Optimization Approach to Force Identification of Prestressed Pin-jointed Structures

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Background

Stiffness

Geometrical

Linear

Geometry

Materials

Prestresses

Prestressed Structures

Light-weight

Long-span

Strong to Eq.

Prestressed Concrete

Suspension Bridges

Cable Domes

Tencho Dome

(Sizuoka Pro., Japan)
Motivation

Force Design

Force Management
- Construction
- Maintenance
  - Assessment
  - Health Monitoring
  - Re-tension

Force Identification
- Measurement Devices
  - Number of Measurements
  - Locations of Measurement
  - Accuracy
  - Efficiency
Basics

Assumptions
- Pin-jointed
- No self-weight
- No external load

Self-equilibrium
- Estimated Forces
  \[ \mathbf{D}_s = 0 \]
  \[ \mathbf{D}^m \mathbf{s}^m + \mathbf{D}^e \mathbf{s}^e = 0 \]
  \[ \mathbf{s}^e = -\left(\mathbf{D}^e\right)^{-1} \mathbf{D}^m \mathbf{s}^m \]

Axial forces only
Self-equilibrium

Measure all nodal coordinates
Measure some member forces

\( \mathbf{D} \): equilibrium matrix
\( \mathbf{s}^m \): measured forces
\( \mathbf{s}^e \): estimated forces
Identification Error

\[ \Delta s = \begin{pmatrix} \Delta s^e_m \\ \Delta s^m \end{pmatrix} = B \begin{pmatrix} \Delta X \\ \Delta s^m \end{pmatrix} \]

- **Total Force Errors**
  - Measurement Errors: Members forces, Nodal coordinates
  - Sensitivity analysis

- **Identification Error**
  - Maximum Euclidean norm of \( \Delta s \)
  - Square root of the largest eigenvalue of the product of \( B \) and its conjugate transpose

\( \Delta s^m \): force errors
\( \Delta X \): coordinate errors
Optimization Problem

Find the optimal measurement members with specific number of measurements

- Mathematical
  - Enumeration
    - Branch-and-Bound
  - Cutting Planes
- Heuristic
  - Genetic Algorithm
  - Tabu Search
  - Stingy (Greedy)
  - . . . . .
- Simulated Annealing (SA)
  - Analogy between metal annealing and optimal solution searching
  - Able to escape from local optima

Analogy between metal annealing and optimal solution searching
Able to escape from local optima
Solutions

Simulated Annealing

START

Initial solution

Local search

Solution transition

Cooling

Terminate

Yes

END

No

Stingy Method

START

All members

Drop the one leading to minimum increase of IE

No

The specified number? Yes

END
Example Cable net 1

- 4 free nodes
- 21 members
- 9 force modes
Example: Cable net 2

**Determining 12 Measurement Members**

### Conventional SA

<table>
<thead>
<tr>
<th>No. of Trials</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean</th>
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<tr>
<td>Identification Error</td>
<td>1.9219</td>
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<td>1.7302</td>
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<td>16626</td>
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### ISA (SA with Stingy)

<table>
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<th>No. of Trials</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean</th>
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### Comparison

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<tr>
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<th>Enumeration</th>
<th>Stingy</th>
<th>SA</th>
<th>ISA</th>
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<td>2.6607</td>
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<td>1.7619</td>
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<td>Relative Error</td>
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<td>53.78%</td>
<td>5.53%</td>
<td>1.83%</td>
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<tr>
<td>Computation Cost</td>
<td>293930</td>
<td>165</td>
<td>20655</td>
<td>17629</td>
</tr>
<tr>
<td>Relative Cost</td>
<td>100%</td>
<td>0.056%</td>
<td>7.03%</td>
<td>6.0%</td>
</tr>
</tbody>
</table>
Example? Cable Dome

30 nodes
60 members
2 force modes

12 Measurements
Measure only cables

(a) measurement members by Stingy
IE = 1.1988
(b) measurement members by ISA
IE = 1.0418
Summary

Identification error
- Measurement errors of nodal coordinates
- Measurement errors of member forces

Optimization
- Minimize Identification error
- S.T. Specified number of measurements

Solutions
- Simulated Annealing
- Stingy Method
- SA + Stingy