Design of deployable structures using limit analysis of partially rigid frames with quadratic yield functions

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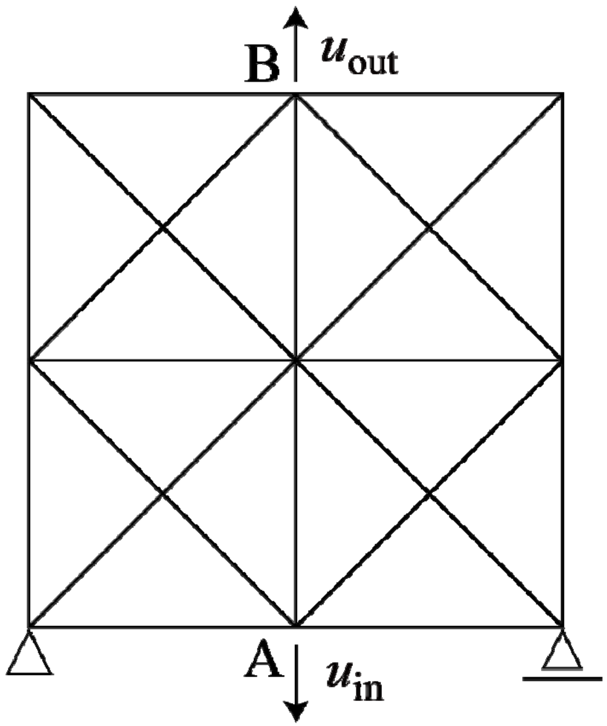
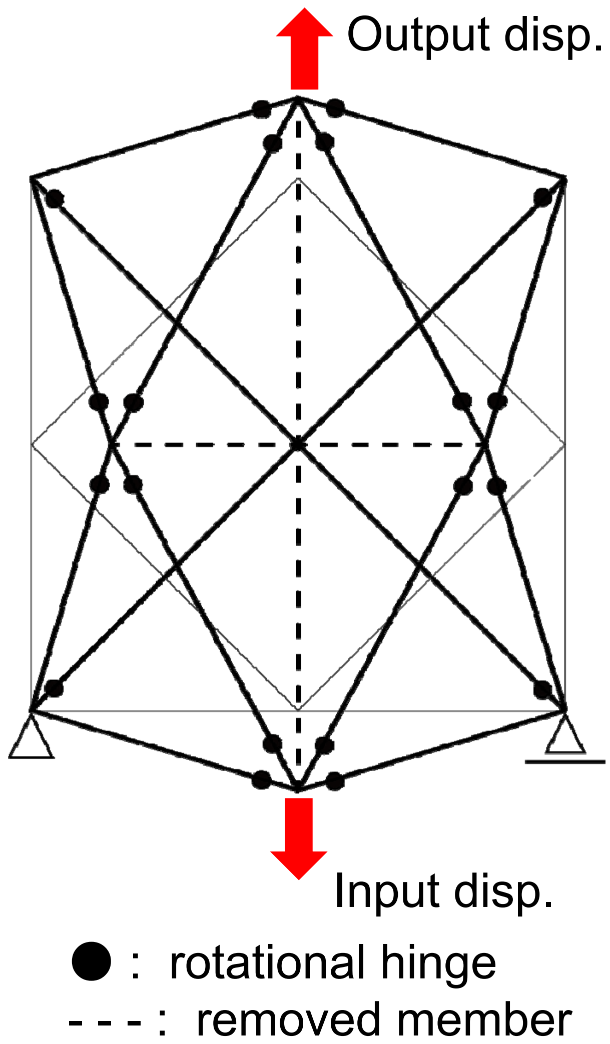
Planar mechanism (deployable structure) with partially rigid joints



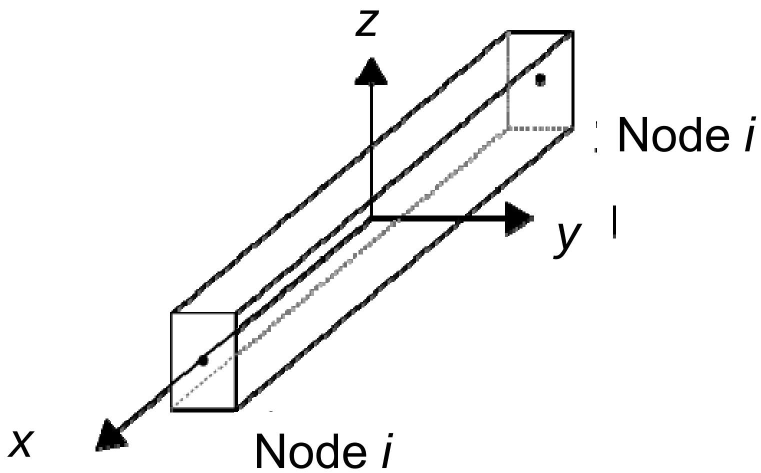


Generate mechanism with partially rigid joints using optimization method

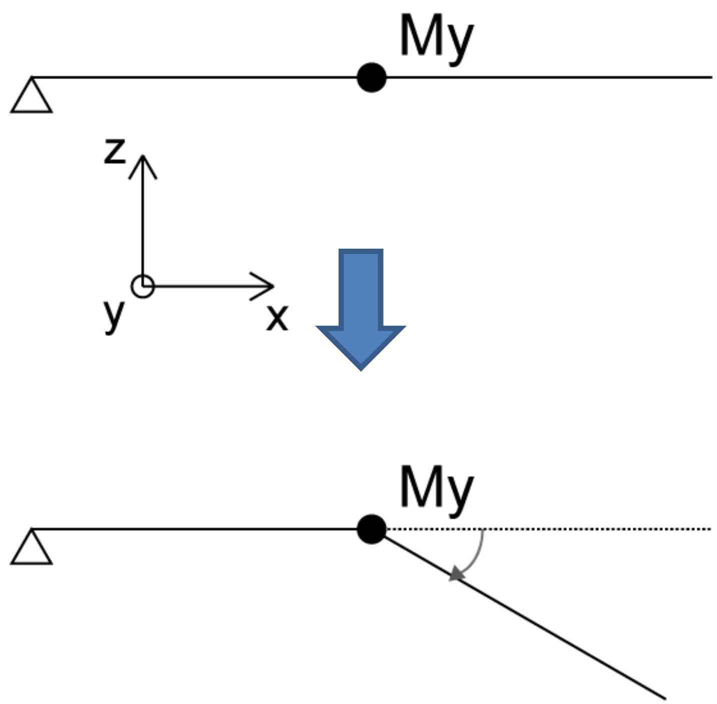
Mechanism with partially rigid joints

- Optimization approaches for generating link mechanisms
  - Truss elements for planar mechanism
  - Ideal (three-axis) pin joints are needed for 3D mechanism
- Ideal pin joints are difficult to manufacture
  → partially rigid connections are preferred
- No systematic approach to design of 3D link mechanism with partially rigid connections

Definition of local coordinates and member-end force



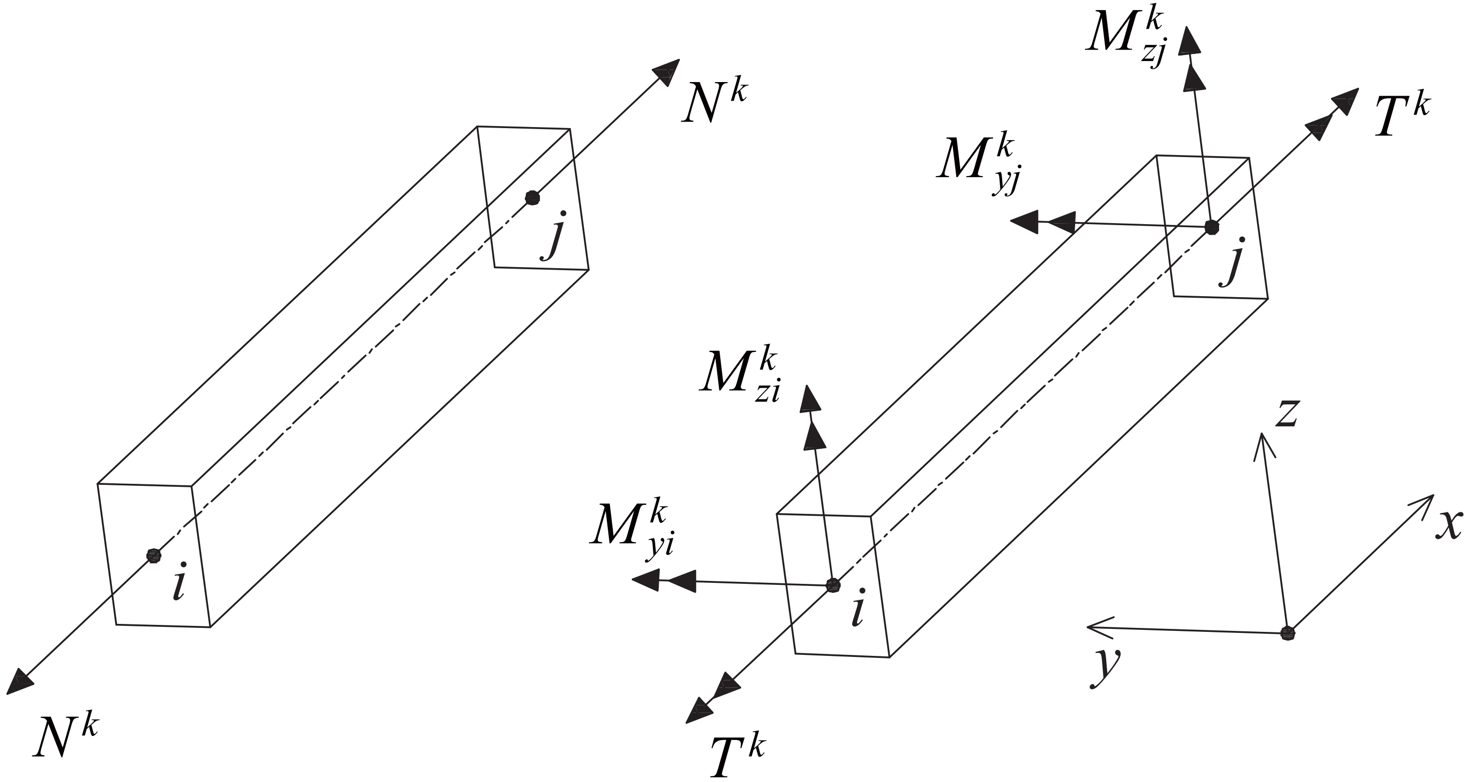
Local coordinates



Release moment	
T	Torsion around *x*-axis
My	Bending around *y*-axis
Mz	Bending around *z*-axis

Rotational hinge around *y*-axis

Definition of local coordinates and member-end force



6 x 2 member-end forces
Six equilibrium equations
→ Six independent components

$$\boldsymbol{f}^k = (N^k, T^k, M_{yi}^k, M_{zi}^k, M_{yj}^k, M_{zj}^k)^T$$

Limit analysis problem with two loading conditions

$$\max_{\lambda_{\mathrm{in}}, \mathbf{y}} . \quad \lambda_{\mathrm{in}}$$

$$\text{s. t.} \quad \sum_{i=1}^{n} f_i \mathbf{h}_i = \lambda_{\mathrm{in}} \mathbf{p}_{\mathrm{in}} + \mathbf{p}_{\mathrm{out}} \qquad \text{equilibrium}$$

$$\alpha w_i \geq | f_i |, \quad (i = 1, \ldots, n) \qquad \text{yield condition}$$

λ_{in} : load factor

$\mathbf{h}_i$: ith row of equilibrium matrix

$\mathbf{p}_{\mathrm{in}}$: load vector at input node

$\mathbf{p}_{\mathrm{out}}$: load vector at output node

αw_i : yield moment or yield member force

Procedure for generating infinitesimal mechanism

- Step 1: Define equilibrium matrix of rigidly jointed frame.
- Step 2: Find release conditions solving limit analysis problem.

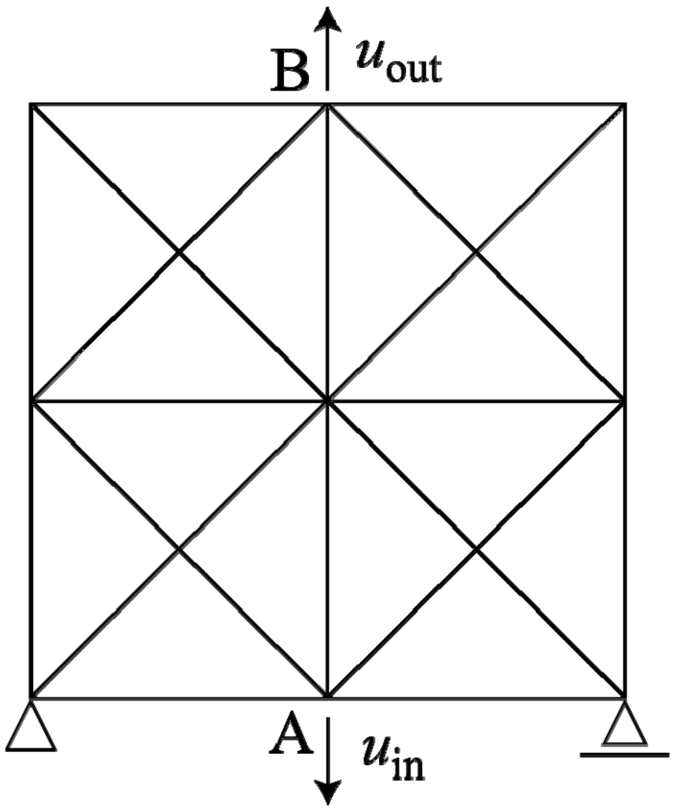
$$\alpha w_i = |f_i| \text{ for bending moment } \rightarrow \text{ rotational hinge}$$
$$\alpha w_i = |f_i| \text{ for torsional moment } \rightarrow \text{ torsional hinge}$$
$$\alpha w_i = |f_i| \text{ for axial force } \rightarrow \text{ remove member}$$

- Step 3: Output nodal displacements (mechanism) given as dual variables.

Procedure for generating finite mechanism

- Step 1: Find release conditions solving limit analysis problem.
- Step 2: Geometrical non-linear analysis.
  - If there exist internal forces, release the corresponding member-end force and continue this process.
- Step 3: Output the release conditions and nodal displacements of mechanism.

Planar model 1



$$\text{unit size} : 1 \times 1$$

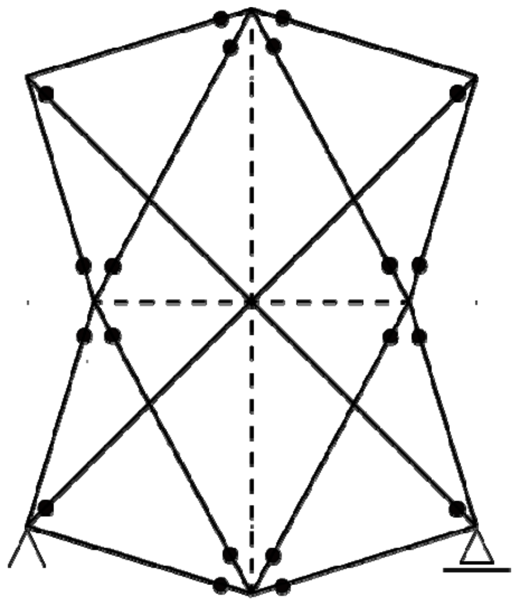
$$\bar{u}_{\text{in}} = 0.3$$

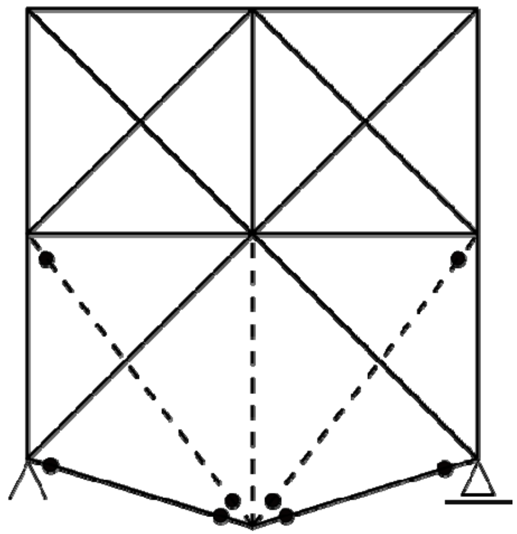
$$w_i = 1.0 \text{ for member extension}$$

$$w_i = 0.0001 \text{ for hinge rotation}$$

Planar model 1

Global mechanism





Local mechanism

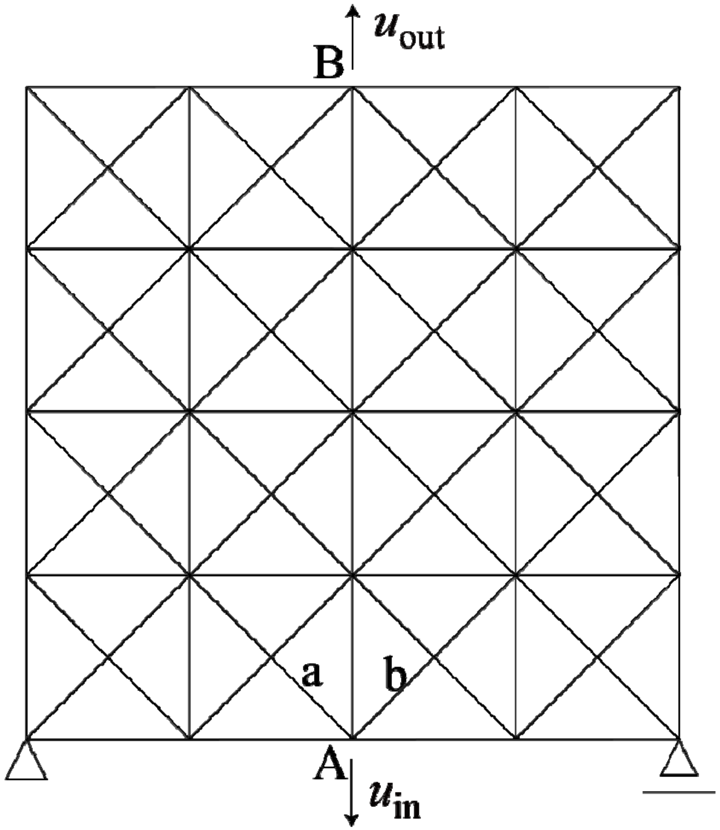
$0.42 \le \alpha \le 0.63$ $\qquad$ $\alpha \ge 0.64$

($\alpha \le 0.41 \rightarrow$ objective function is not bounded below)

● : rotational hinge

- - - : removed member

Planar model 2



$\text{unit size}: 1 \times 1$

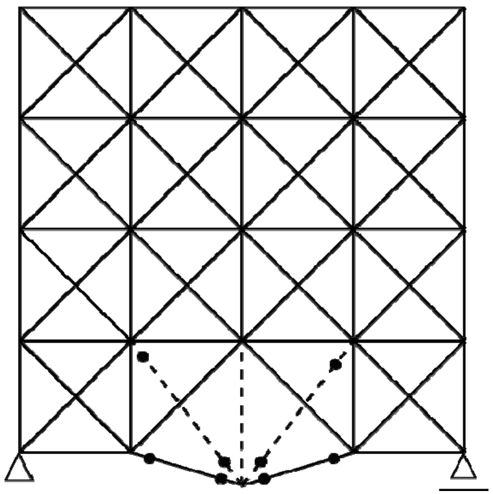
$\bar{u}_{\text{in}} = 0.3$

$w_i = 1.0$ for member extension

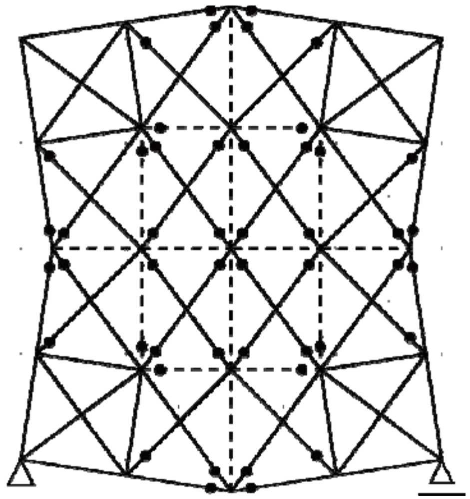
$w_i = 0.0001$ for hinge rotation

Planar model 2

Local mechanism



$$\alpha \geq 0.42$$



Global mechanism

$$\alpha \geq 0.42,$$

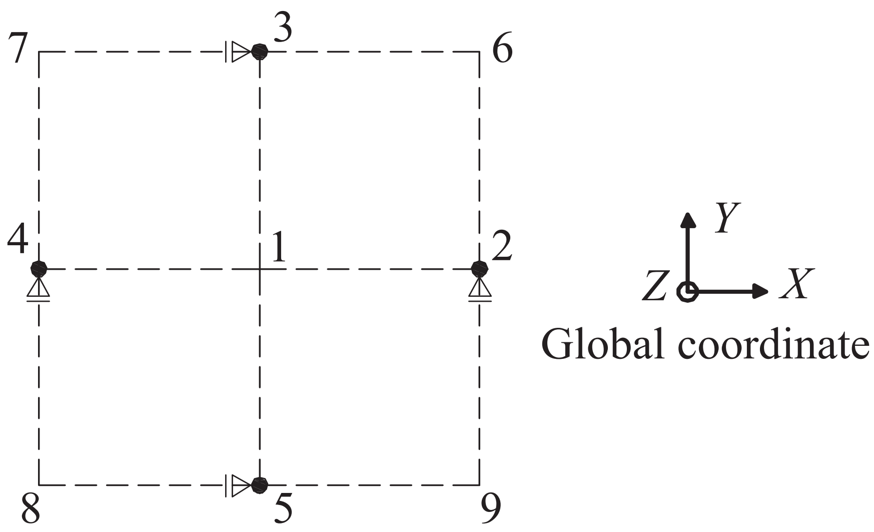
$$w_i = 10000.0 \text{ for extension of member A}$$

($\alpha \leq 0.41 \rightarrow$ objective function is not bounded below)

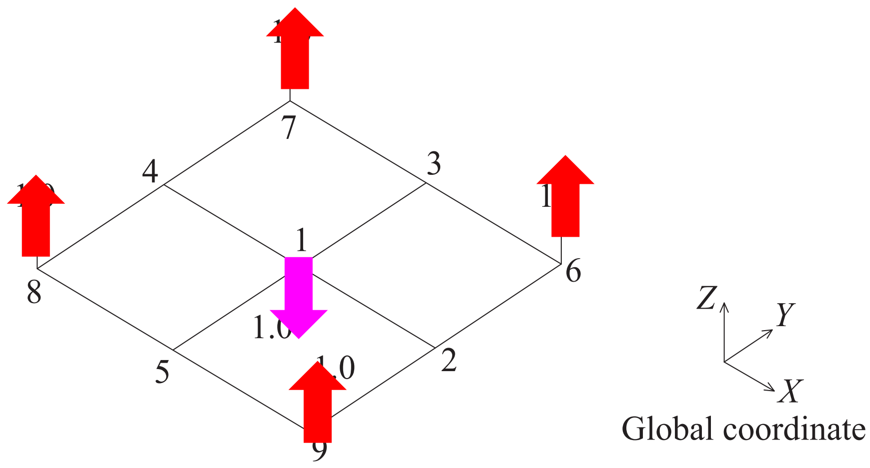
● : rotational hinge

- - - : removed member

3D mechanism of grid model



Boundary condition



Specified deformation

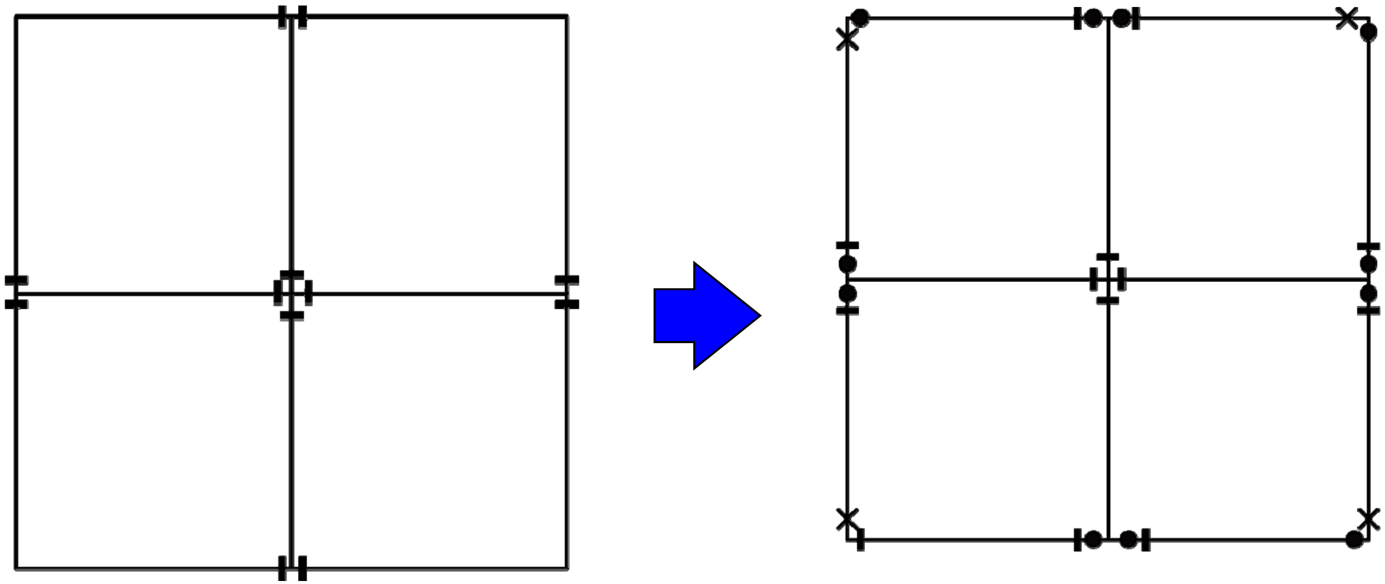
Symmetric w.r.t. XZ- and YZ-planes

Node 1: fix rotations around X- and Y-axes

Nodes 2 and 4: Fix displacements in Y- and Z-directions

Nodes 3 and 5: Fix displacements in X- and Z-directions

3D mechanism of grid model



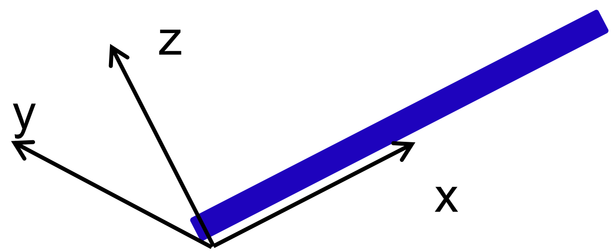
Infinitesimal mechanism

Finite mechanism

- : rotational hinge around local y-axis (bending)

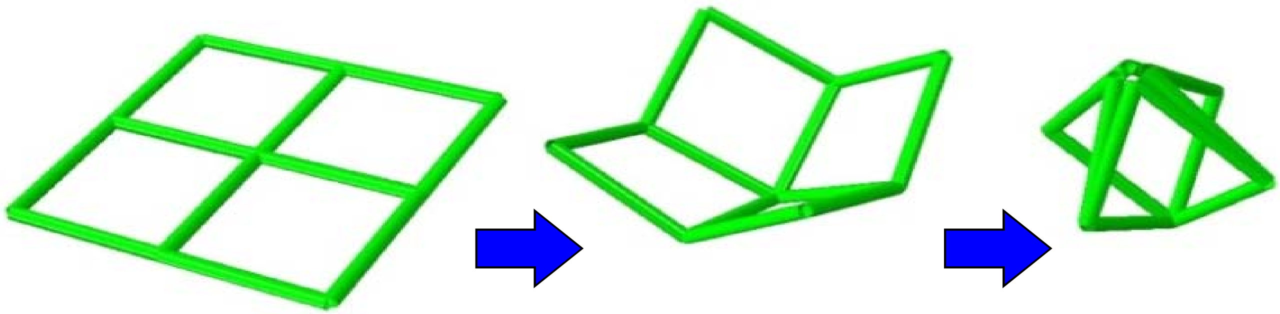
● : rotational hinge around local z-axis (bending)

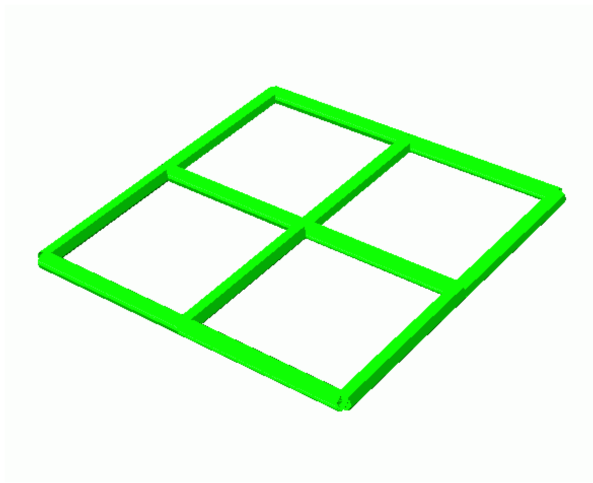
X : rotational hinge around local x-axis (torsion)



Local axis

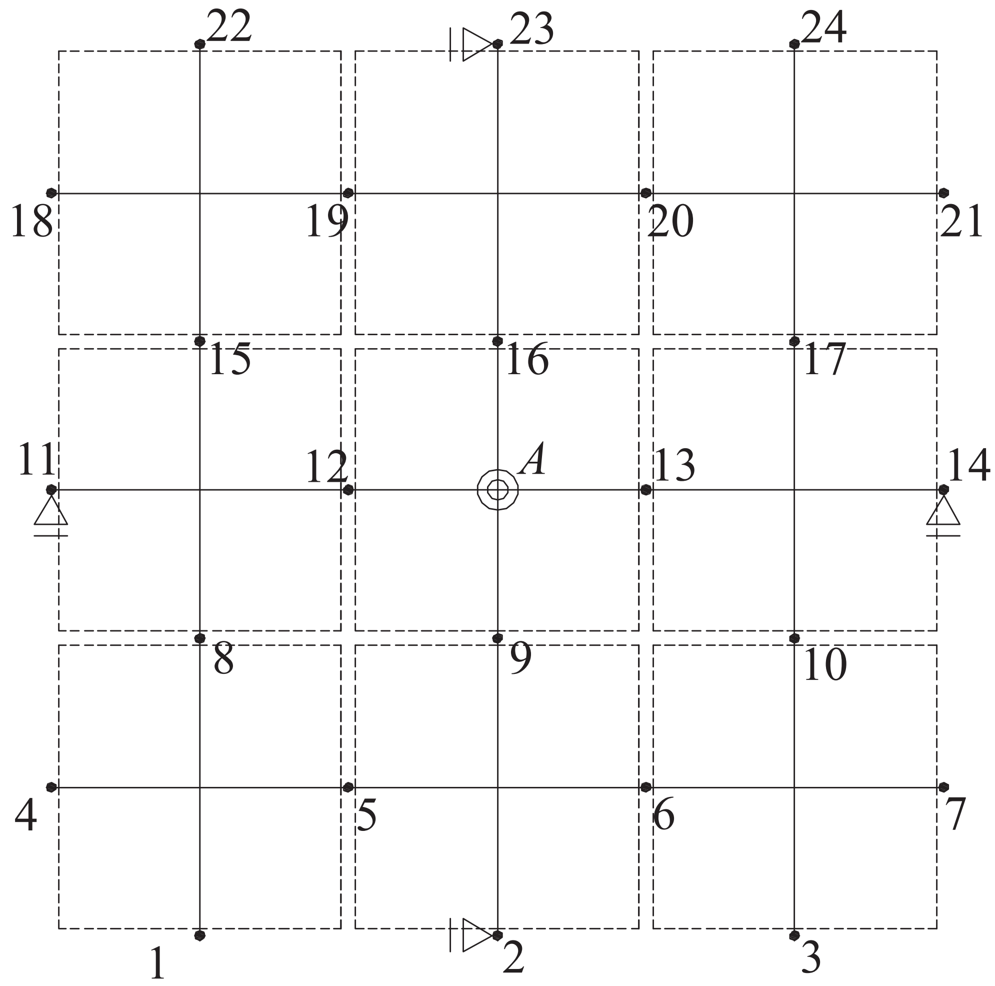
3D mechanism of grid model

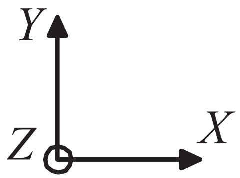




Deformation of 3D mechanism
without external load

Connect 9-units





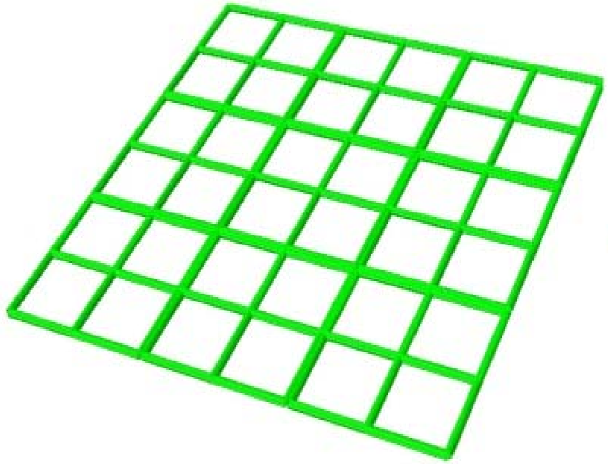
Global coordinate

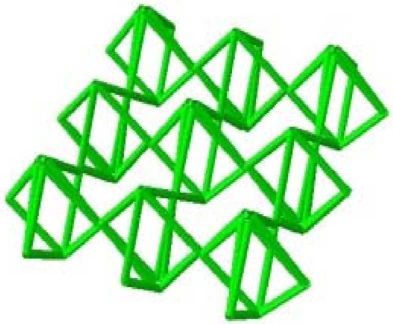
Boundary condition

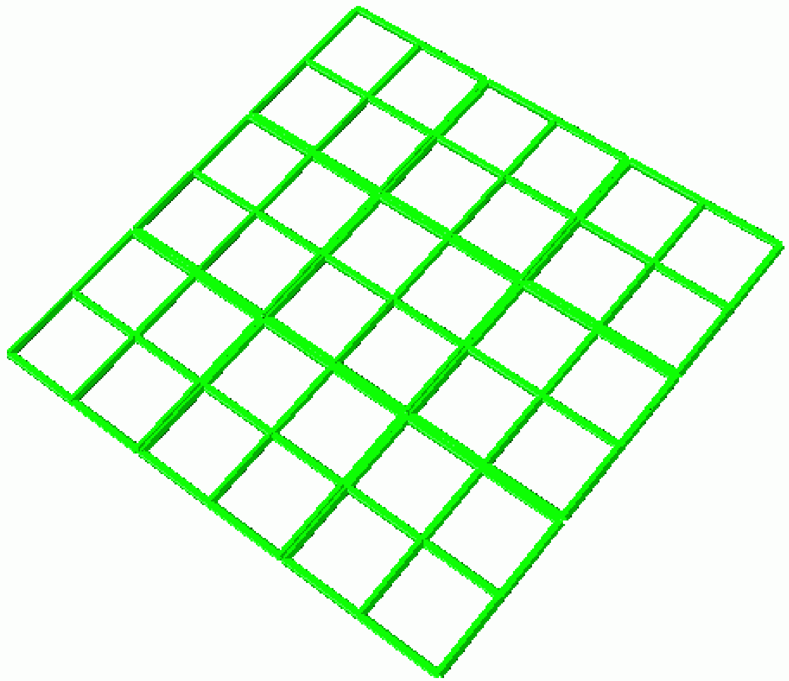
- fixed in Z-direction
- fixed in X-direction
- fixed in Y-direction

node 1, 2 and 3: fixed around Y-direction

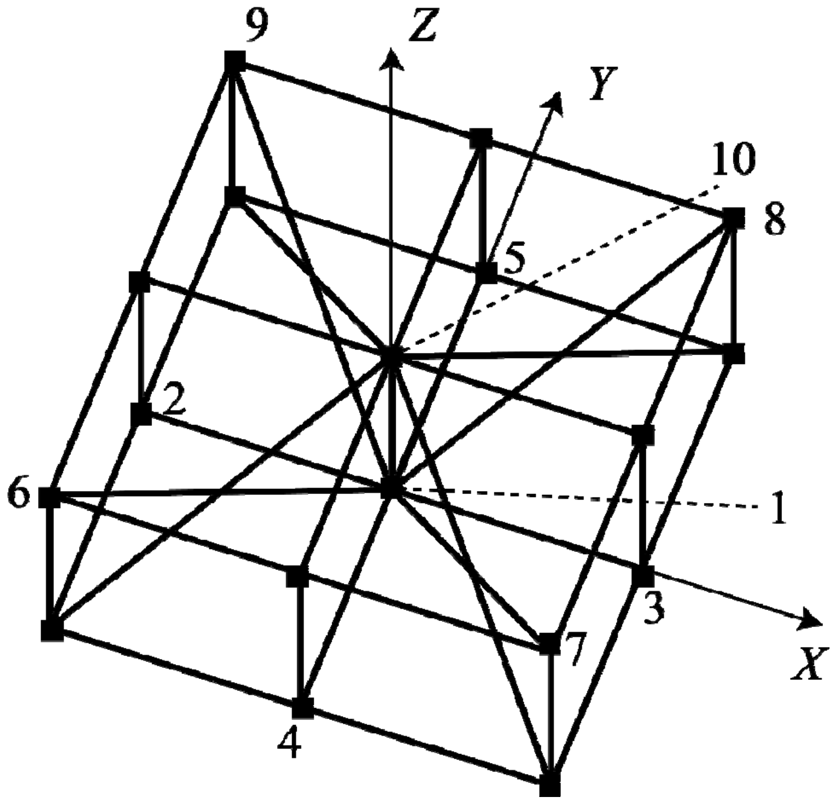
node 4, 11 and 18: fixed around X-direction







3D model 1



Pull node 1 downward
→
Node 10 moves upward

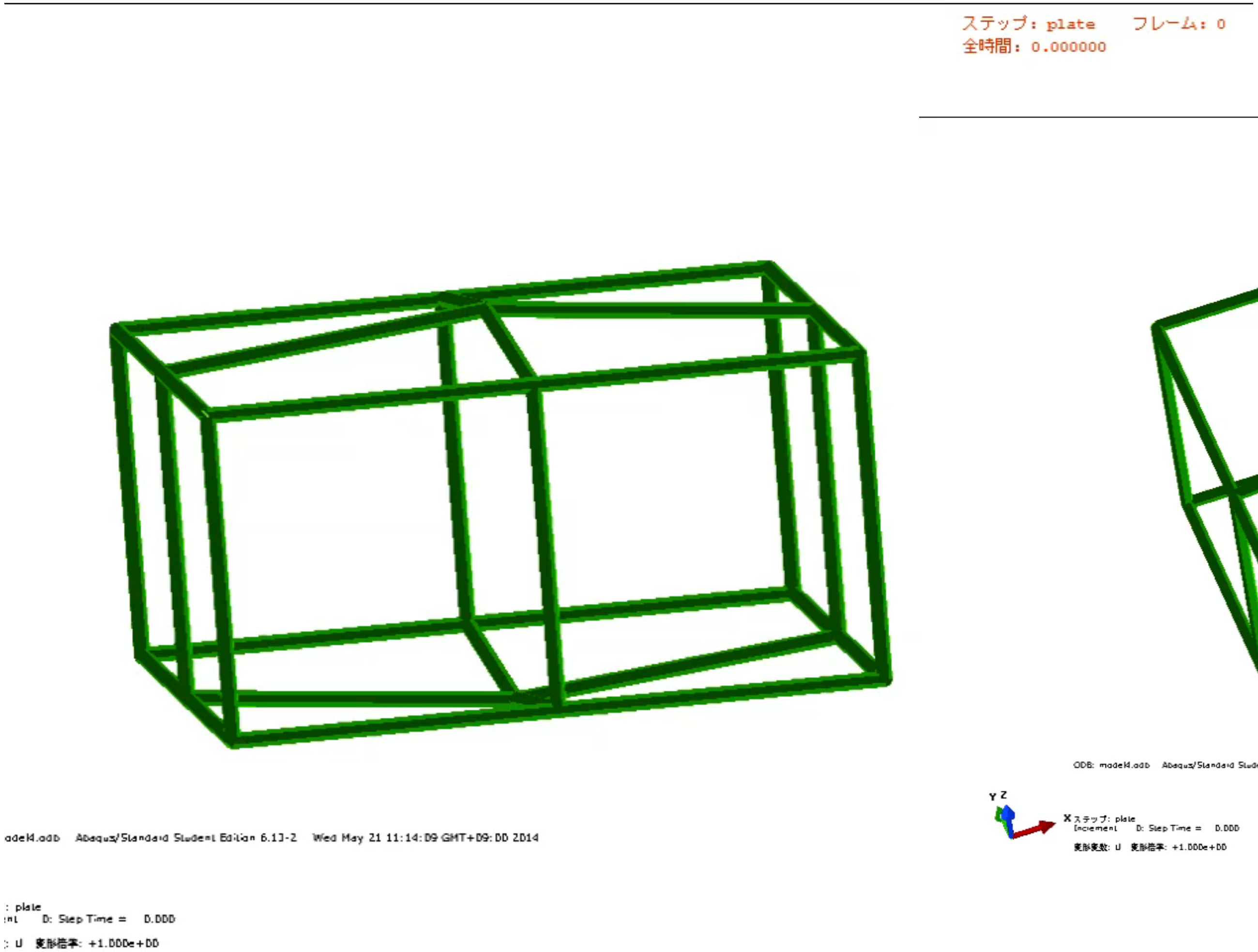
unit size : 1×1

$\beta = 1.0$

$w_i = 1.0$ for member extension

$w_i = 0.1$ for bending and torsion

3D model 2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全時間: 0.000000
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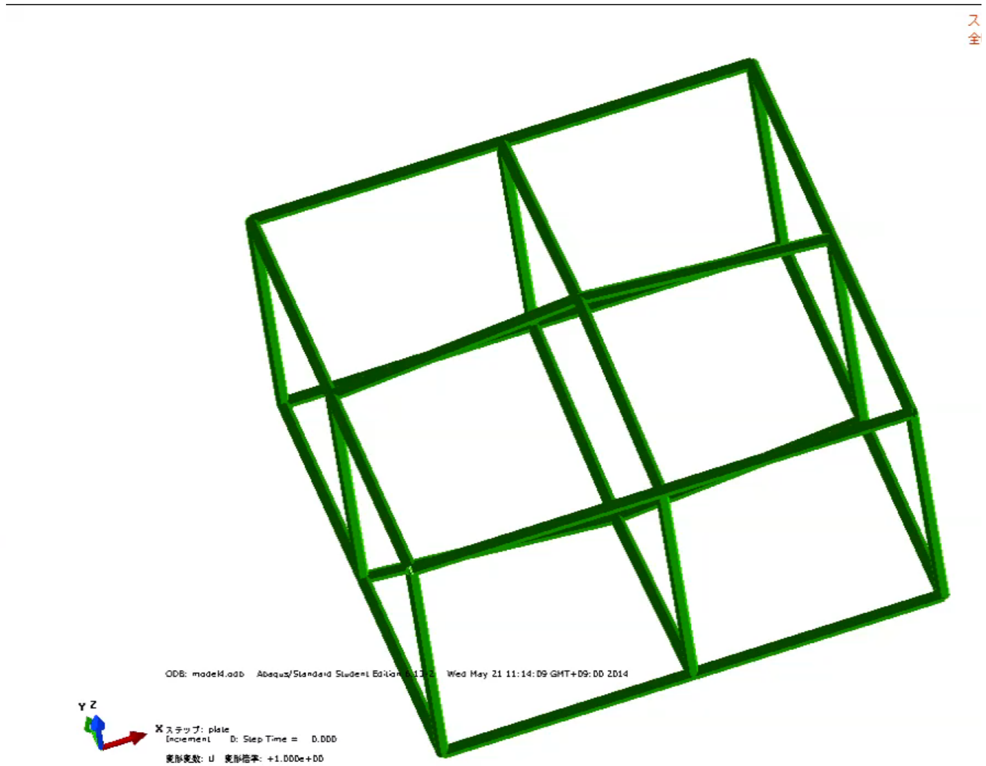
D: Step Time = 0.000
変形倍率: +1.000e+00

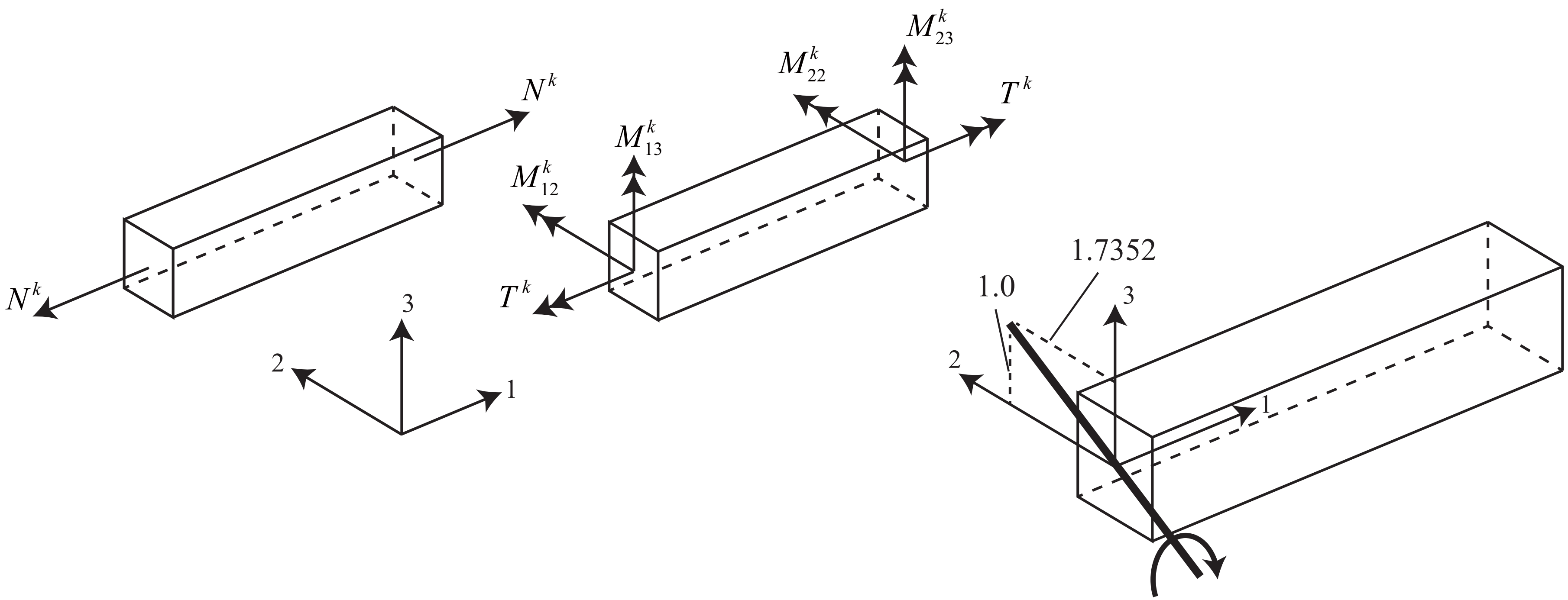

ODB: model4.odb Abaqus/Standard Student Edition
Wed May 21 11:14:09 GMT+09:00 2014
ステップ: plate
Increment 0: Step Time = 0.000
変形変数: U 変形倍率: +1.000e+00

Partially rigid frame with diagonal hinges

Diagonal hinge (arbitrary direction)

→ More diverse deformation

Reduce number of hinges



Optimization problem for mechanism with diagonal hinges

maximize λ_{in}

load factor

subject to $\displaystyle\sum_{i=1}^{6m} f_i \mathbf{h}_i = \mathbf{p}_{\text{out}} + \lambda_{\text{in}} \mathbf{p}_{\text{in}}$

equilibrium

yield condition for moment

$$(T^k(\mathbf{f}))^2 + (M_{j2}^k(\mathbf{f}))^2 + (M_{j3}^k(\mathbf{f}))^2 \leq \alpha w^{\mathrm{b}},$$

$$(k = 1, \dots, m;\ j = 1, 2)$$

yield condition for axial force

$$(N^k(\mathbf{f}))^2 \leq \alpha w^{\mathbf{a}},\ \ (k = 1, \dots, m)$$

quadratic yield condition conditions

Optimality conditions

Normalization of u

$$1 - \mathbf{p}_{\text{in}}{}^{T}\mathbf{u} = 0$$

Bending moment

$$\mathbf{h}_i{}^{T}\mathbf{u} + 2M_{jp}^{k}(\mathbf{f})c_j^{k} = 0,$$

$$(k = 1, \ldots, m;\ j = 1, 2;\ p = 2, 3)$$

Torsional moment

$$\mathbf{h}_i{}^{T}\mathbf{u} + 2T^{k}(\mathbf{f})(c_1^{k} + c_2^{k}) = 0,$$

$$(k = 1, \ldots, m;\ j = 1, 2)$$

j : member-end

p : axis

c_j^k : rotation of member-end j

M_{jp}^k : bending moment at member-end j around axix p

T^k : torsional moment

Optimality conditions

Axial force

$$\mathbf{h}_i^T \mathbf{u} + 2N^k(\mathbf{f}) c_0^k = 0,$$
$$(k = 1, \ldots, m)$$

j : member-end

p : axis

c_0^k : extension

N^k : axial force

Complemenrarity condition

$$[(T^k(\mathbf{f}))^2 + (M_{j2}^k(\mathbf{f}))^2 + (M_{j3}^k(\mathbf{f}))^2 - \alpha w^{\mathrm{b}}]\, c_{kj} = 0,$$
$$c_{kj} \geq 0, \qquad (k = 1, \ldots, m;\ j = 1, 2)$$

$$[(N^k(\mathbf{f}))^2 - \alpha w^{\mathbf{a}}]\, c_0^k = 0,$$
$$c_0^k \geq 0, \qquad (k = 1, \ldots, m;\ j = 1, 2)$$

Rotation or extension is non-zero only when yield condition is satisfied with equality

Optimality condition

Bending moment

$$\theta_{jp}^{k} = 2M_{jp}^{k} c_{j}^{k}, \ (k = 1, \ldots, m; \ j = 1, 2; \ p = 1, 2)$$

Rotation θ_{jp}^{k} around axis p is proportional to

bending moment M_{jp}^{k}

(norm of rotation c_{j}^{k} does not depend on axis)

Torsional moment

$$\theta_{1}^{k} = \theta_{j1}^{k} - \theta_{i1}^{k} = 2T^{k}(c_{1}^{k} + c_{2}^{k}), \ (k = 1, \ldots, m; \ j = 1, 2)$$

$$\Rightarrow \quad \mathbf{R}_{1}^{k} = \begin{pmatrix} -\theta_{1}^{k} \\ \theta_{12}^{k} \\ \theta_{13}^{k} \end{pmatrix} = c_{k1} \begin{pmatrix} -T^{k} \\ M_{12}^{k} \\ M_{13}^{k} \end{pmatrix} \qquad \mathbf{R}_{2}^{k} = \begin{pmatrix} \theta_{1}^{k} \\ \theta_{22}^{k} \\ \theta_{23}^{k} \end{pmatrix} = c_{k2} \begin{pmatrix} T^{k} \\ M_{22}^{k} \\ M_{23}^{k} \end{pmatrix}$$

Auxiliary problem for determination of parameter alpha

maximize μ load factor

subject to $\sum_{i=1}^{6m} f_i \mathbf{h}_i = \mu \mathbf{p}_{\mathrm{out}}$ equilibrium

yield condition for moment

$$(T^k(\mathbf{f}))^2 + (M_{j2}^k(\mathbf{f}))^2 + (M_{j3}^k(\mathbf{f}))^2 \leq w^{\mathrm{b}},$$

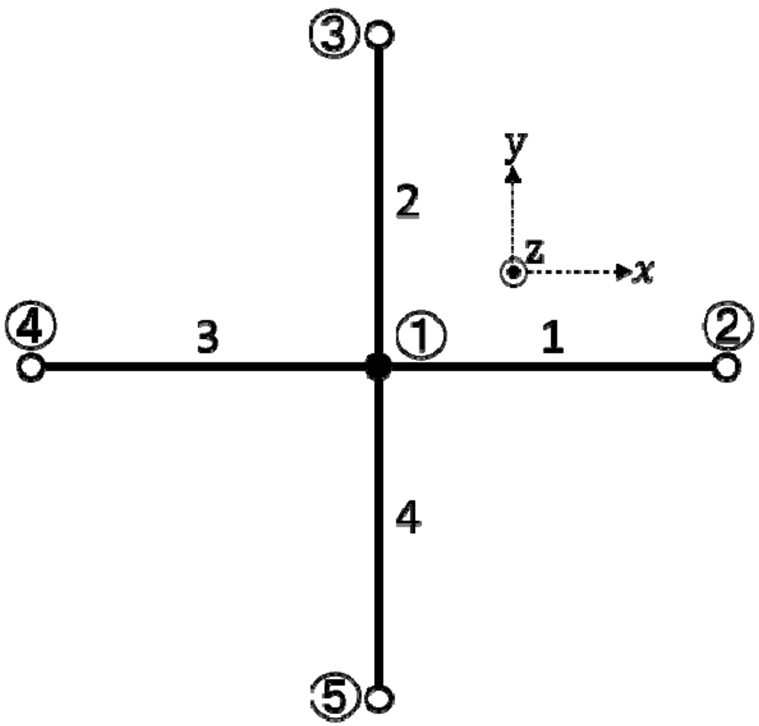
$$(k = 1, \ldots, m;\ j = 1, 2)$$

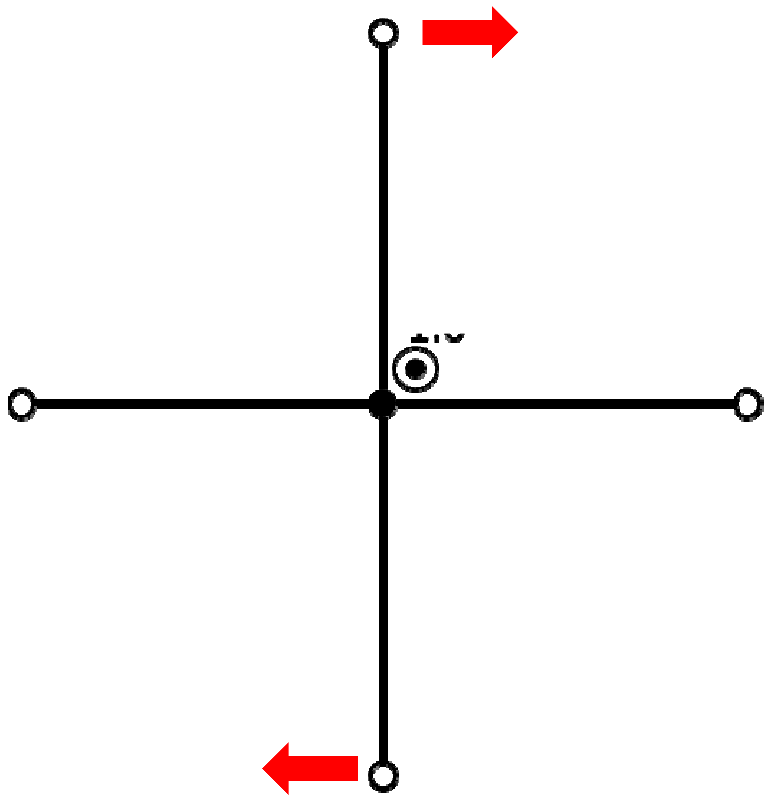
yield condition for axial force

$$(N^k(\mathbf{f}))^2 \leq w^{\mathbf{a}},\ (k = 1, \ldots, m)$$

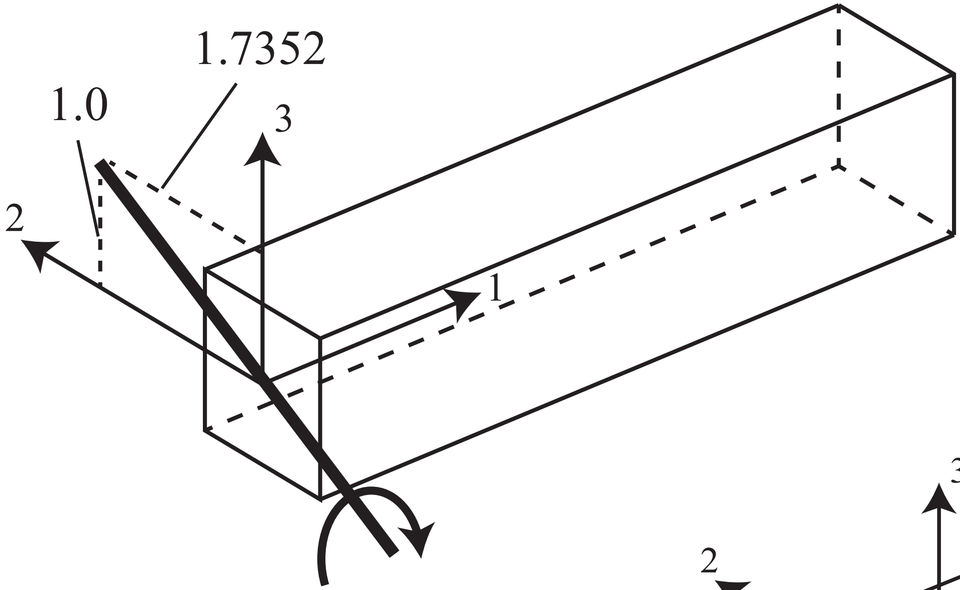
→ lower bound of $\alpha = 1/\mu$

Example 1

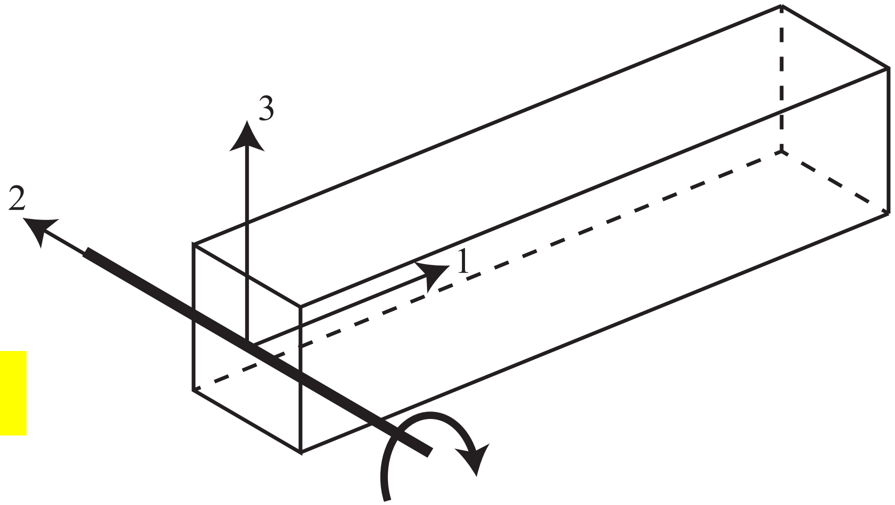




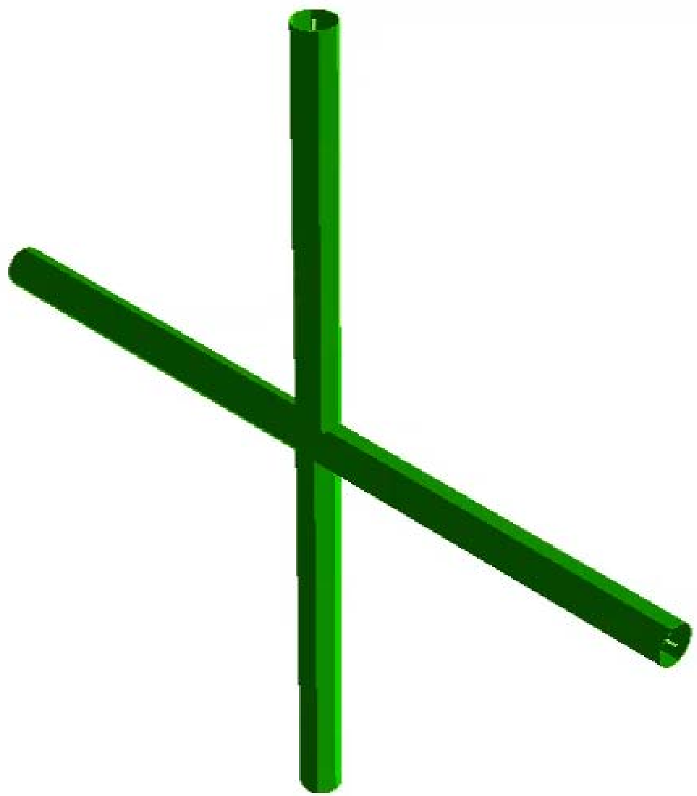
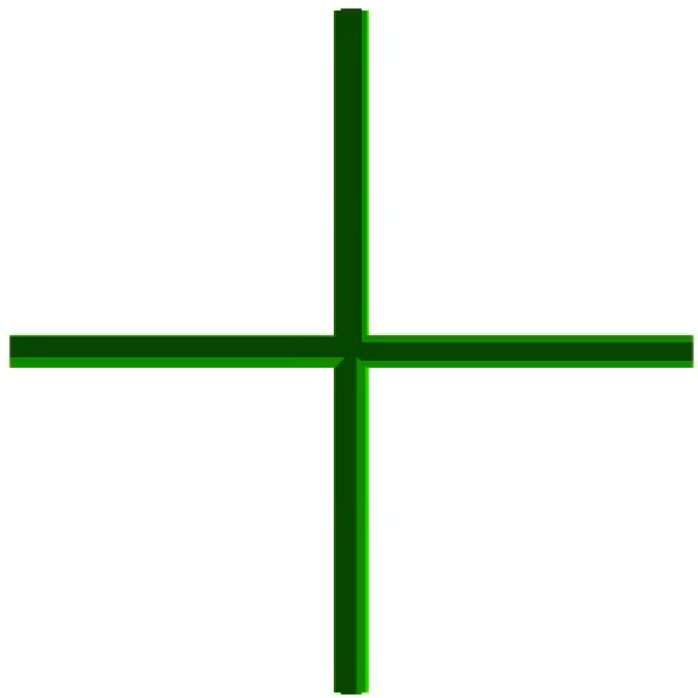
Example 1



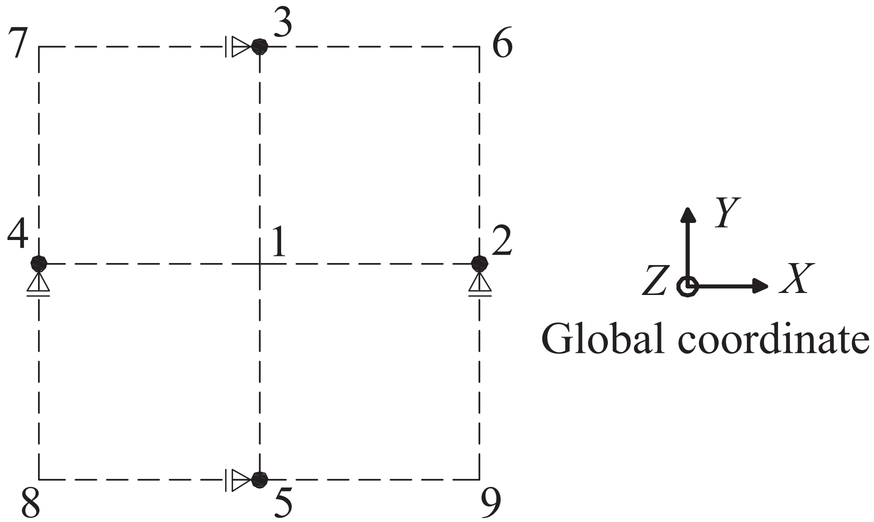
Vertical members 1-3 and 1-5



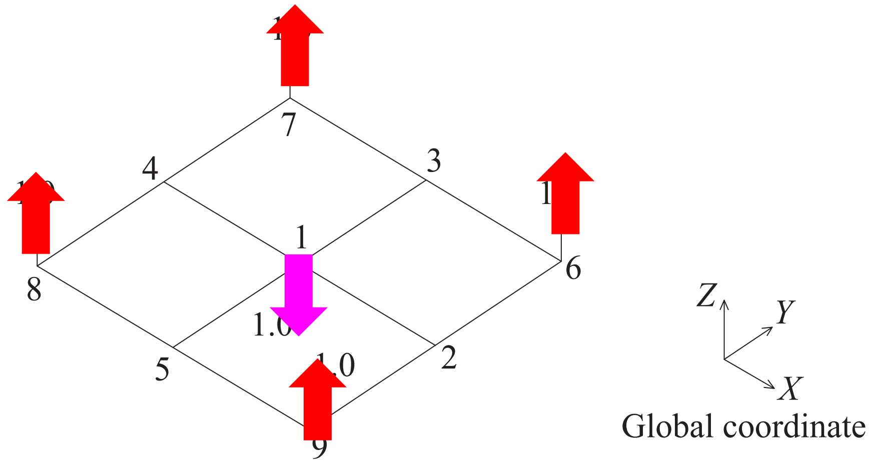
Horizontal members 1-2 and 1-4

Example of 3D-mechanism with partially rigid joints



Boundary condition



Specified deformation

Symmetric w.r.t. XZ- and YZ-planes

Node 1: fix rotations around X- and Y-axes

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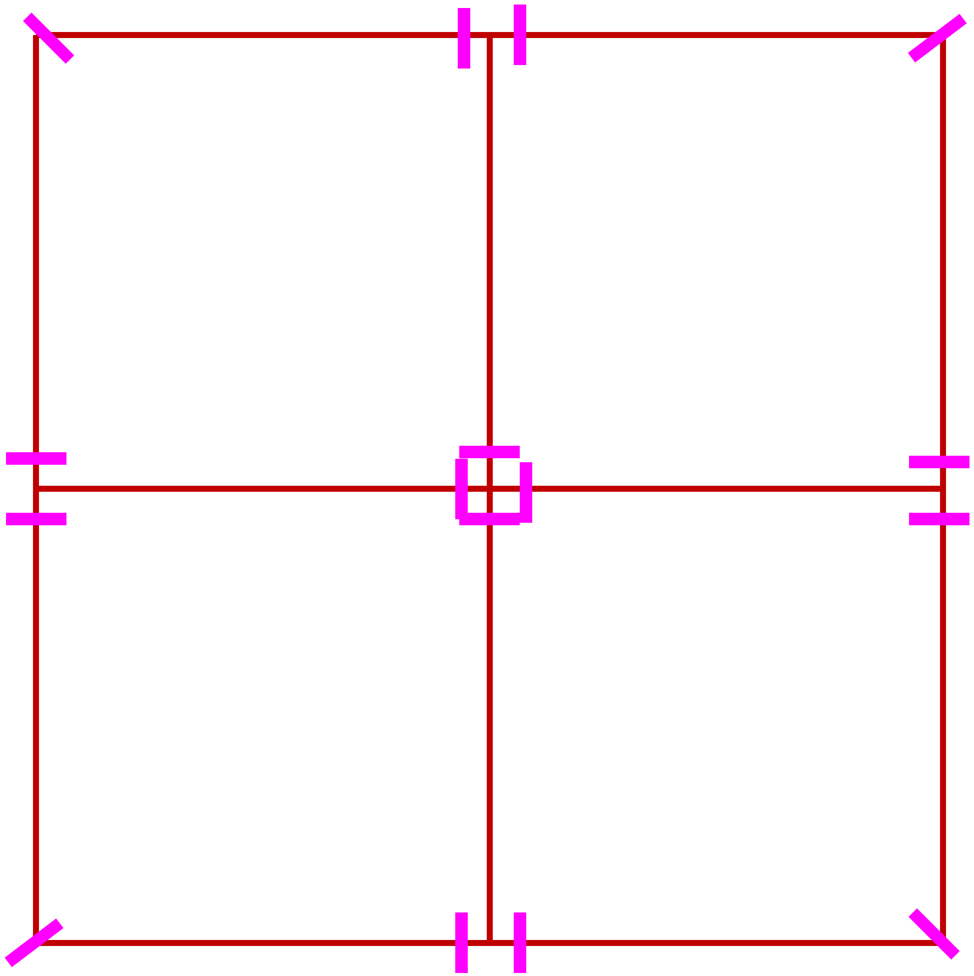
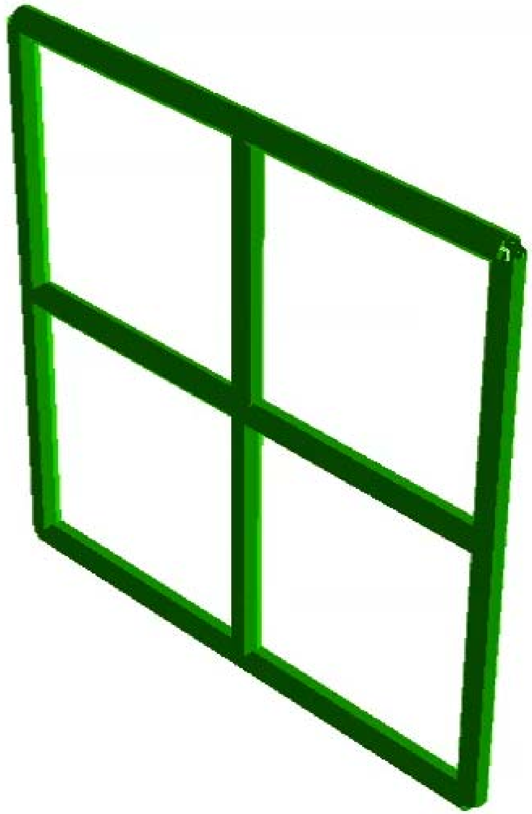
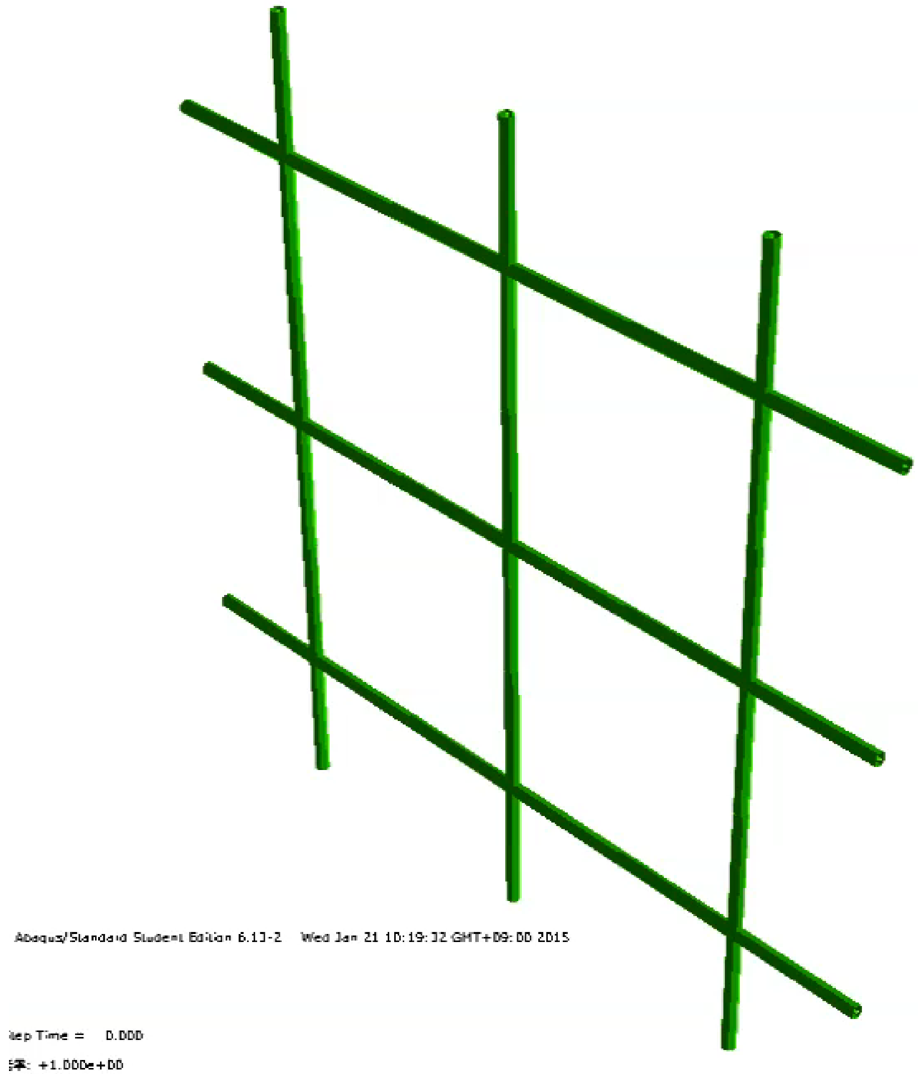
Nodes 3 and 5: Fix displacements in X- and Z-directions

Example 2



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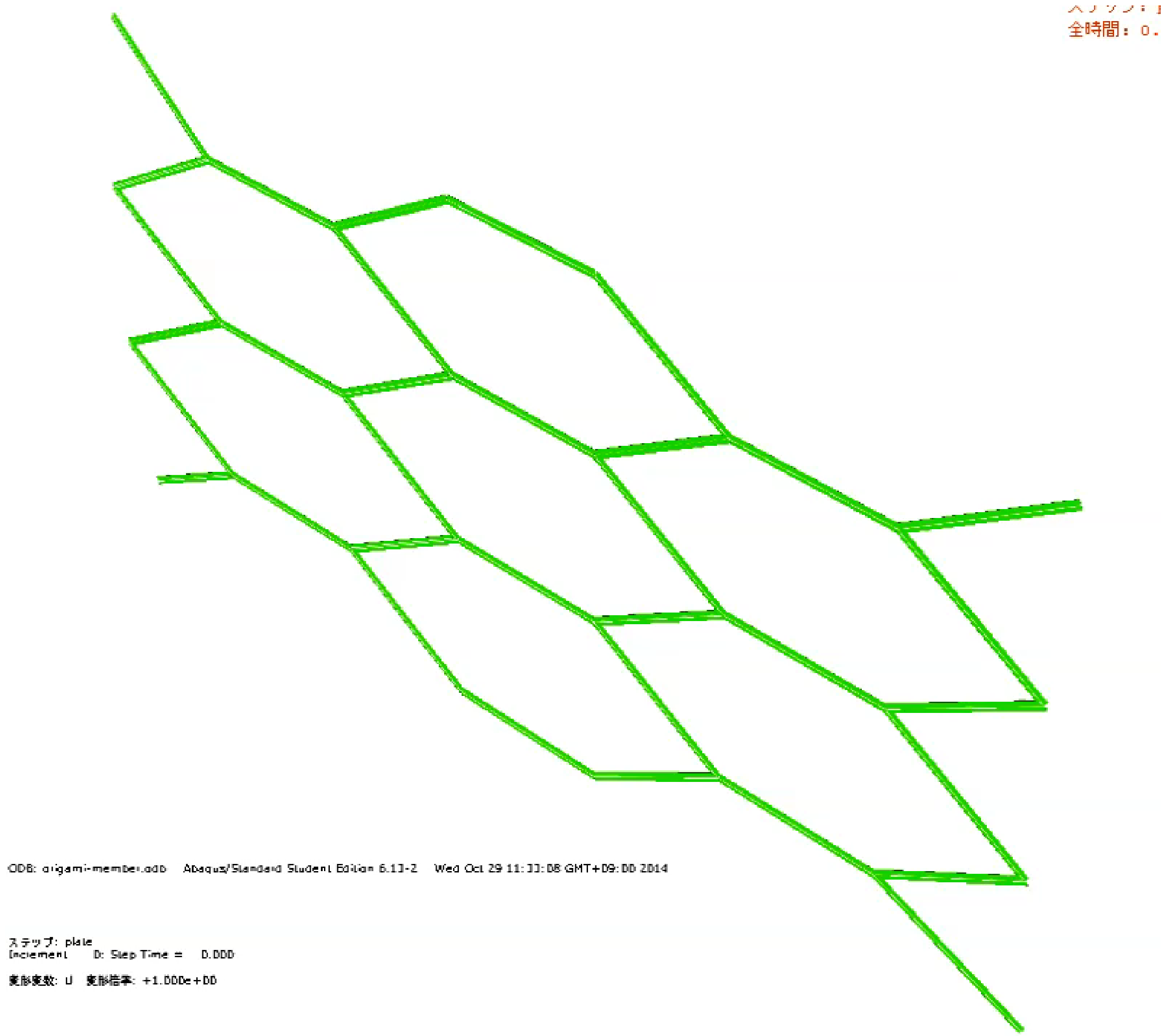
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Conclusions

- New method for generating a deployable structure by solving a quadratic programming problem.
- Limit analysis problem with a quadratic yield function of the member-end moments.
- Mechanism with diagonal hinges.
- By allowing a diagonal hinge, the number of hinges can be reduced compared with the previous study, where only the hinges around the local axes are allowed.
- Additional hinges may be needed to generate a finite mechanism.