An optimization method for generating self-equilibrium shape of curved surface from developable surface

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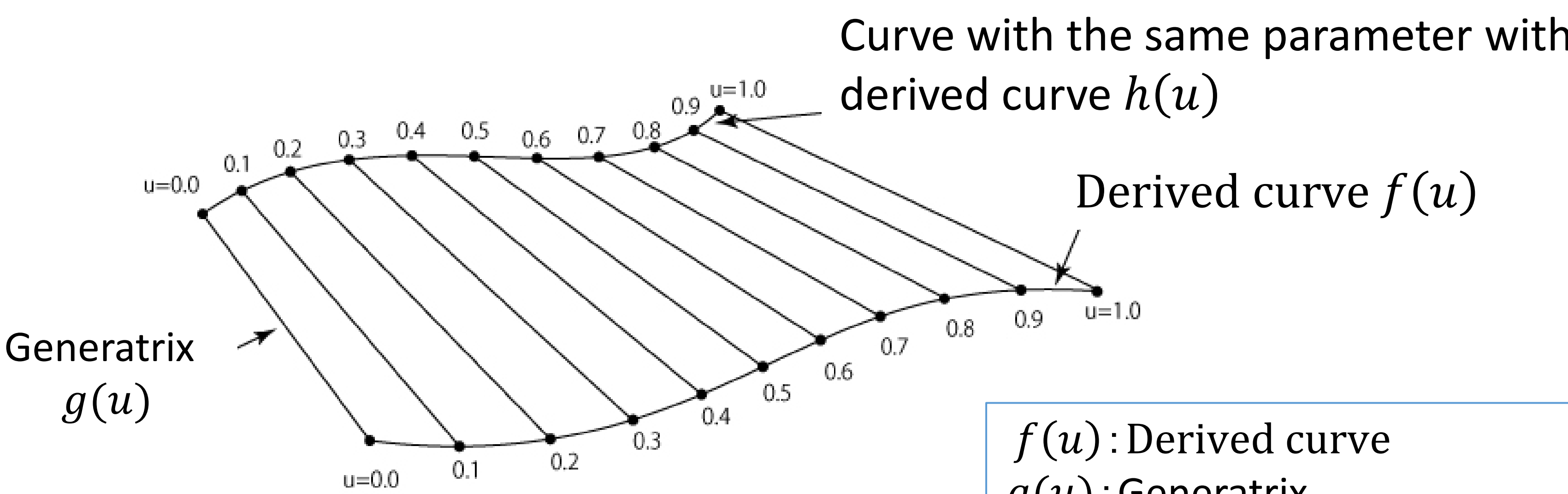
✓ Target surface

① Ruled surface

$$\mathbf{s}(u,v) = \mathbf{f}(u) + v\mathbf{g}(u)$$

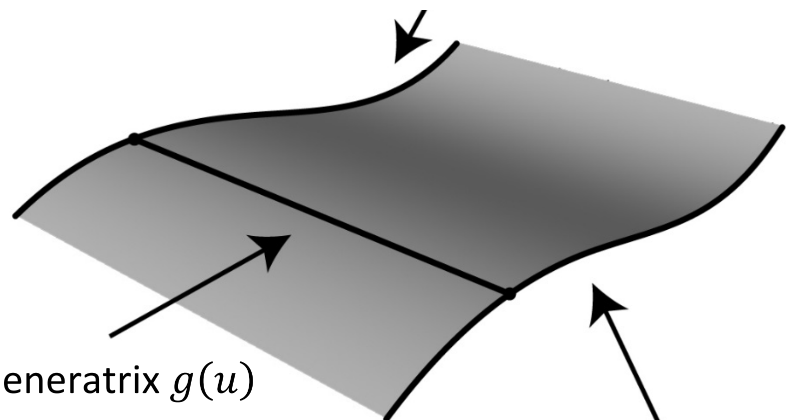
$$\mathbf{s}(u,v) = (1-v)\mathbf{f}(u) + v\mathbf{h}(u)$$



$f(u)$: Derived curve
$g(u)$: Generatrix
$h(u)$: Curve with the same parameter with derived curve
u, v : Scalar parameter

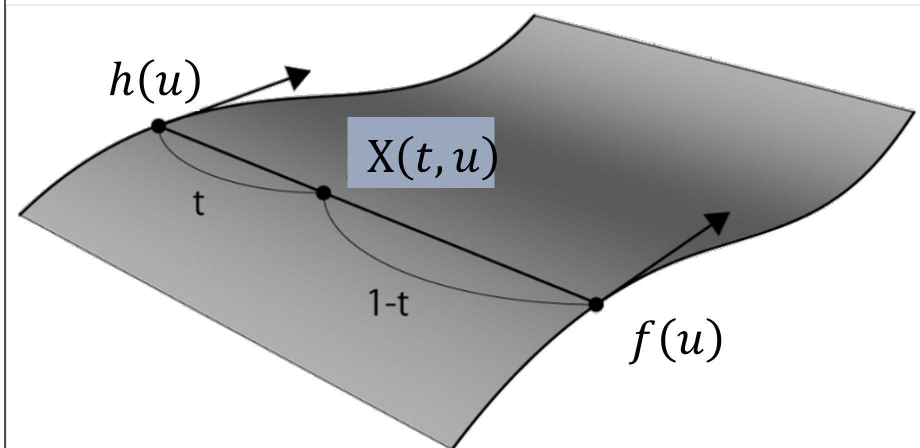
② Bézier curve and developability condition

Define $f(u)$、$h(u)$ with Bezier curves



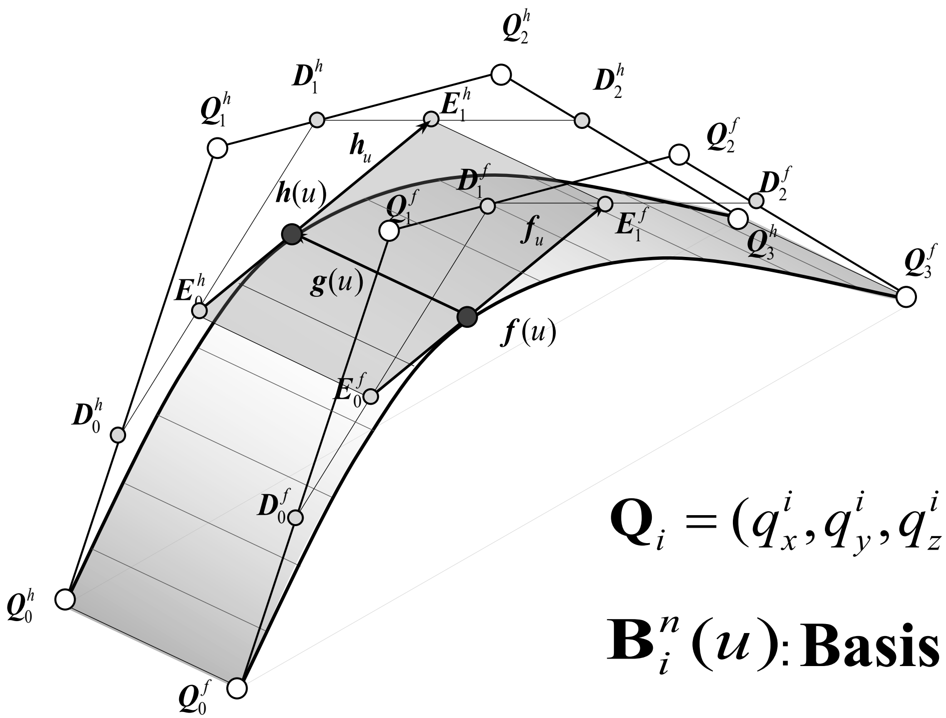
Various curved surface shapes can be effectively described with few control points

$$\det(\mathbf{f}_u, \mathbf{h} - \mathbf{f}, \mathbf{h}_u) = 0$$



Can be expanded on a plane surface

③ Representation of the developable surface by Bézier curve



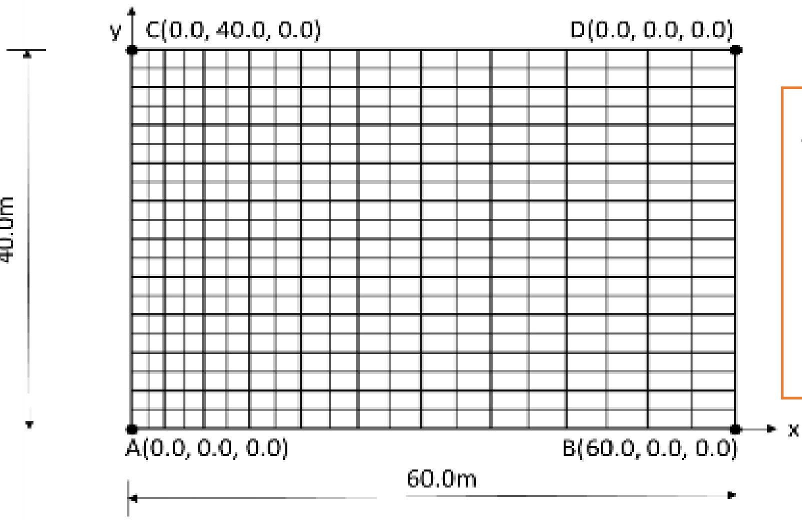
$$\mathbf{P}^n(u) = \sum_{i=0}^{n} \mathbf{Q}_i \mathbf{B}_i^n(u)$$

$\mathbf{Q}_i = (q_x^i, q_y^i, q_z^i)^{\mathrm{T}}$: **Fixed point of Bezier curve**

$\mathbf{B}_i^n(u)$: **Basis function**

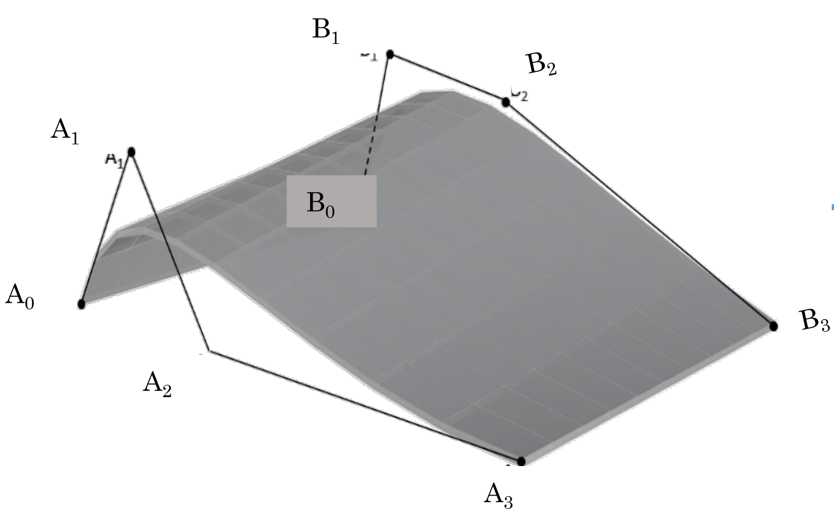
$$\mathbf{s}(u,v) = (1-v)\sum_{i=0}^{n} \mathbf{Q}_i^f \mathbf{B}_i^n(u) + v\sum_{i=0}^{n} \mathbf{Q}_i^h \mathbf{B}_i^n(u)$$

④Developable surface

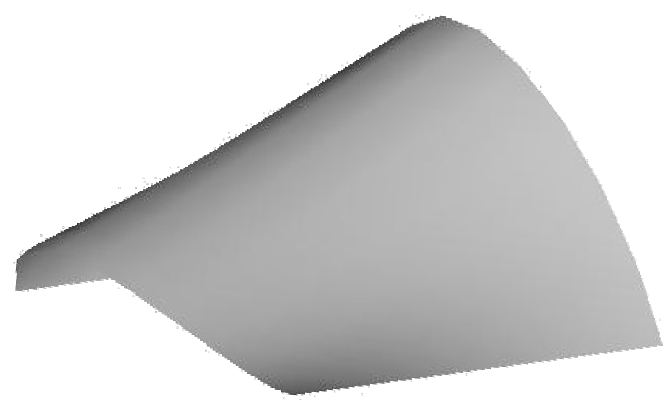


Plan view of initial shape

Jinglan CUI , Makoto OHSAKI and Keigo NAKAMURA , Shape Optimization of Free-Form Surface Shells Consisting of developable Surfaces, Journal of Structural and Construction Engineering , Vol. 82, No. 737, pp. 1137-1143, 2017.



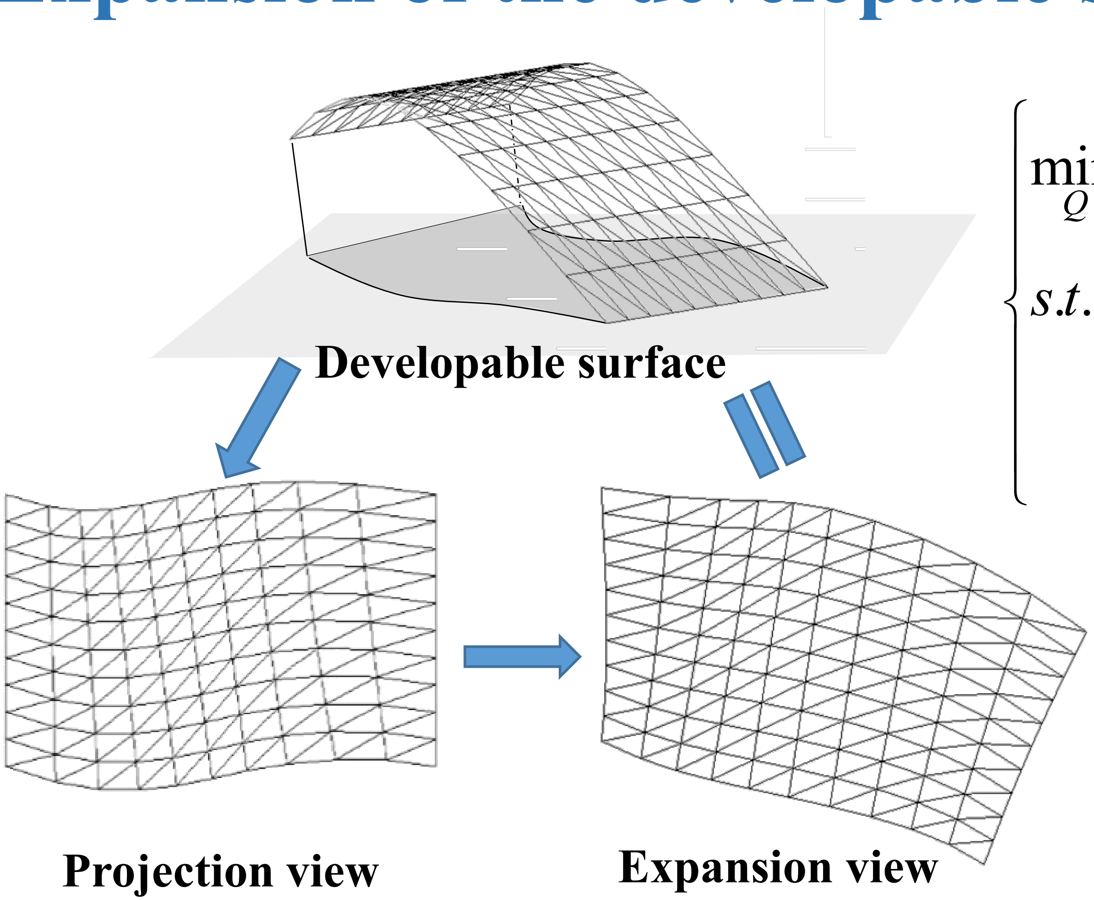
Oblique projection of initial shape



Developable surface

✓ Plane projection

Expansion of the developable surface to the plane



$$\begin{cases} \min\limits_{Q} \ F(P) = \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{3} \omega_{ij} \cdot (l_{ij} - l_{ij}^{0})^2 \\ s.t. \quad x_k^L \le x_k \le x_k^U \\ \qquad\ \ y_k^L \le y_k \le y_k^U \qquad (k = 1, 2, \cdots m) \end{cases}$$

$P(x, y)$: **Nodes on the plane**

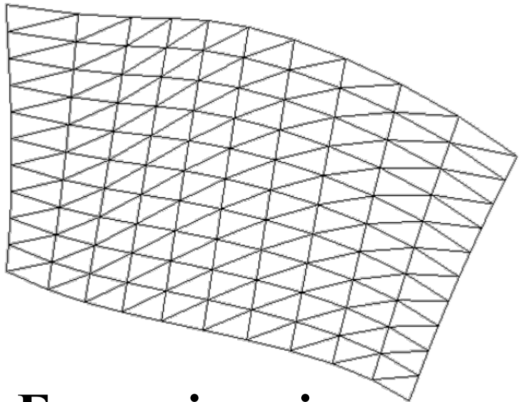
ω_{ij} : **Weight coefficient**

(x_k^L, y_k^L) : **Lower bounds**

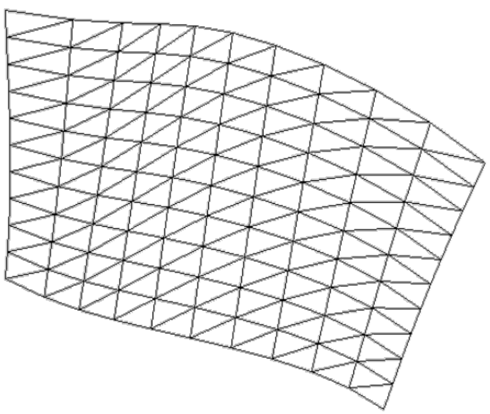
(x_k^U, y_k^U) : **Upper bounds**

l_{ij}^0 : **Edge lengths on the developable surface** l_{ij} : **Edge lengths on the cutting pattern**

✓ Cutting pattern



Expansion view



Cutting pattern view

Desired uniform stress

$$\boldsymbol{\sigma}_0 = \left\{ \sigma_{\mathrm{x}}^{e}, \sigma_{\mathrm{y}}^{e}, 0 \right\}$$

$$\sigma = D\varepsilon$$

$$\boldsymbol{\varepsilon}_0 = \left\{ \varepsilon_{x}^{e}, \varepsilon_{y}^{e}, 0 \right\}$$

$$\boldsymbol{x}_p = \boldsymbol{x}(1-\varepsilon_x)$$
$$\boldsymbol{y}_p = \boldsymbol{y}(1-\varepsilon_y)$$

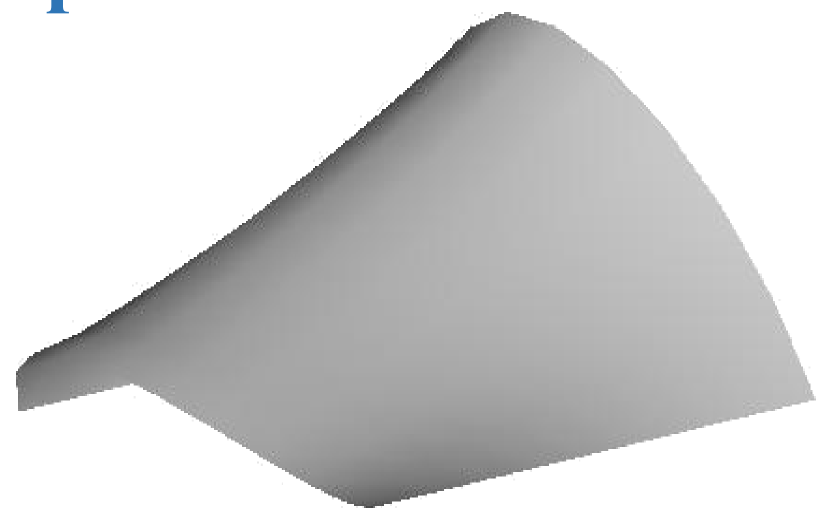
(x, y) **: Expansion nodes**

(x_p, y_p)**: Cutting pattern nodes**

✓ Equilibrium surface

① An optimization problem

$$\begin{cases} \min\limits_{Q} \; E(\mathrm{H}) = \dfrac{1}{2}\displaystyle\int_{v} \left(\boldsymbol{\varepsilon}^{T} \boldsymbol{D}\boldsymbol{\varepsilon}\right) dV \\ s.t. \quad \mathrm{X}_k^L \le \mathrm{X}_k \le \mathrm{X}_k^U \\ \qquad \mathrm{Y}_k^L \le \mathrm{Y}_k \le \mathrm{Y}_k^U \\ \qquad \mathrm{Z}_k^L \le \mathrm{Z}_k \le \mathrm{Z}_k^U \qquad (k = 1, 2, \cdots m) \end{cases}$$



$\mathrm{H(X,Y,Z)}$: **Node coordinates of equilibrium surface**

$\left(\mathrm{X}_k^L, \mathrm{Y}_k^L, \mathrm{Z}_k^L\right)$: **Lower bounds**

$\left(\mathrm{X}_k^U, \mathrm{Y}_k^U, \mathrm{Z}_k^U\right)$: **Upper bounds**

Material constants of membrane

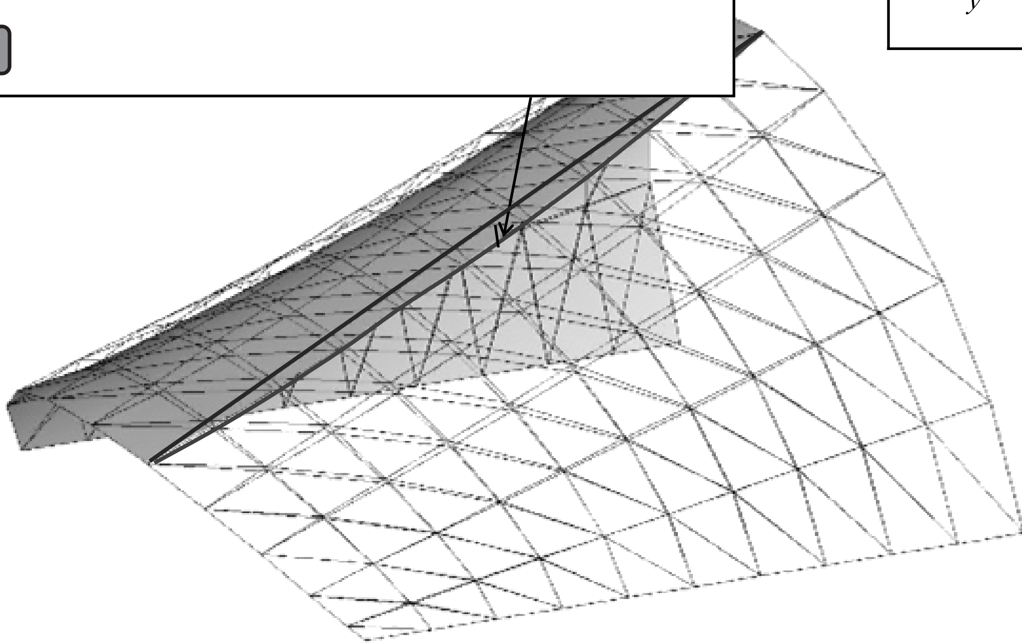
Elastic coefficient	E_x=243KN/m	E_y=227KN/m
Poisson's ratio	V_{xy}=0.55	V_{yx}=0.51
Shear rigidity	G_{xy}=76.84KN/m	G_{yx}=76.84KN/m

librium surface

Target tensile stress of 3.0 kN/m

Stress (kN/m) of the equilibrium surface

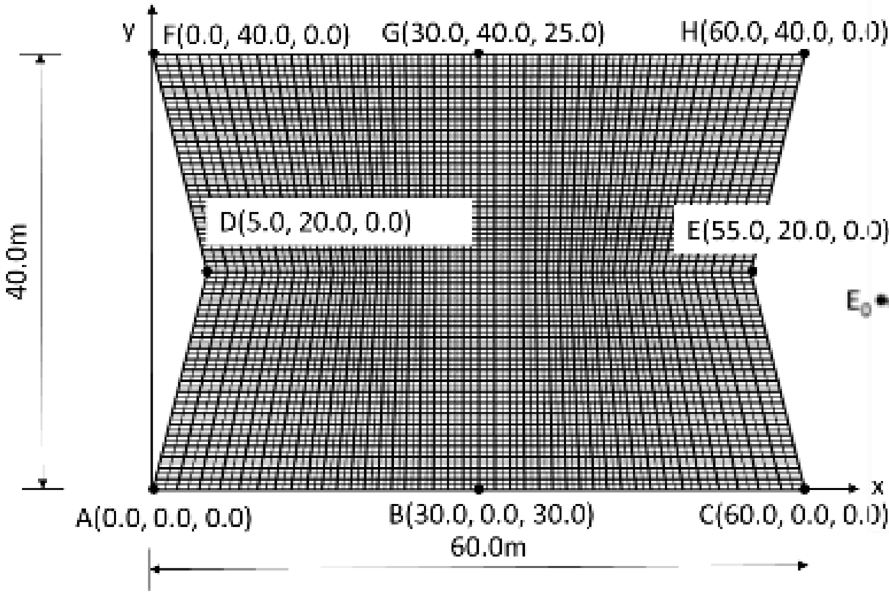
	Minimum	Maximum	Average
σ_x	1.012	9.621	4.387
σ_y	0.127	8.724	2.892



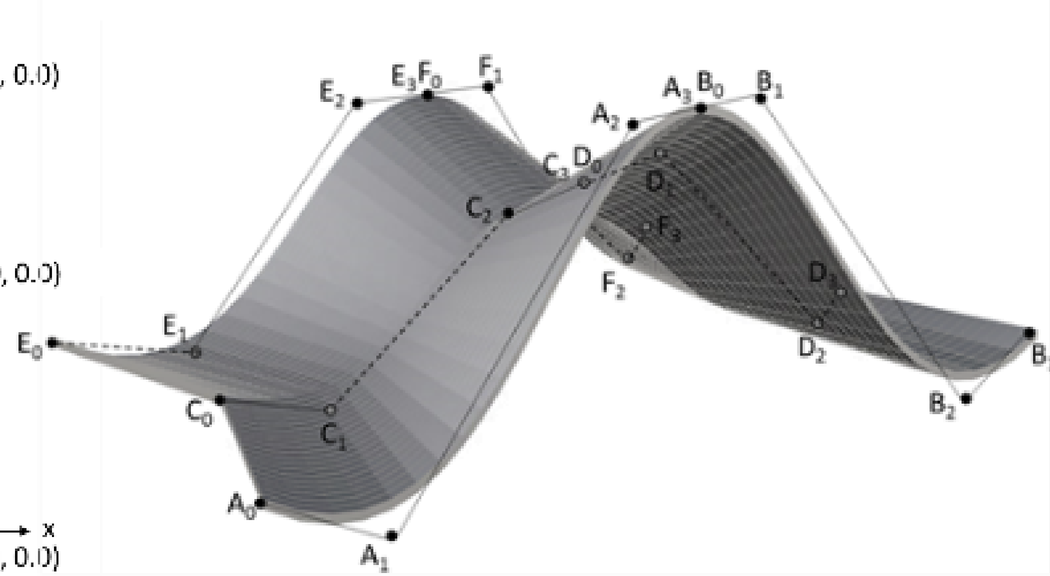
about 2.6 % of the span length.

Comparison of developable surface and equilibrium surface

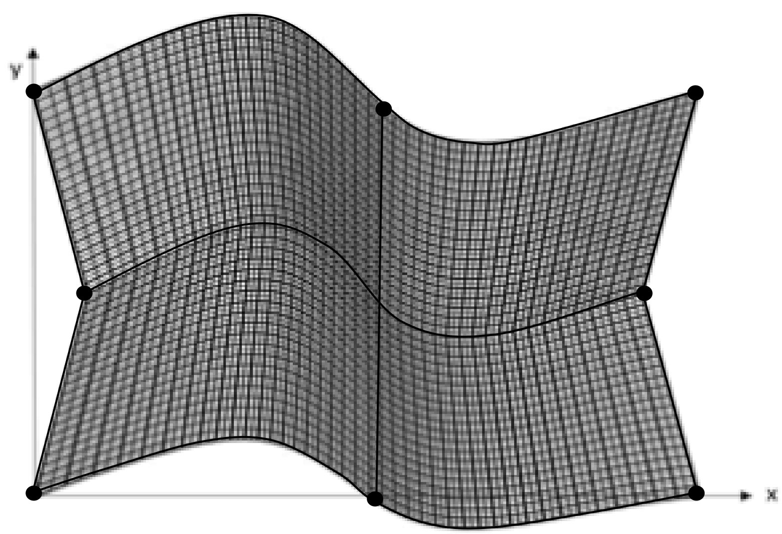
4. Examples



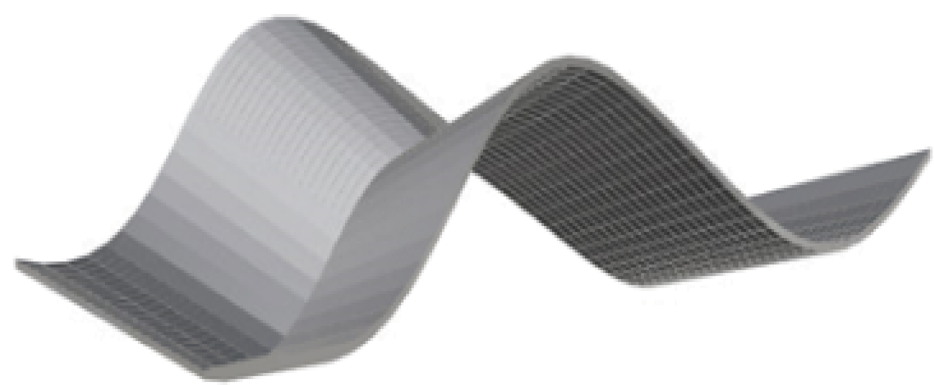
Plan view of initial shape



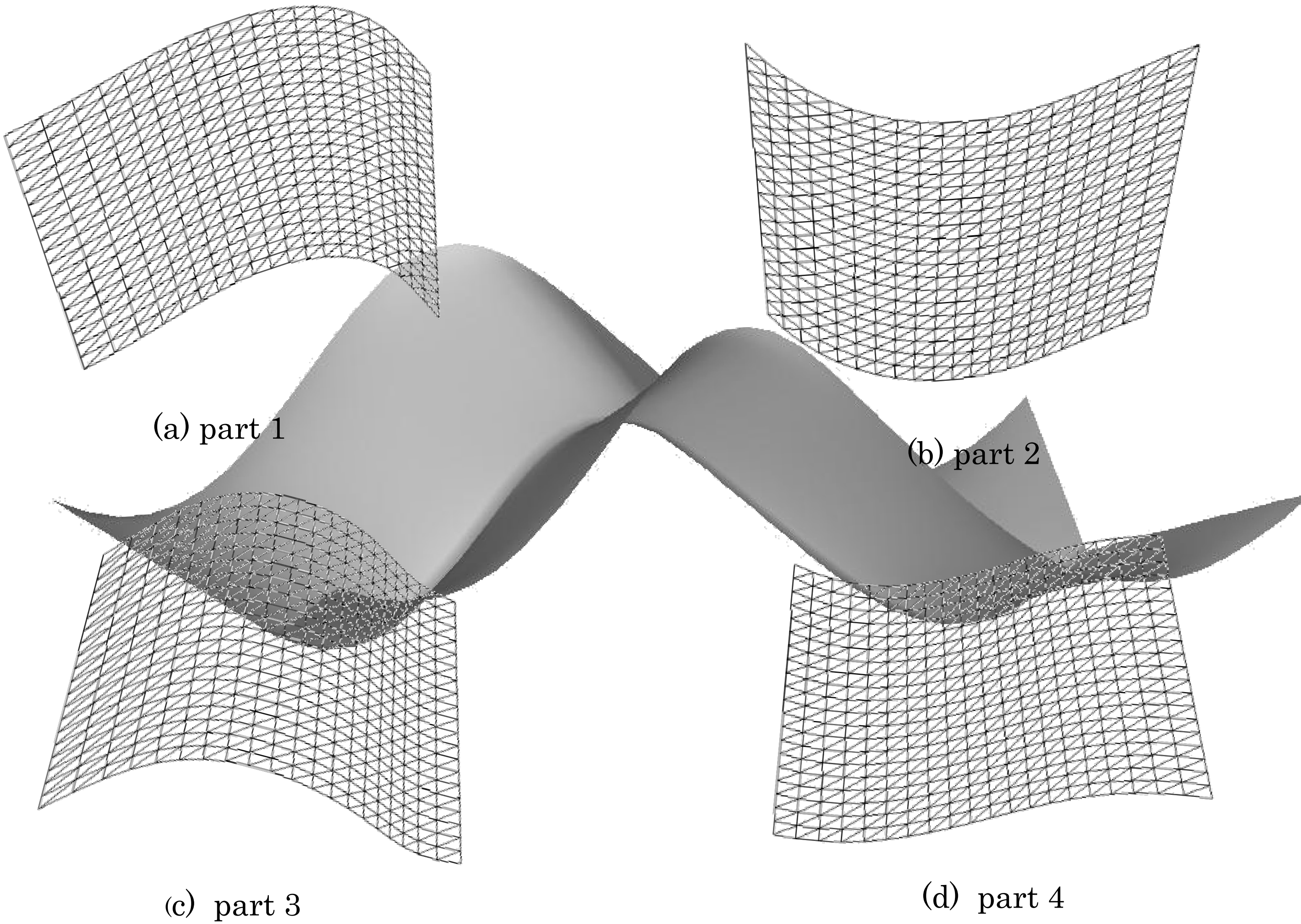
Oblique projection of initial shape



Plan view of developable surface



Diagonal view of developable surface



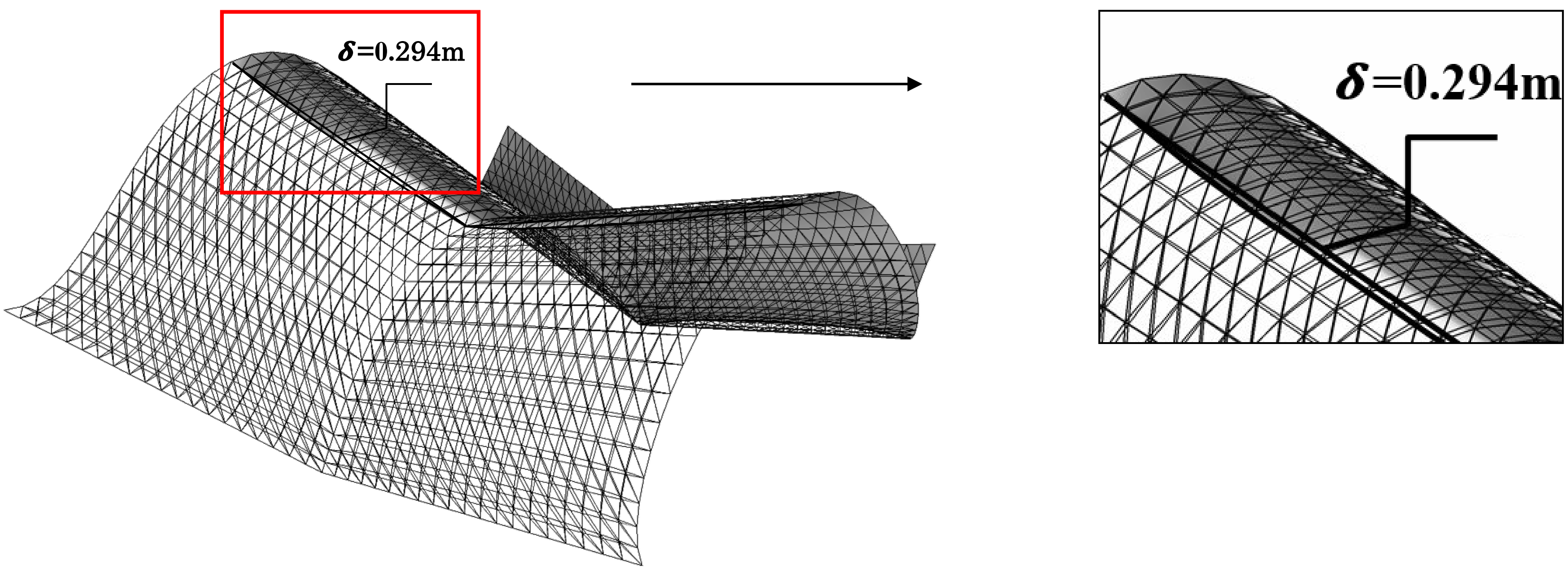
Cutting pattern of four parts

about **0.7 %** of the span length.

Stress (kN/m) of the equilibrium surface

	Minimum	Maximum	Average
σ_x	1.106	8.065	3.206
σ_y	0.599	7.400	2.589

Target tensile stress of 3.0 kN/m



Comparison of developable surface and equilibrium surface

5.Conclusion

- **Focusing on the geometric characteristics and mechanical rationality of the developable surface , a reasonable surface shape of the developable surface was obtained through optimization.**
- **A method was proposed that for generating framework structure morphology to convert to equilibrium surface.**
- **The generation of reasonable developable surface and the projection from developable surface to plane were realized.**
- **Stabilized equilibrium membrane structure was generated through operations such as minimizing strain energy.**