3-dimensional elastic beam model for large-deformation analysis of bending-active gridshells

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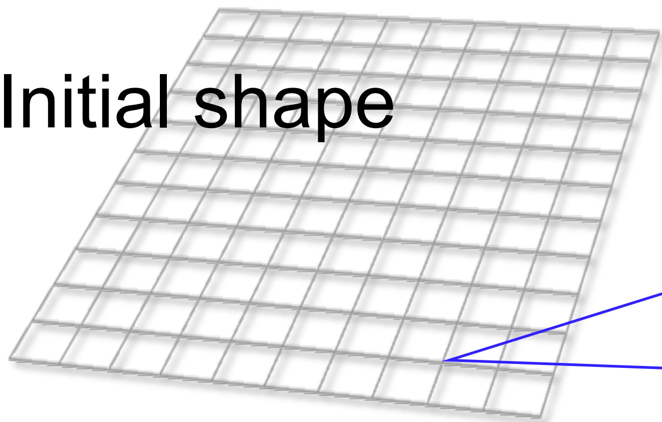
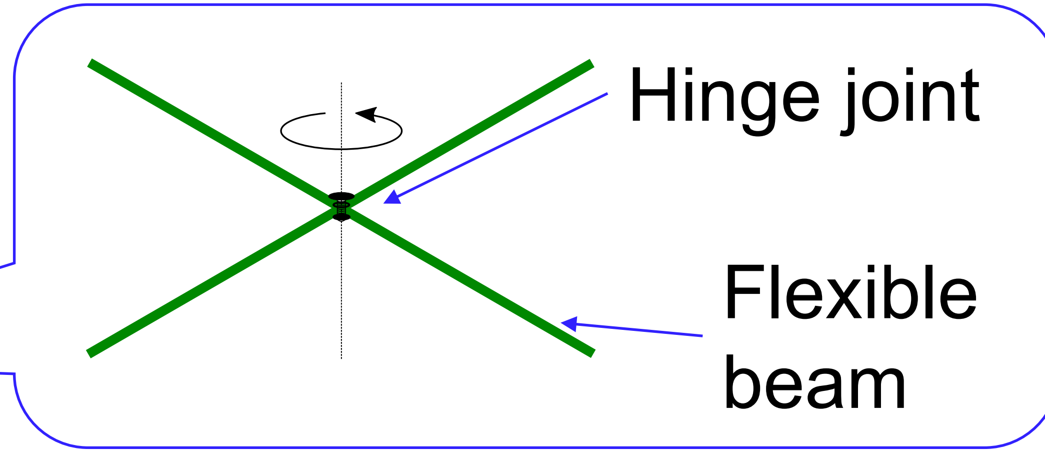
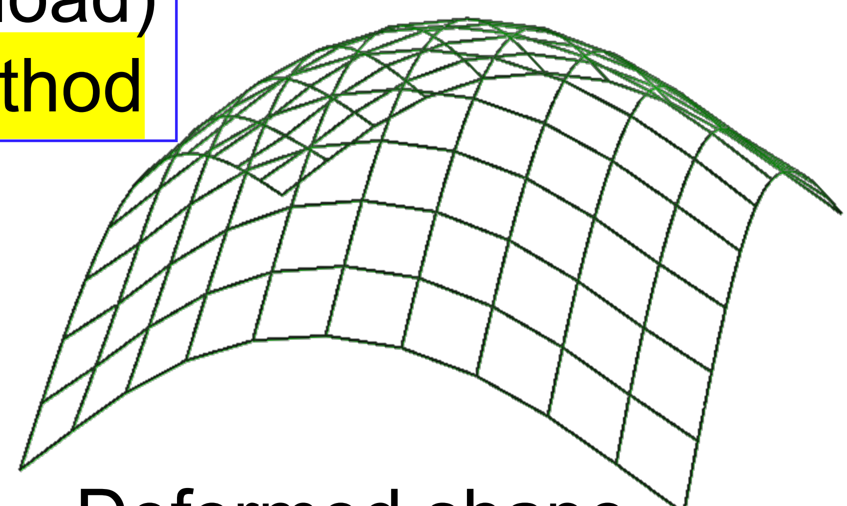
Bending-Active Gridshell





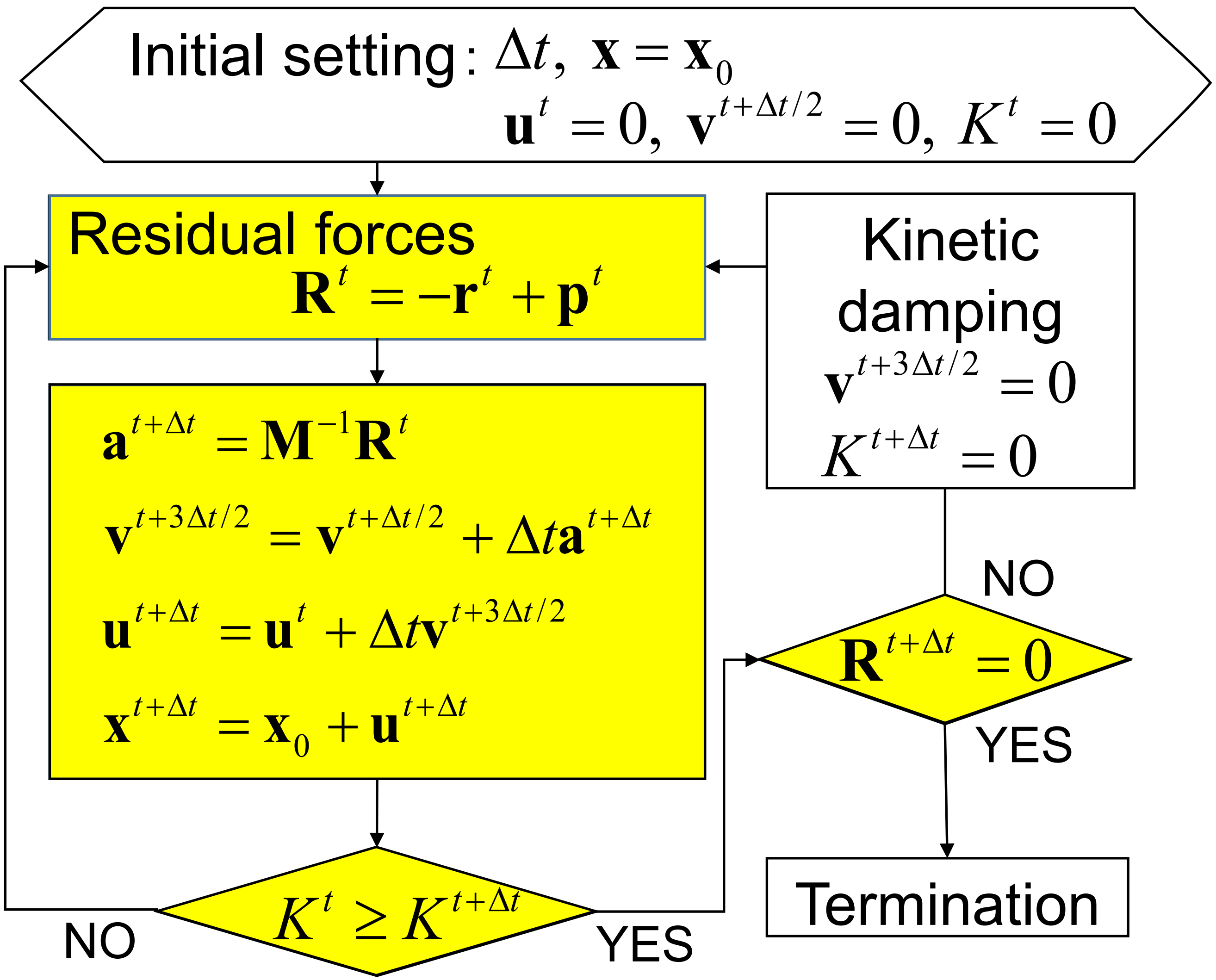
Large-deformation analysis
(Forced displacement, load)
e.g.) Dynamic Relaxation method



Deformed shape

Dynamic Relaxation method

for dynamically solving static problems



Scalar

K : Kinetic energy

Vector

$\mathbf{x}$: position

$\mathbf{u}$: displacement

$\mathbf{v}$: velocity

$\mathbf{a}$: acceleration

$\mathbf{r}$: internal forces

$\mathbf{p}$: external forces

Matrix

$\mathbf{M}$: mass

Our goal:

Proposing 3-dimensional elastic beam model with 6 degrees of freedom

→ Simple tool for form-finding of flexible beams

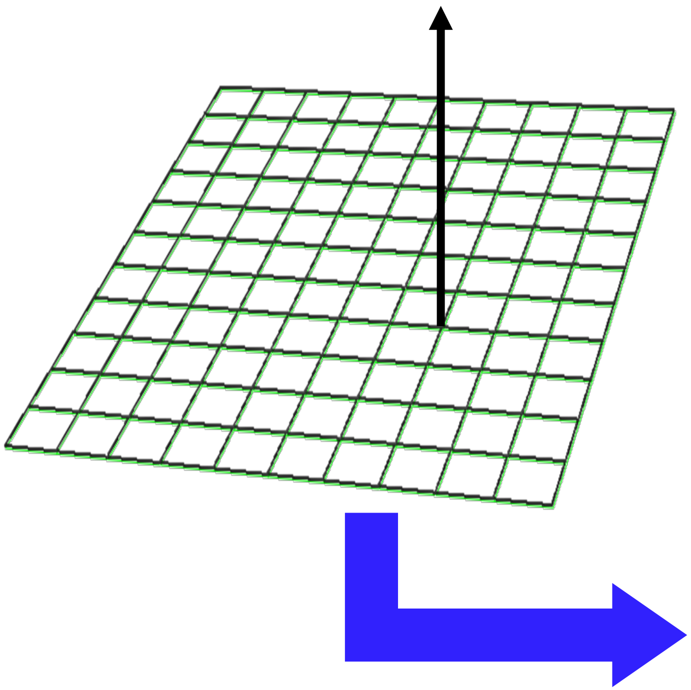
What's the feature?

Energy-based formulation & DRM algorithm → No tangent stiffness matrix

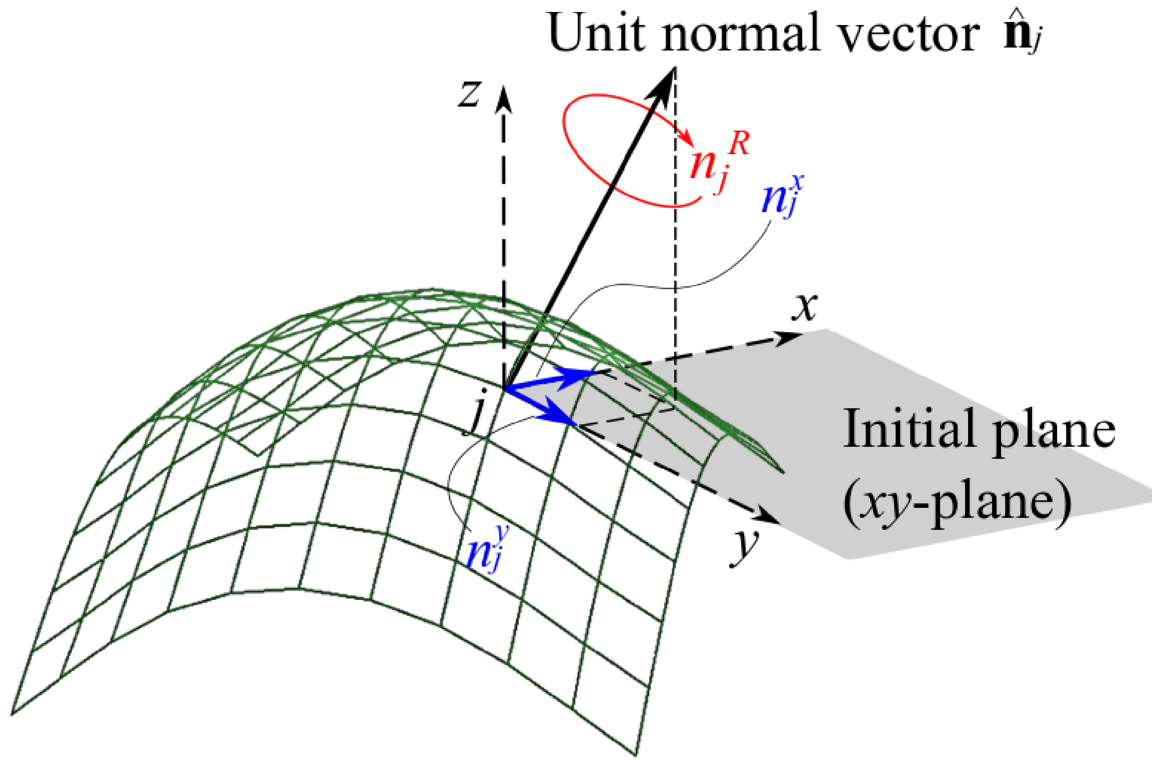
Unit normal vector → No large-rotation formulation

→ No need sophisticated software packages

3D elastic beam model using Unit normal vector



Perpendicular to tangent plane of a curved surface



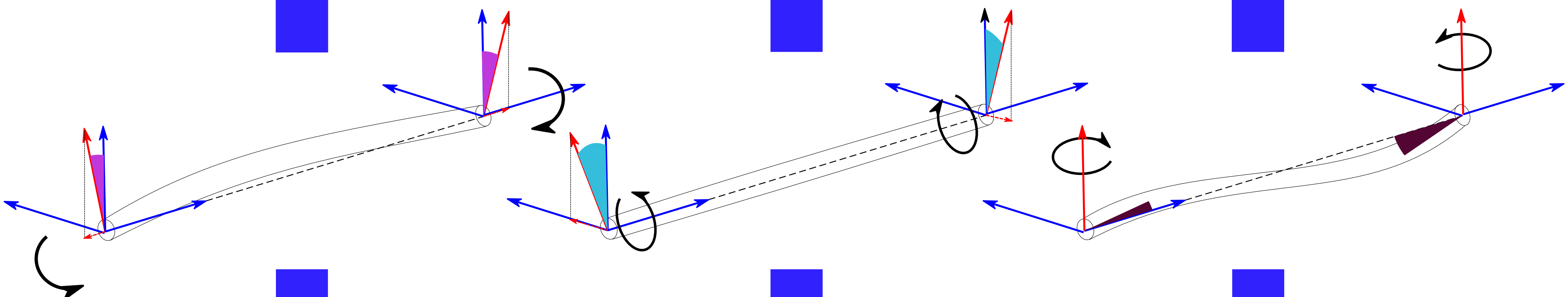
Describe the deformation of a beam element by using the unit normal vector at deformed state

Six variables at each end node of a beam element

$U_j^x \quad U_j^y \quad U_j^z$ Translational displacements

+

$n_j^x \quad n_j^y \quad n_j^R$

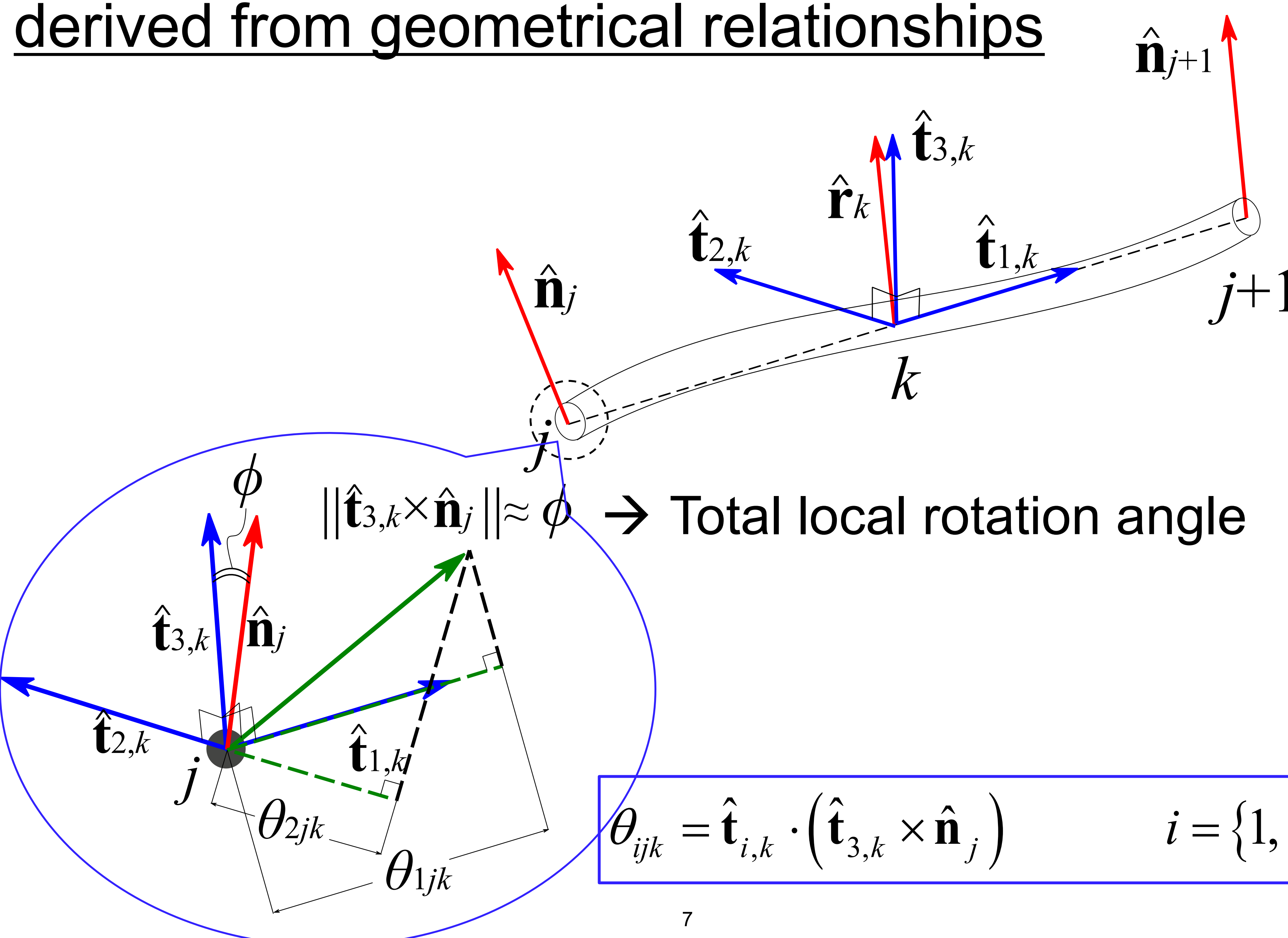


Two inclinations of the vector + Rotation around the vector

Out-of-plane Bending Torsion In- plane Bending

Unknown

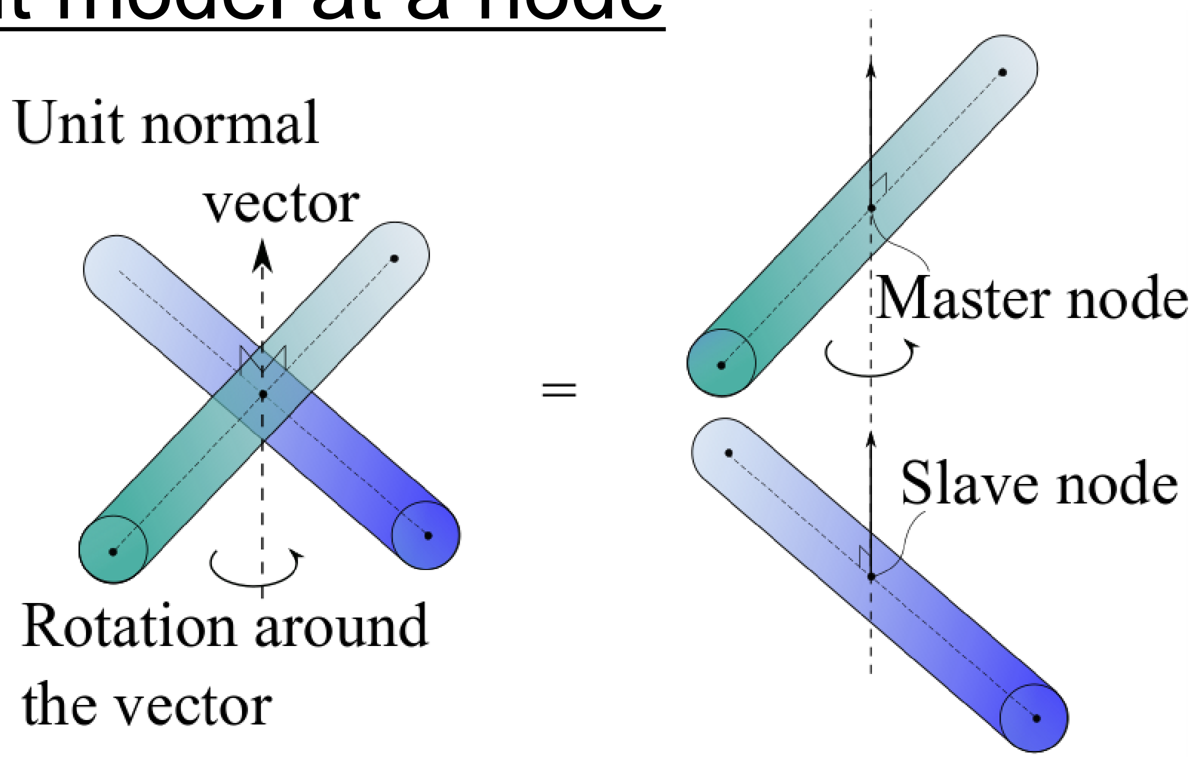
The other two rotational displacements derived from geometrical relationships



$\|\hat{\mathbf{t}}_{3,k} \times \hat{\mathbf{n}}_j\| \approx \phi$ → Total local rotation angle

$$\theta_{ijk} = \hat{\mathbf{t}}_{i,k} \cdot \left(\hat{\mathbf{t}}_{3,k} \times \hat{\mathbf{n}}_j \right) \qquad i = \{1,\ 2\}$$

Hinge joint model at a node



							DOF
Master node:	U^x_{mst}	U^y_{mst}	U^z_{mst}	n^x_{mst}	n^y_{mst}	n^R_{mst}	→ 6
Slave node:	U^x_{mst}	U^y_{mst}	U^z_{mst}	n^x_{mst}	n^y_{mst}	n^R_{slv}	→ 1

Total potential energy

$$\Pi_{\text{total}} = \sum_{k=1}^{m} \Pi_{\text{int}}^{k} - \sum_{j=1}^{s} \Pi_{\text{ext}}^{j}$$

$$\Pi_{\text{int}}^{k} = \frac{EA}{2\bar{L}_k} e_k^{\,2} + \frac{GJ}{2\bar{L}_k}\left(\theta_{1(j+1)k} - \theta_{1jk}\right)^2$$
$$+ \frac{EI_2}{2\bar{L}_k}\left[4\left(\theta_{2jk}\right)^2 + 4\theta_{2jk}\theta_{2(j+1)k} + 4\left(\theta_{2(j+1)k}\right)^2\right]$$
$$+ \frac{EI_3}{2\bar{L}_k}\left[4\left(\theta_{3jk}\right)^2 + 4\theta_{3jk}\theta_{3(j+1)k} + 4\left(\theta_{3(j+1)k}\right)^2\right]$$

$$\Pi_{\text{ext}}^{j} = \mathbf{P}_j \cdot \begin{bmatrix} U_j^x \\ U_j^y \\ U_j^z \end{bmatrix}$$

EA Axial stiffness

EI_i Bending stiffness

GJ Torsional stiffness

$\bar{L}_k$ Initial length

U_j Translational disp.

θ_{ijk} Rotational disp. around i at node j of element k

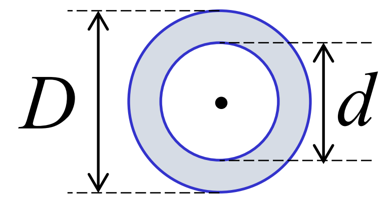
u_i Variables

Residual force: Sensitivity of Π_{total}

$$R_i^t = -r_i^t + p_i = -\sum_{k=1}^{m} \frac{\partial \Pi_{\text{int}}^{k}}{\partial u_i} + \sum_{k=1}^{s} \frac{\partial \Pi_{\text{ext}}^{k}}{\partial u_i}$$

Example (material properties and methods)

$E = 200$ GPa, D=0.03 m, d=0.026 m



Implement the proposed model

1. Dynamic Relaxation method (DRM)

2. Optimization (OPT) → Minimize total potential energy (quasi Newton method)

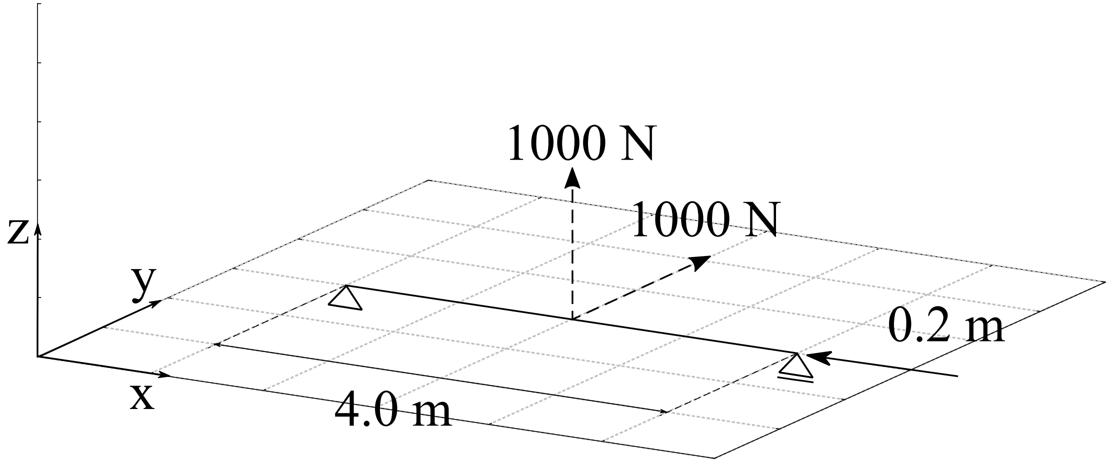
3. Finite Element method (FEM)

$0.0 \leq t_{\mathrm{FEM}} \leq 1.0$ Upward virtual load equal to self-weight

$1.0 \leq t_{\mathrm{FEM}} \leq 2.0$ Forced disp. and External loads

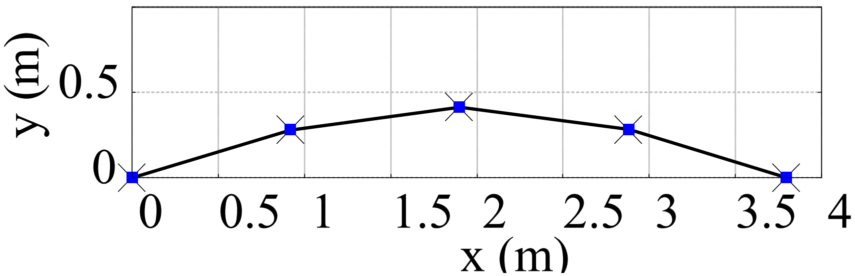
Example 1 : Simple beam model

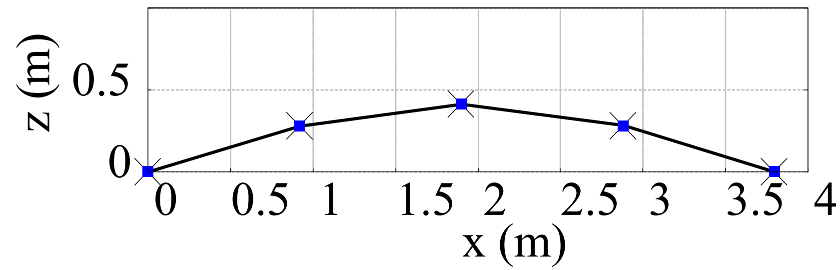
DRM
OPT
FEM

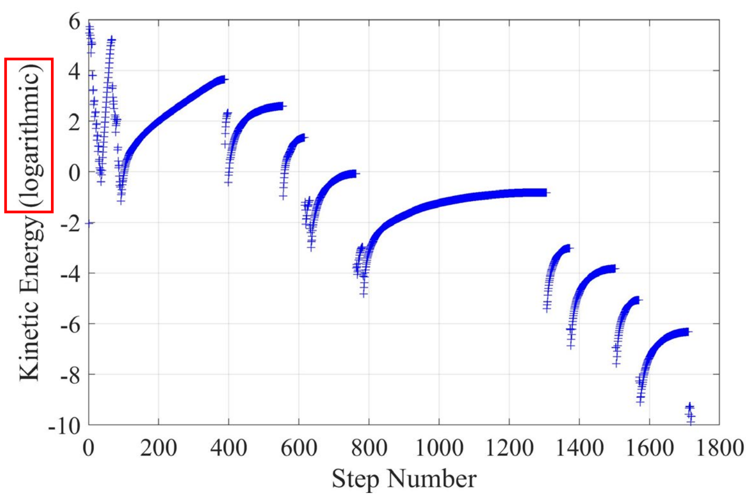


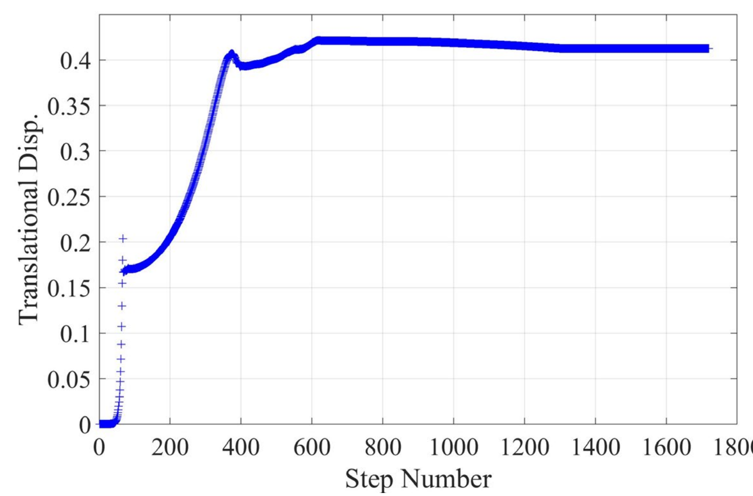
Number of steps

DRM	1720
OPT	860

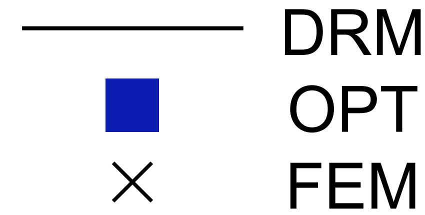






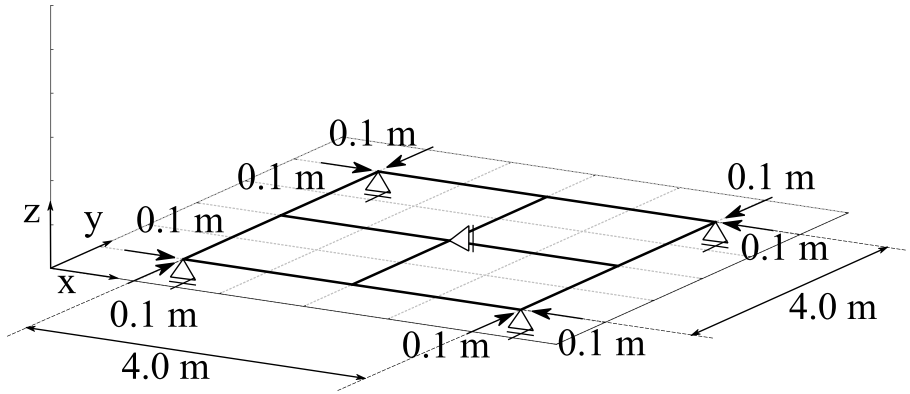


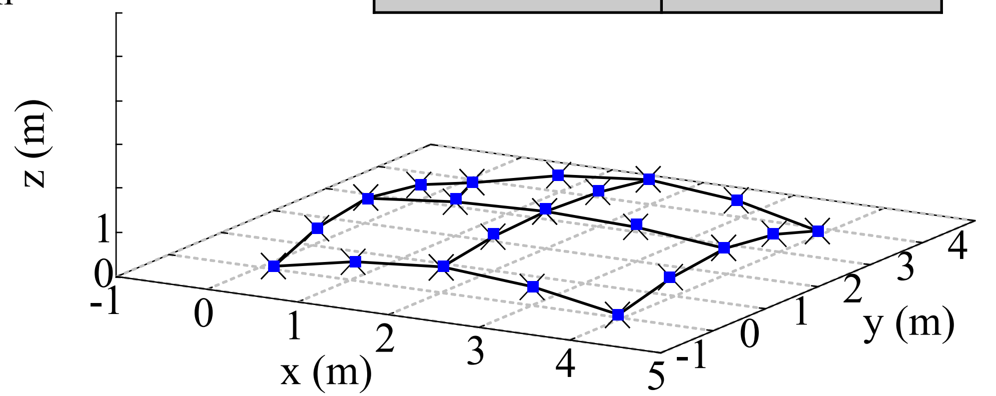
Example 2: Short span gridshell

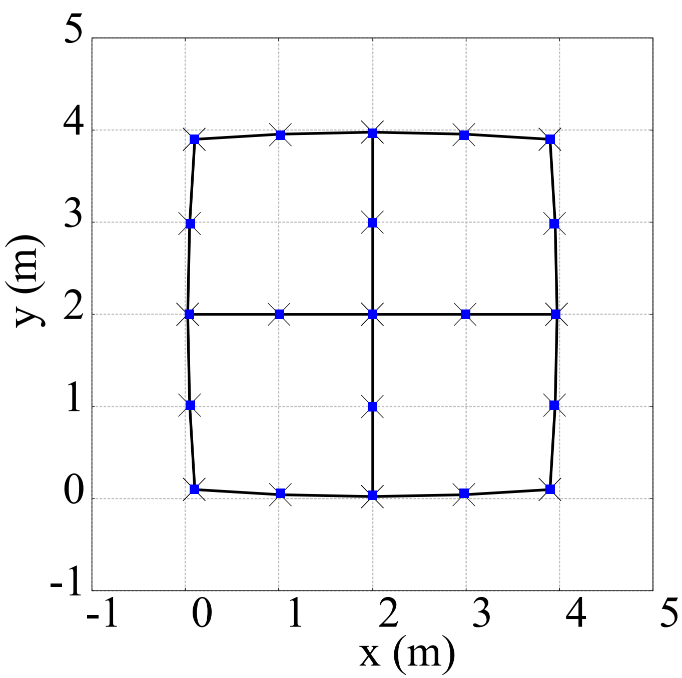


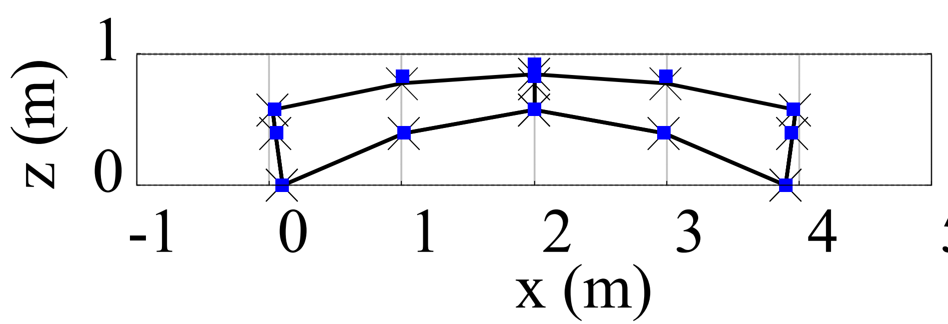
Number of steps

DRM	3410
OPT	1845

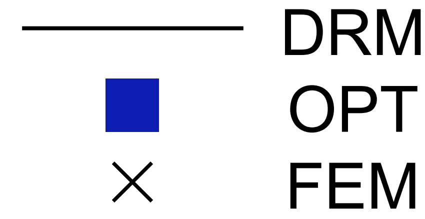






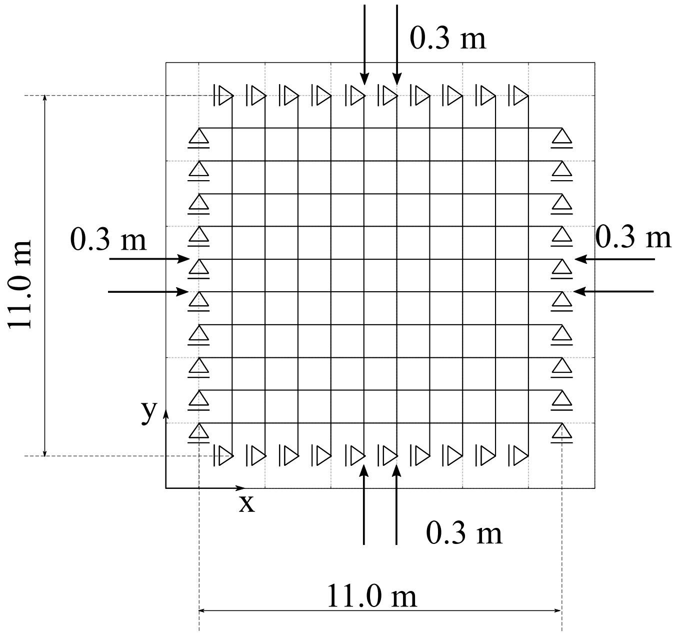


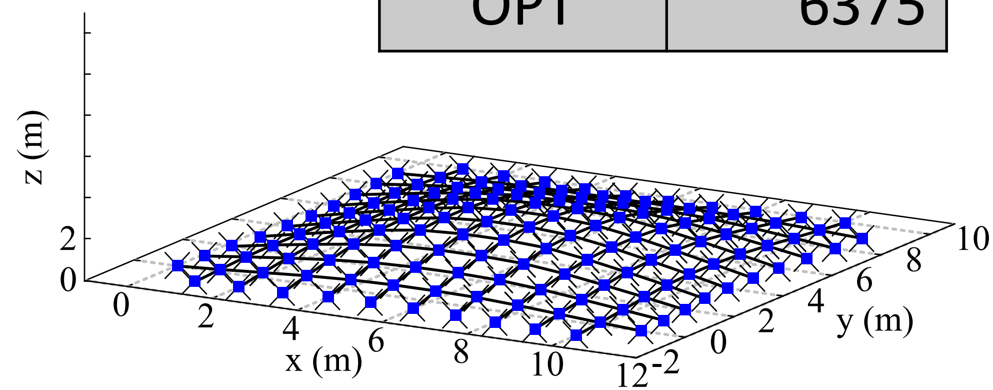
Example 3: Large span gridshell

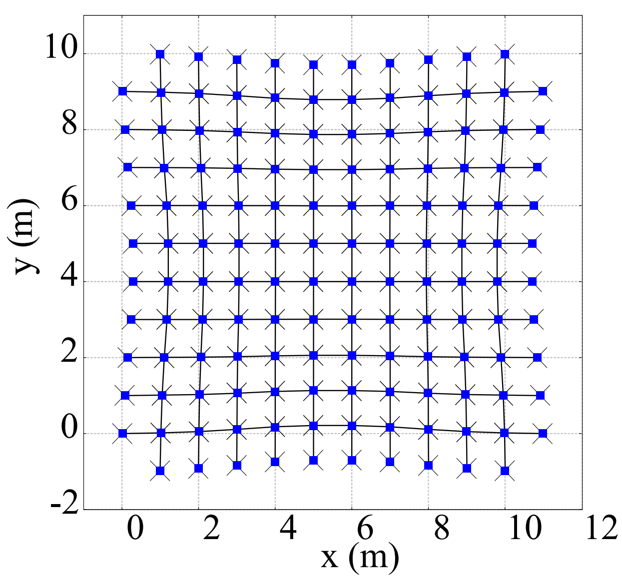


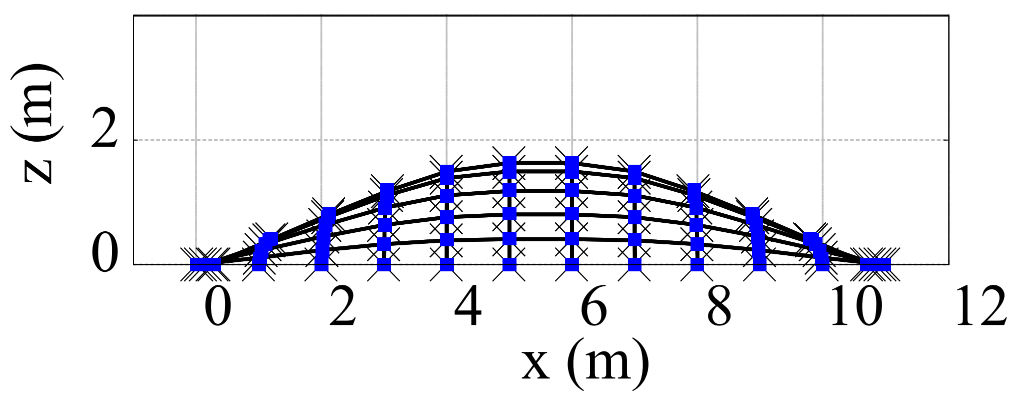
Number of steps

DRM	8449
OPT	6375









Conclusions :

3-dimensional elastic beam model

1. Energy-based formulation + DRM algorithm
   >> No need tangent stiffness matrix

2. Unit normal vector at each end node
   >> No need large-rotation formulation

3. Proposed model + DRM algorithm
   >> can carry on large-deformation analysis of bending-active gridshells
   without sophisticated software packages