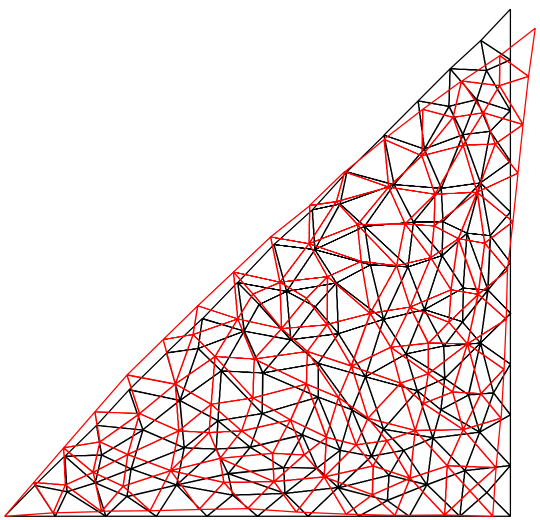
Boundary shape

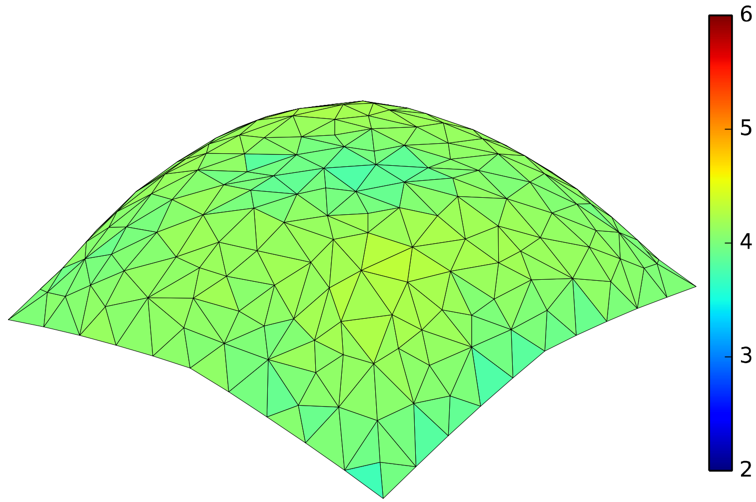


ETFE

➤ Target stress $\bar{\sigma} = 4.0\ [kN/m]$

	Step 0		Step 2		Step 4		Step 7		Step 10	
Direction	1	2	1	2	1	2	1	2	1	2
Average	4.435	4.437	4.108	4.108	4.016	4.017	4.036	4.037	4.072	4.073
Max.	5.100	5.076	4.266	4.283	4.154	4.179	4.503	4.402	4.313	4.449
Min.	2.311	2.360	3.168	3.632	3.721	3.694	3.499	3.316	3.712	3.719
Std. Dev.	0.379	0.362	0.094	0.091	0.085	0.088	0.156	0.160	0.091	0.097



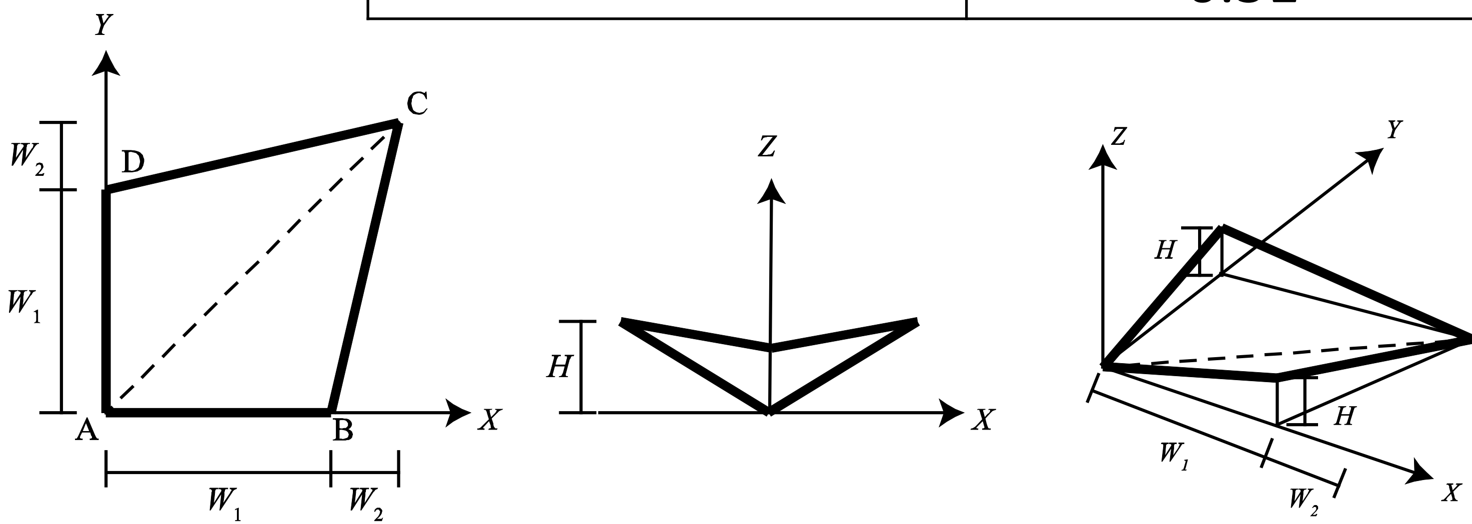


Example of frame-supported membrane

PVC

➢ Target stress $\bar{\sigma} = 3.0\ [kN/m]$

Young's modulus (Warp)	243 [kN/m]
Young's modulus (Weft)	227 [kN/m]
Shear modulus	24.2 [kN/m]
Poisson's ratio	0.51

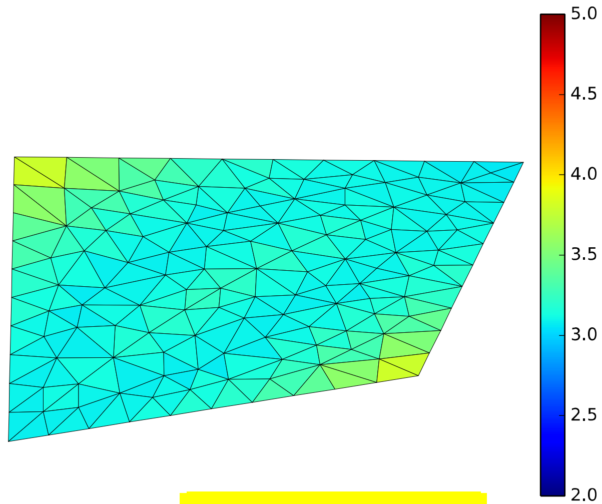


Boundary shape

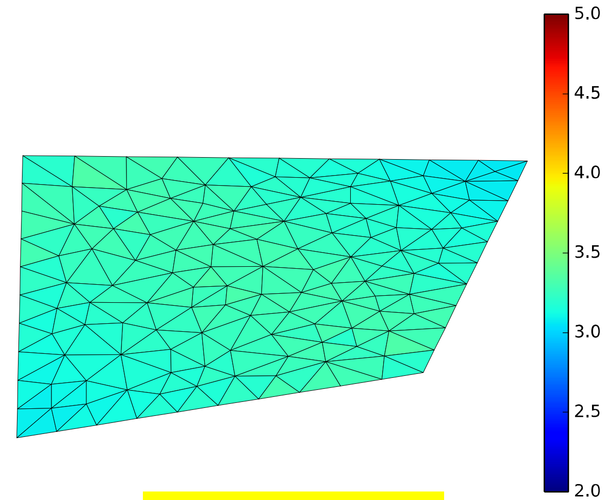
PVC

➢ Target stress $\bar{\sigma} = 3.0\ [kN/m]$

	Step 0		Step 5		Step 10		Step 20	
Direction	1	2	1	2	1	2	1	2
Average	0.577	0.102	3.838	3.896	3.479	3.634	3.167	3.218
Max.	3.320	2.811	5.341	4.230	5.008	4.011	3.803	3.335
Min.	-16.619	-2.680	3.326	3.318	3.170	3.175	3.063	3.063
Std. Dev.	2.202	0.621	0.386	0.218	0.314	0.199	0.128	0.064



Warp



Weft

Conclusions

- New method for approximate cutting pattern optimization.
- Iterative update of target stress.
- Equilibrium shape analysis by energy minimization.
- Applicable to pneumatic membrane and ETFE sheet.