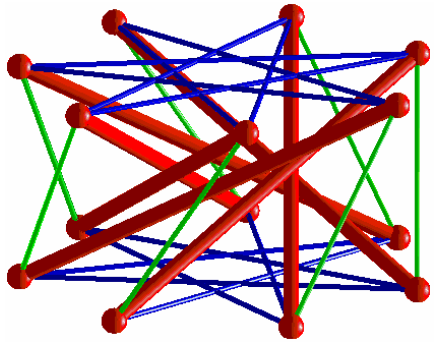
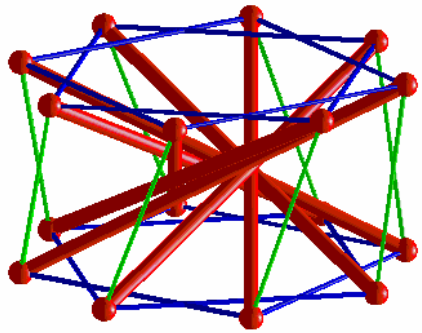
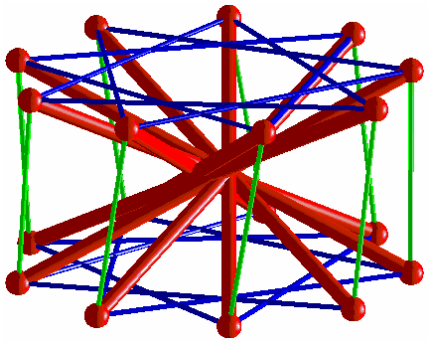
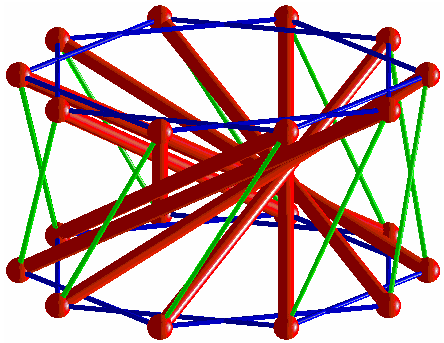
Symmetric Prismatic Tensegrity Structures

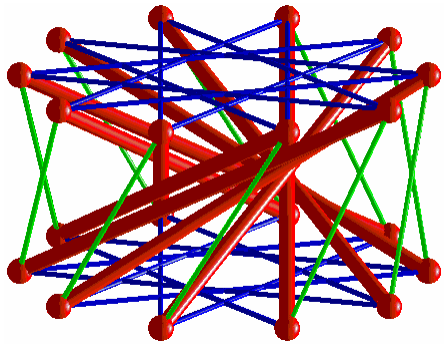




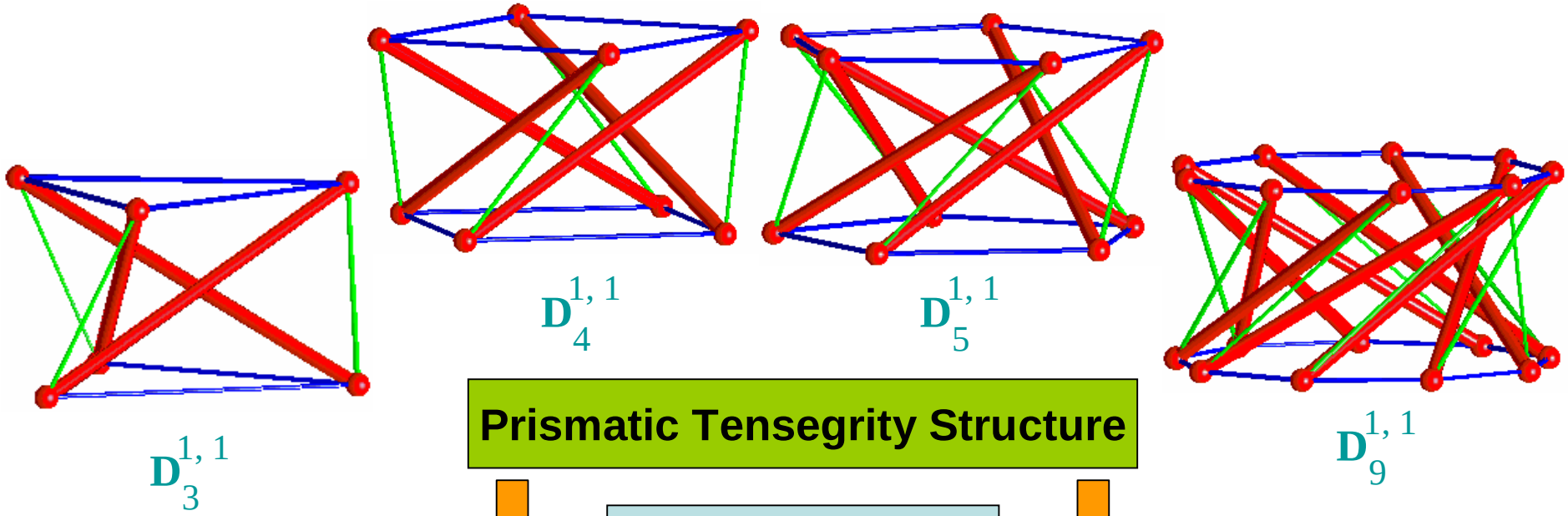
Jingyao Zhang
Simon D. Guest
Makoto Ohsaki







Objective



Prismatic Tensegrity Structure

Dihedral Symmetry

Self-equilibrated Configuration

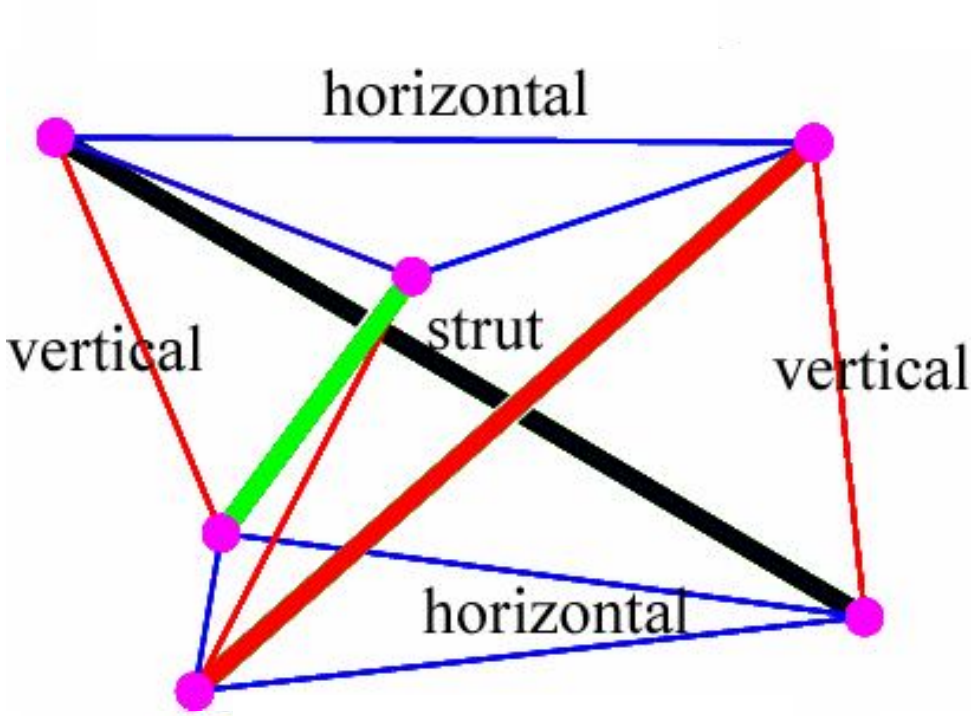
Stability Properties

Connectivity

Connectivity

Configuration

Configuration $\mathbf{D}_n^{h,v}$



6 Nodes

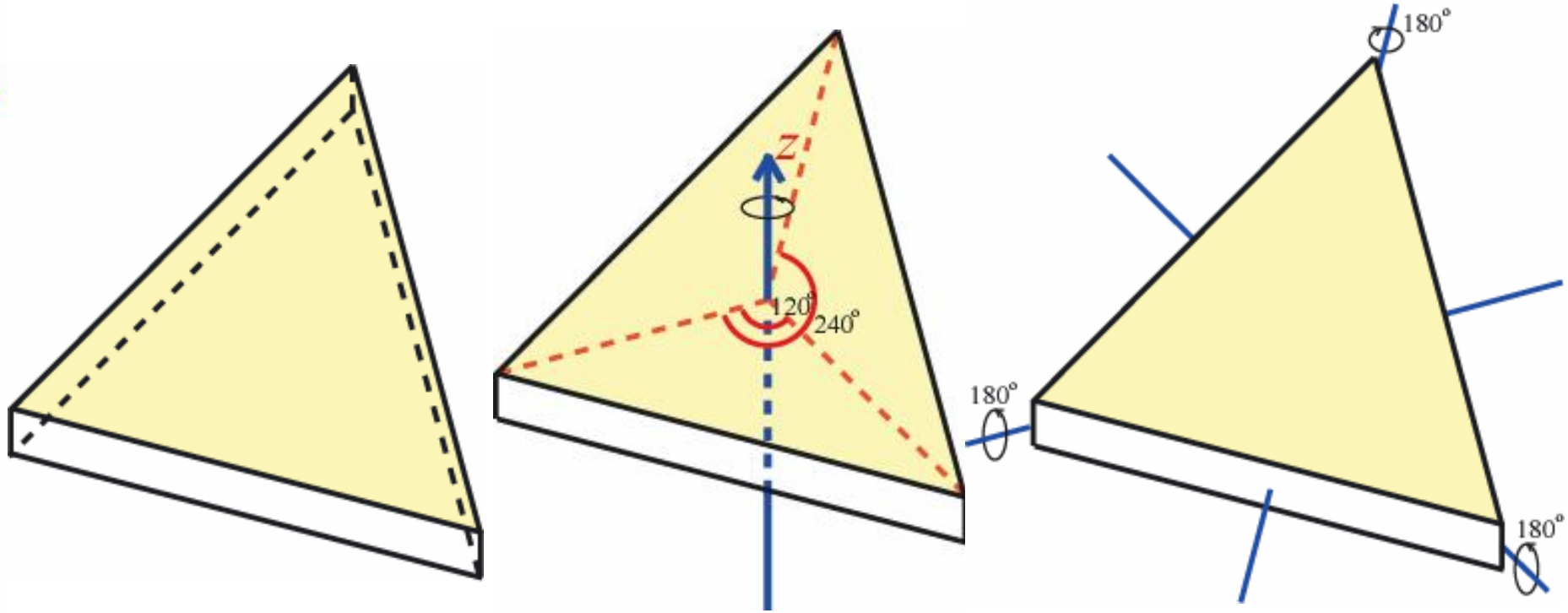
3 Struts

6 Horizontal

3 Vertical

The simplest prismatic tensegrity structure $\mathbf{D}_3^{1,1}$

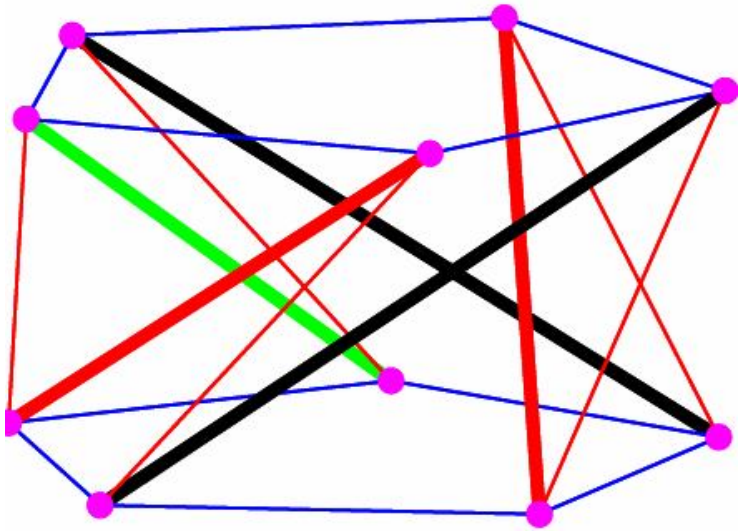
Dihedral Symmetry $\mathbf{D}_{n}^{h,v}$

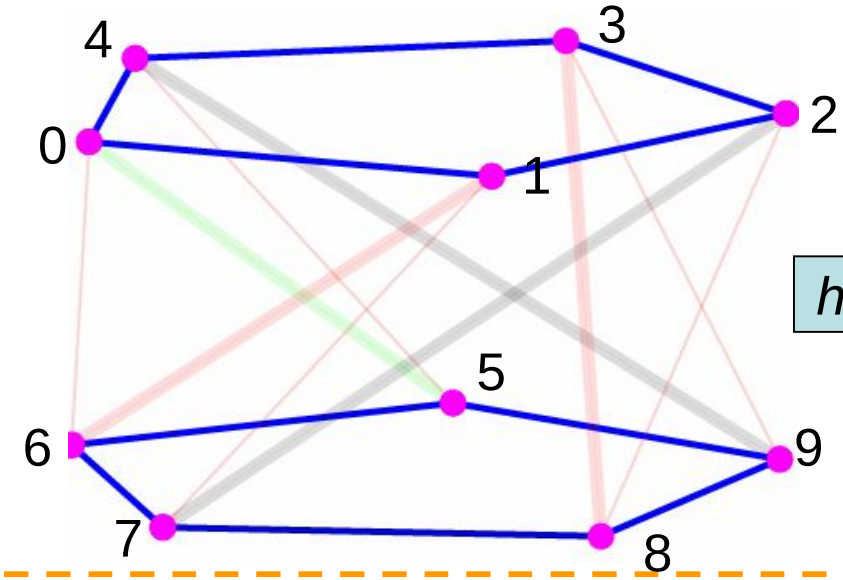


$E\ (C_3^0)$ | $C_3^1,\ C_3^2$ | $C_{21},\ C_{22},\ C_{23}$

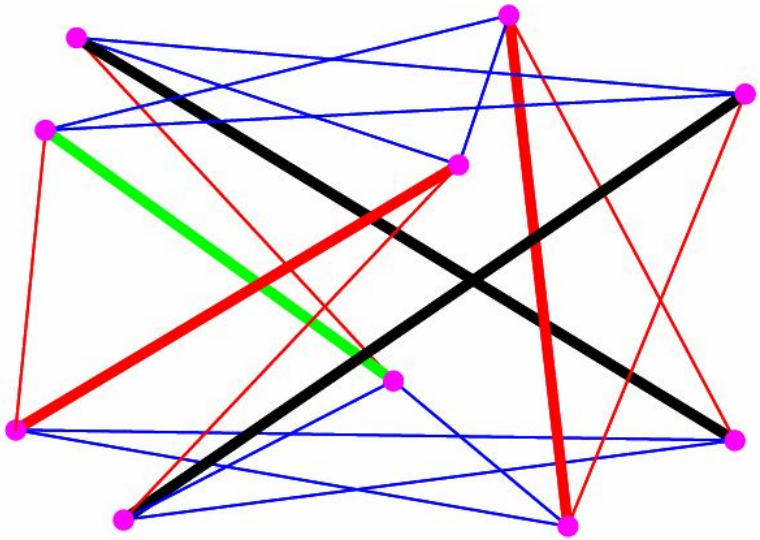
D3 Symmetry

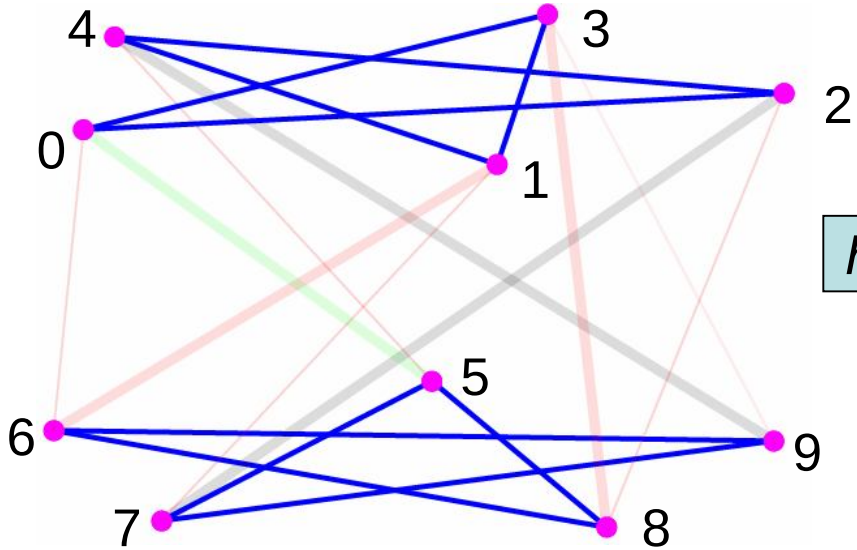
Connectivity ? Horizontal Cables $\mathbf{D}_n^{h,v}$





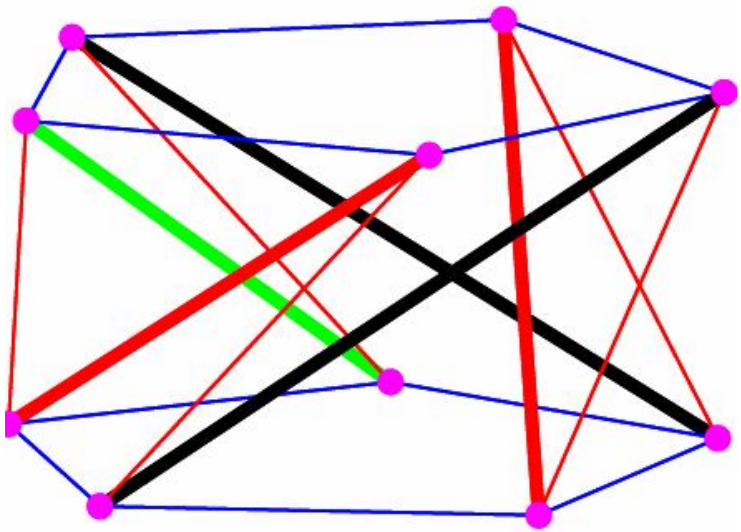
h=1



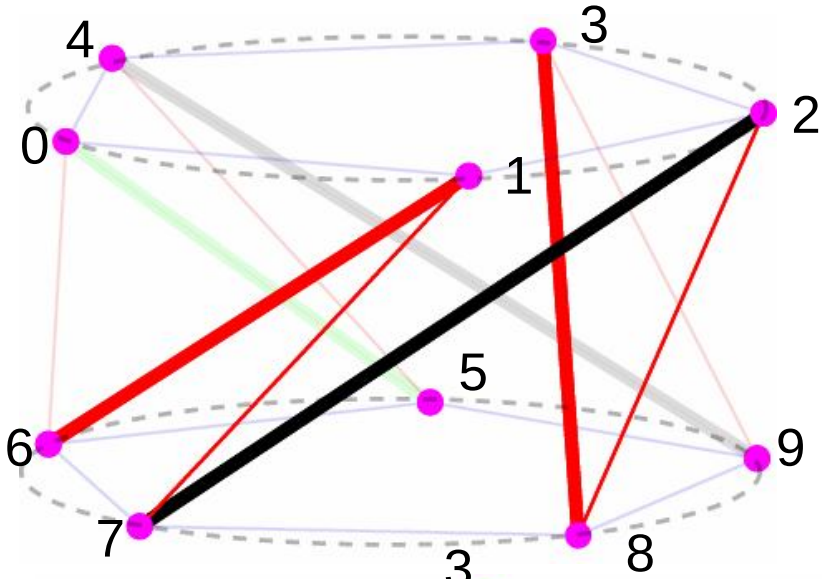


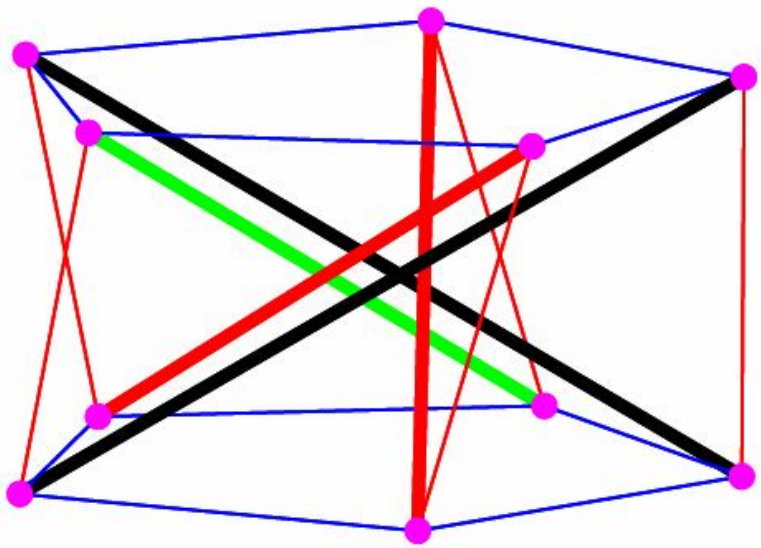
h=2

Connectivity ? Vertical Cables $\mathbf{D}_n^{h,v}$

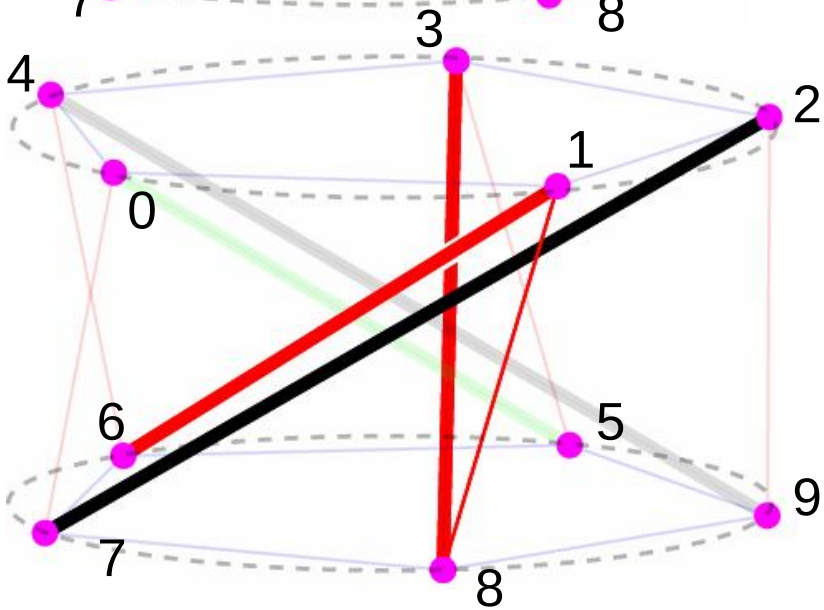


v=1

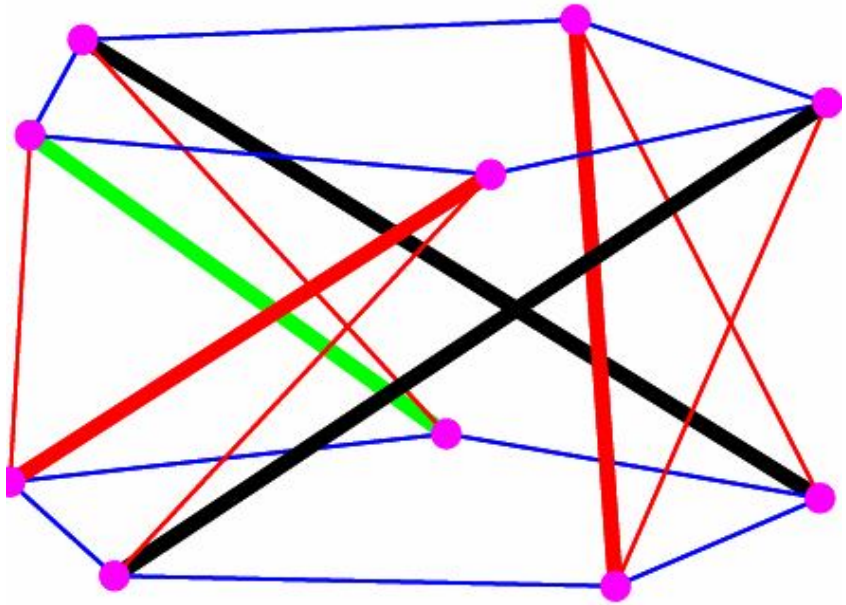


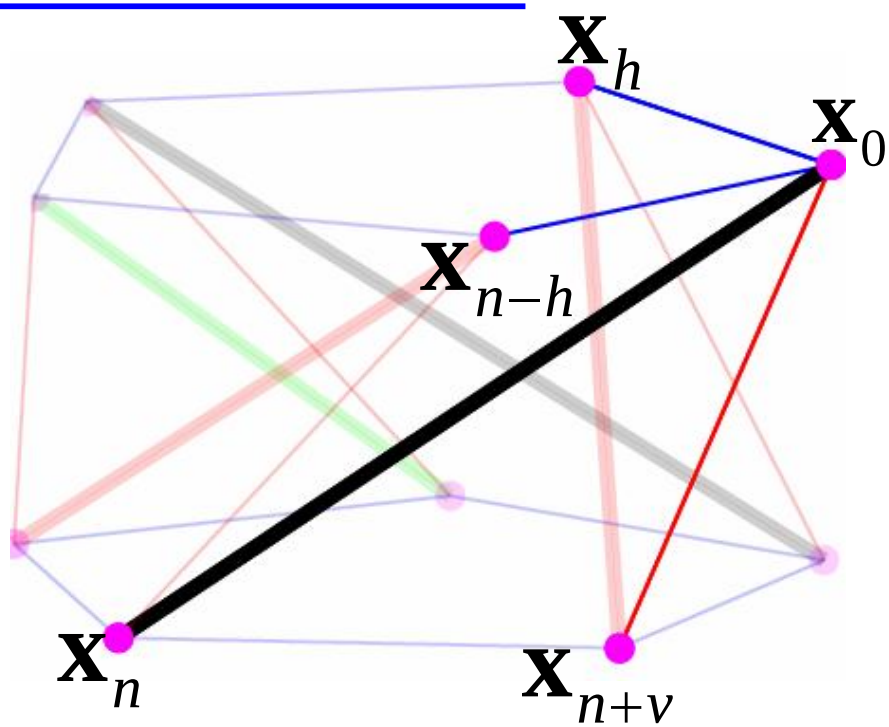


v=2



Self-equilibrium

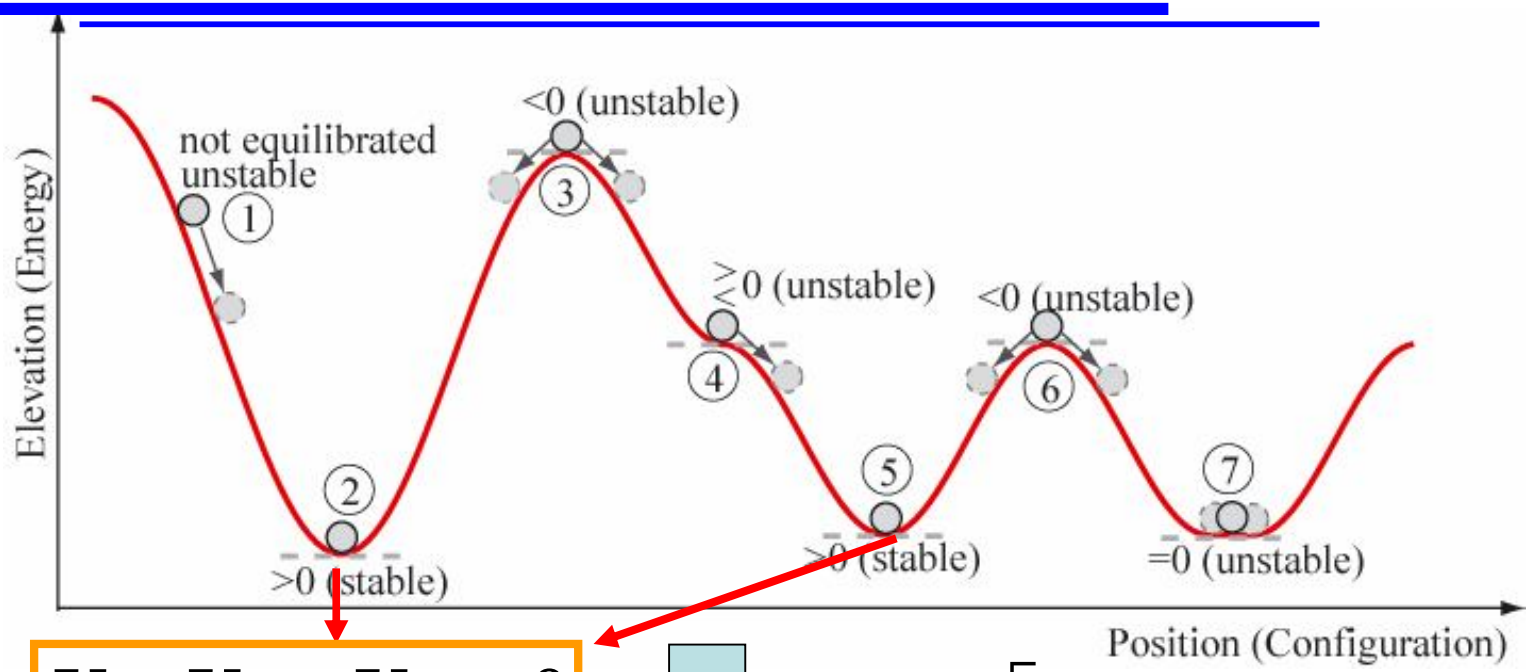




$$\mathbf{A}\mathbf{x}_0 = \mathbf{0} \xrightarrow{\text{A singular}} \begin{cases} q_v = -q_s \\ q_h / q_v = \sqrt{2(1-\cos(2v\pi/n))}\,/(1-\cos(2h\pi/n)) \end{cases}$$

$$\mathbf{x}_0 \xrightarrow{\text{symmetry}} \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_{2n-1}$$

Stability Criterion



$$\mathbf{K} = \mathbf{K}_E + \mathbf{K}_G > 0$$

infinite stiffness

$$\mathbf{K}_E \to 0 \text{ or } \infty$$

$$\mathbf{Q} = \mathbf{M}^{\mathrm{T}}\mathbf{K}_G\mathbf{M} > 0$$

M ? mechanism

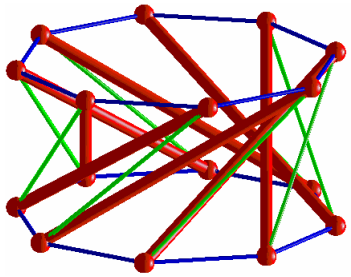
Block Diagonalisation

$$\tilde{\mathbf{Q}} = \begin{bmatrix} > 0 & & & & & \\ & > 0 & & & & \\ & & > 0 & & & \\ & & & > 0 & & \\ & & & & > 0 & \\ & & & & & > 0 \end{bmatrix}$$

Stability

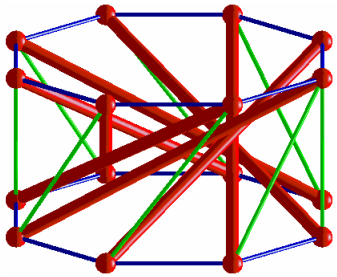
- $h = 1 \Rightarrow \mathbf{K}_G \geq 0 \rightarrow \mathbf{Q} > 0 \Rightarrow$ Stable

$\mathbf{D}_8^{1,1}$



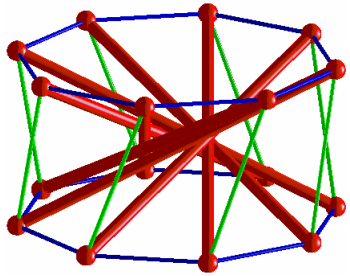
Stable

$\mathbf{D}_8^{1,2}$



Stable

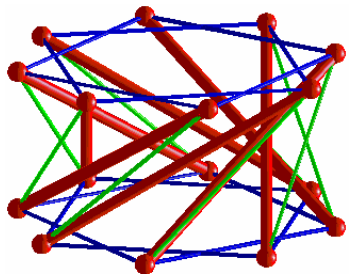
$\mathbf{D}_8^{1,3}$



Stable

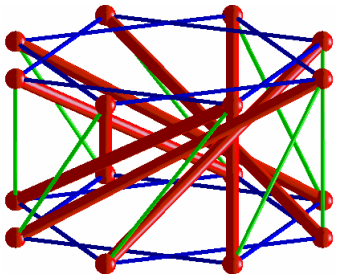
- $h \neq 1 \Rightarrow \mathbf{K}_G \,?\, 0 \rightarrow \mathbf{Q} \,?\, 0 \Rightarrow$?

$\mathbf{D}_8^{2,1}$



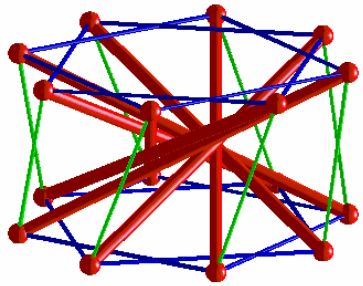
Unstable

$\mathbf{D}_8^{2,2}$



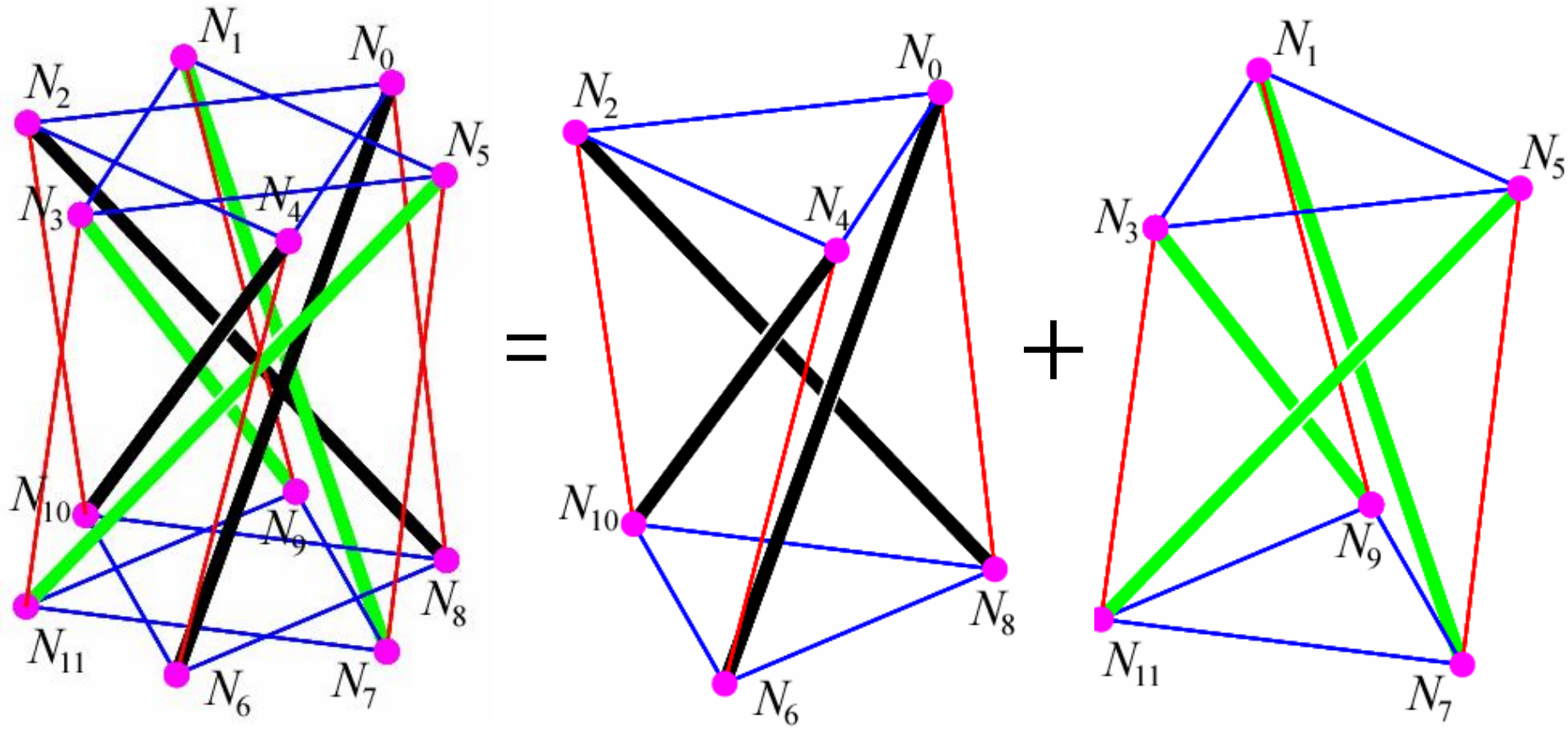
Divisible

$\mathbf{D}_8^{2,3}$



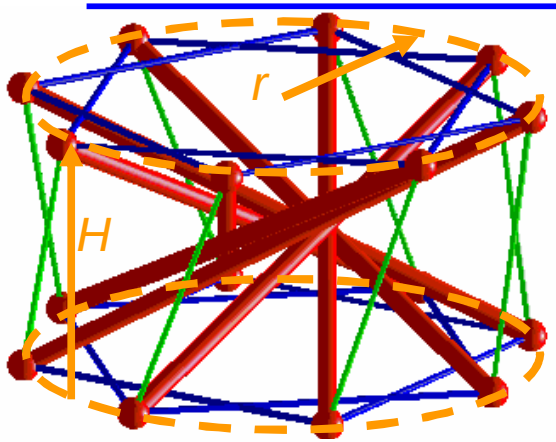
Conditionally Stable

Divisible Structures $\mathbf{D}_6^{2,2}$

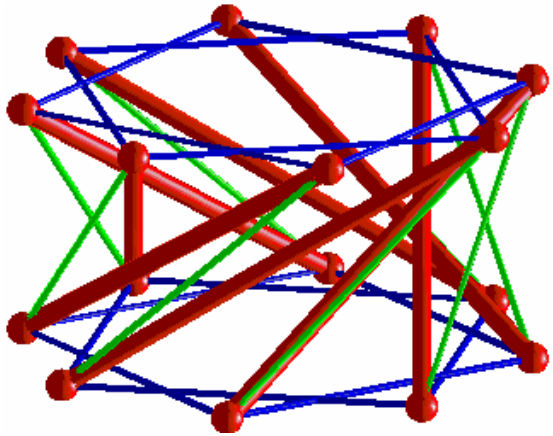


$$\mathbf{D}_6^{2,2} = \mathbf{D}_3^{1,1} + \mathbf{D}_3^{1,1}$$

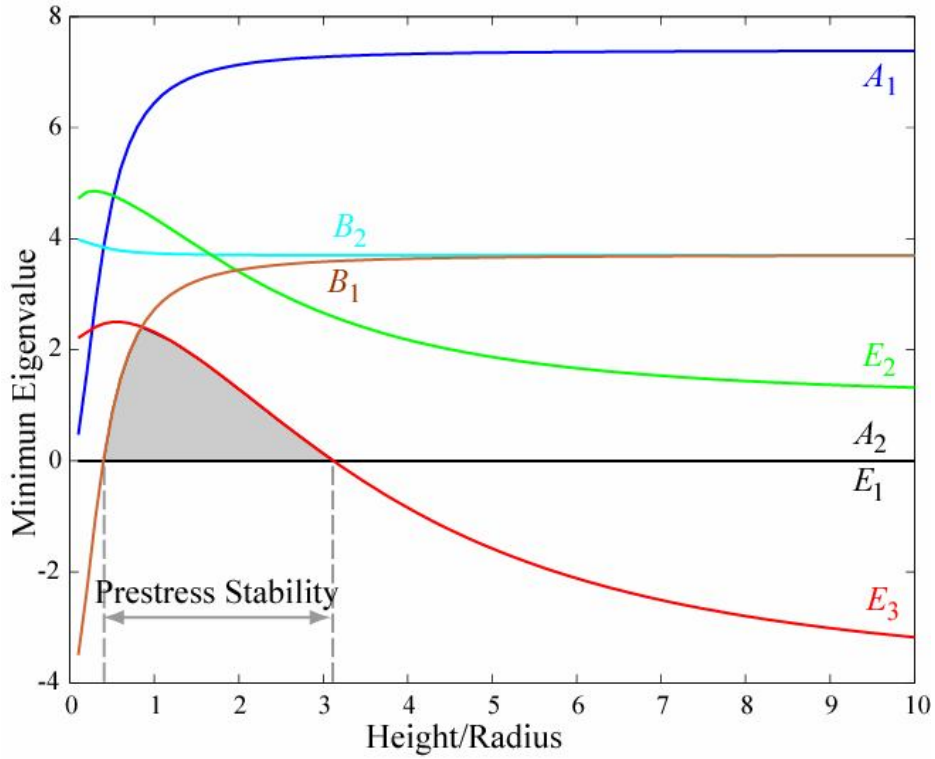
Numerical Investigation

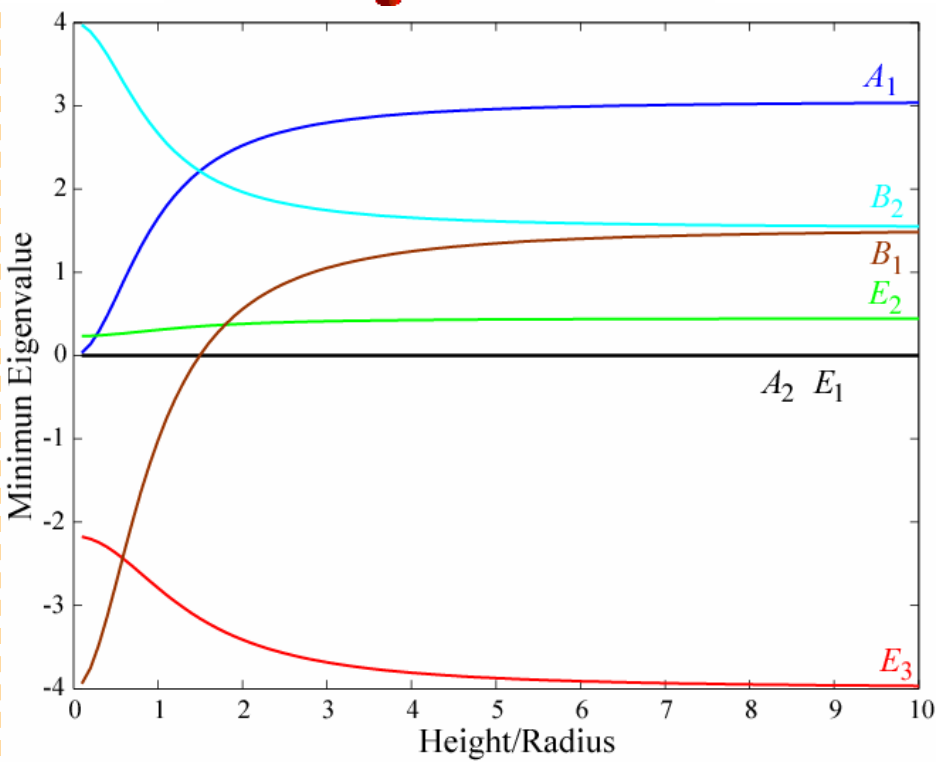


$\mathbf{D}_8^{2,3}$



$\mathbf{D}_8^{2,1}$





Catalogue

$n=3$		h
		1
v	1	s

$n=4$		h	
		1	2
v	1	s	u
	2	s	$2D_2^{1,1}$

$n=5$		h	
		1	2
v	1	s	u
	2	s	u

$n=6$		h		
		1	2	3
v	1	s	u	u
	2	s	$2D_3^{1,1}$	u
	3	s	p	$3D_2^{1,1}$

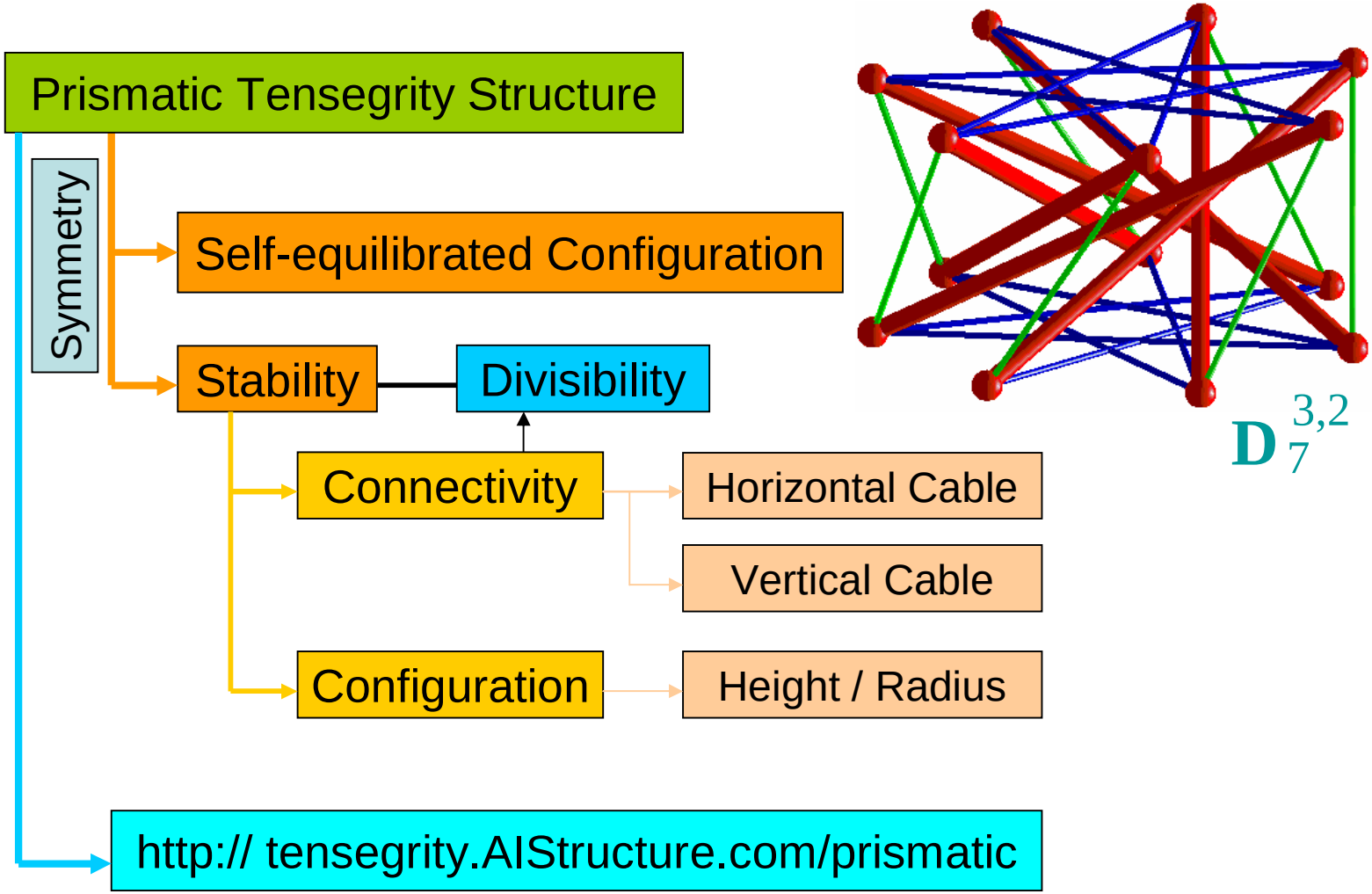
$n=7$		h		
		1	2	3
v	1	s	u	u
	2	s	u	[0.75,1.05]
	3	s	u	u

$n=8$		h			
		1	2	3	4
v	1	s	u	u	u
	2	s	$2D_4^{1,1}$	u	$2D_4^{2,1}$
	3	s	[0.40,3.10]	u	u
	4	s	$2D_4^{1,2}$	[0.35,2.35]	$4D_2^{1,1}$

$n=9$		h			
		1	2	3	4
v	1	s	u	u	u
	2	s	u	u	u
	3	s	u	$3D_3^{1,1}$	u
	4	s	u	[0.20,1.60]	u

Please note in the paper that there are some mistakes on *n*, *h* and *v*.

Summary



$D_7^{3,2}$