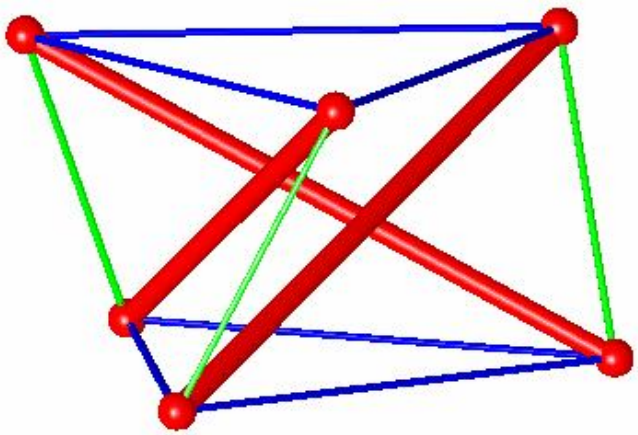
Multi-objective Optimization for Force Design of Tensegrity Structures



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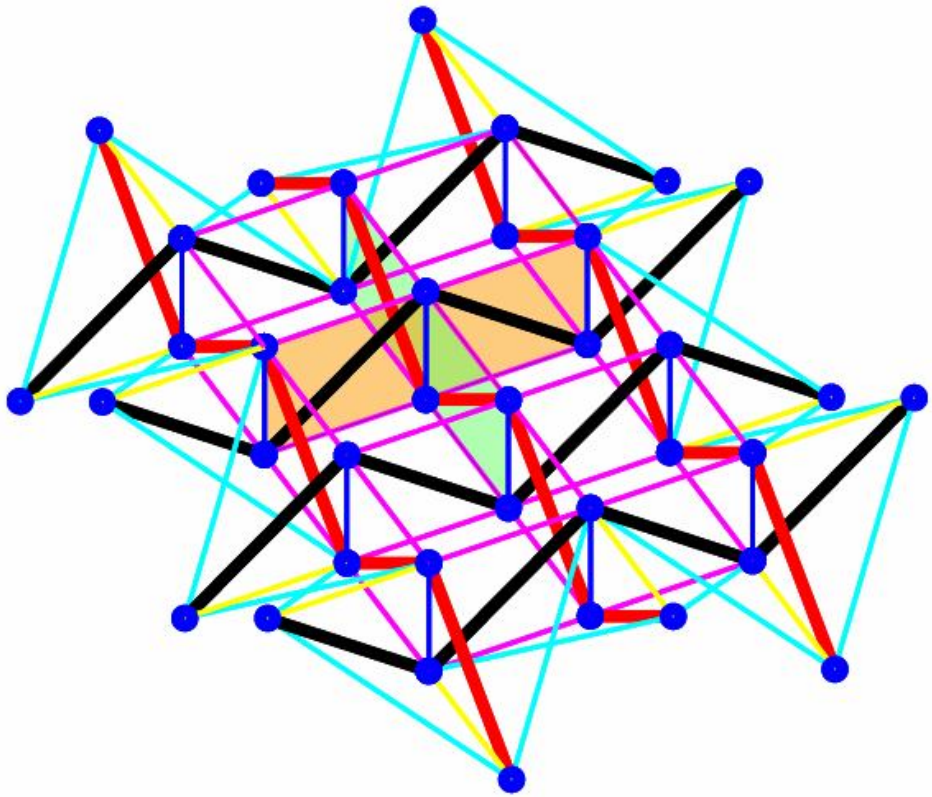
Content

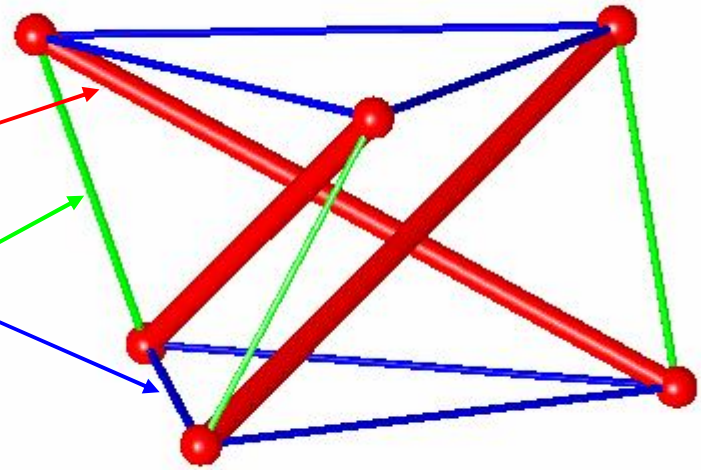


Tensegrity Grid

What is Tensegrity?

Tensegrity = Tension + Integrity (R.B. Fuller 1975)

- Self-equilibrated
- Member Force ⇔ Configuration
- Member Type
  - Strut in Compression
  - Cable in tension
  - Bar without force



The struts in compression are floating in the sea of tension constructed by cables.

Force Design & Optimization

Self-equilibrium Equation

$$\mathbf{Ds} = \mathbf{0}$$

D ? Equilibrium Matrix ← Configuration

s ? Member Force

$$\mathbf{s} = \alpha_1 \mathbf{f}_1 + \alpha_2 \mathbf{f}_2 + \cdots + \alpha_i \mathbf{f}_i \cdots + \alpha_{m-R} \mathbf{f}_{m-R}$$

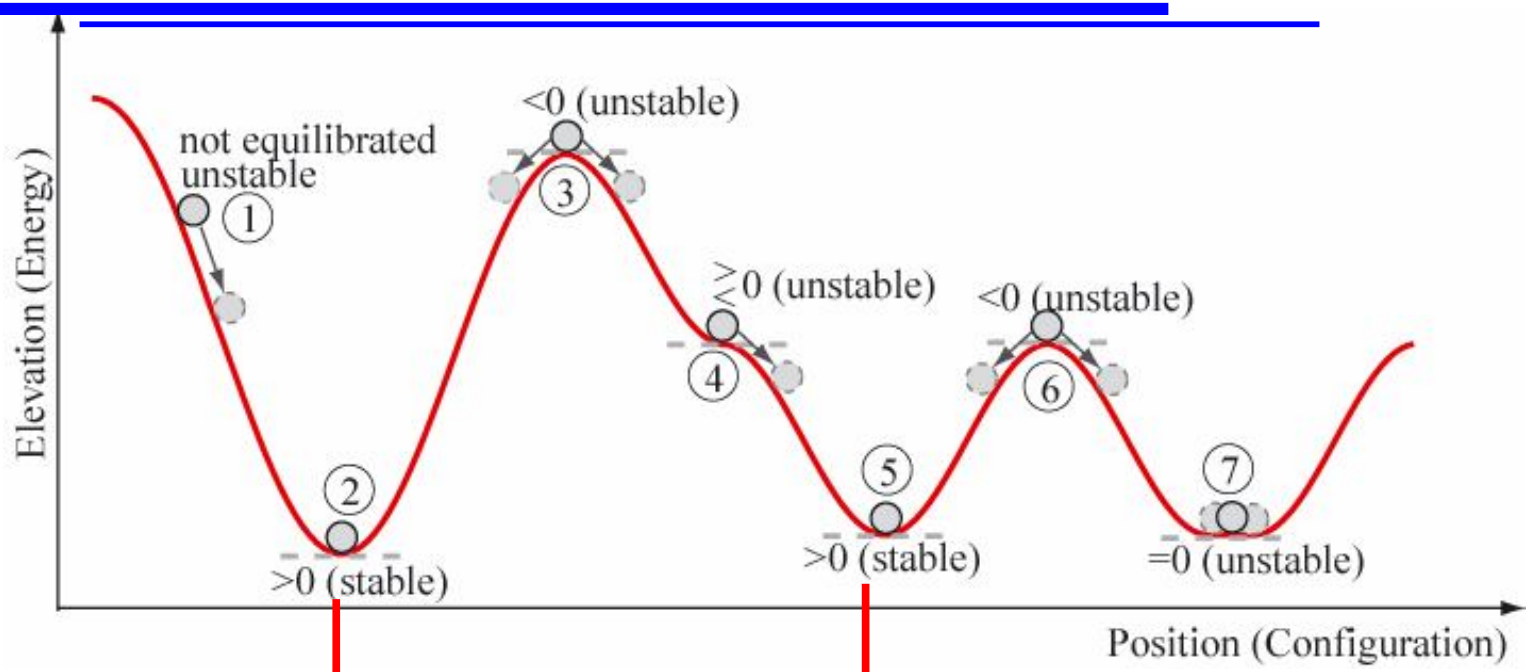
$\mathbf{f}_i$? Force Mode

Unknown Force ← Unknown Coefficient

Force Deviation ↑ Stiffness

Optimization Problem

Stiffness



$$\mathbf{K} = \mathbf{K}^G + \mathbf{K}^E > 0$$

$$\mathbf{K}^G \,?\, 0$$

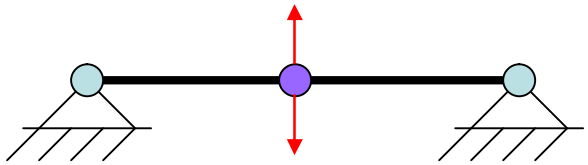
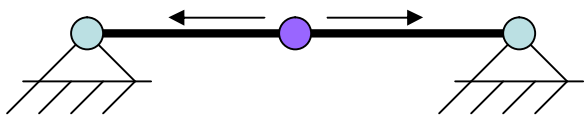
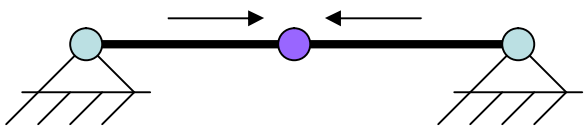
$$\mathbf{K}^E \geq 0$$

Member Force

Configuration

Member Stiffness

Prestress & Stiffness

No prestress	Mechanism  Neutrally Stable	$\mathbf{K}^G = 0$ $\mathbf{K}^E \geq 0$ → $\mathbf{K} \geq 0$
Tension	 Stable	$\mathbf{K}^G > 0$ $\mathbf{K}^E \geq 0$ → $\mathbf{K} > 0$
Compression	 Unstable	$\mathbf{K}^G < 0$ $\mathbf{K}^E \geq 0$ → $\mathbf{K} < 0$

Optimization 1

$$\mathbf{K} = \mathbf{K}^G + \mathbf{K}^E$$

$+\infty$ stiffness

$$\mathbf{K}^E \to 0 \quad \text{or} \quad +\infty$$

$$\mathbf{Q} = \mathbf{M}^{\mathrm{T}}\mathbf{K}^G\mathbf{M} > 0$$

M ? mechanism

Structure collapses in the weakest direction!

Objective 1

Strengthen the Weakest

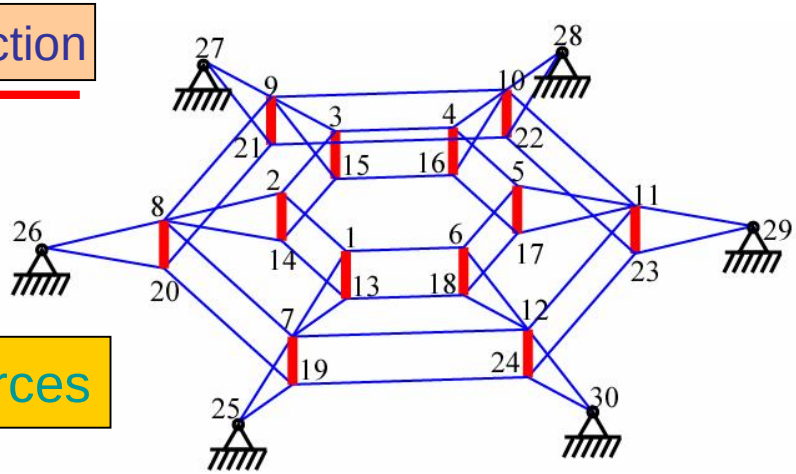
Maximize the Minimum Eigenvalue of Q

Optimization 2 & Constraints

Design | Analysis | Manufacture | Construction

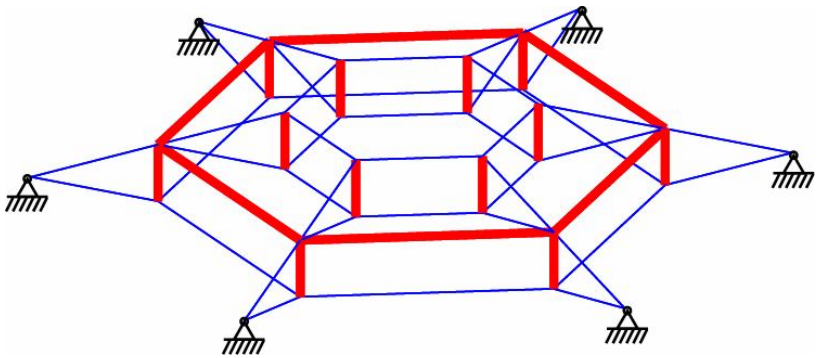
Objective 2

Minimize deviation of member forces



Constraints

- Positive force for cable
- Negative force for strut
- Given strain energy



Optimization Problem

Objectives

Maximize the Minimum Eigenvalue of Q

Minimize deviation of member forces

Constraints

Positive force for cable

Negative force for strut

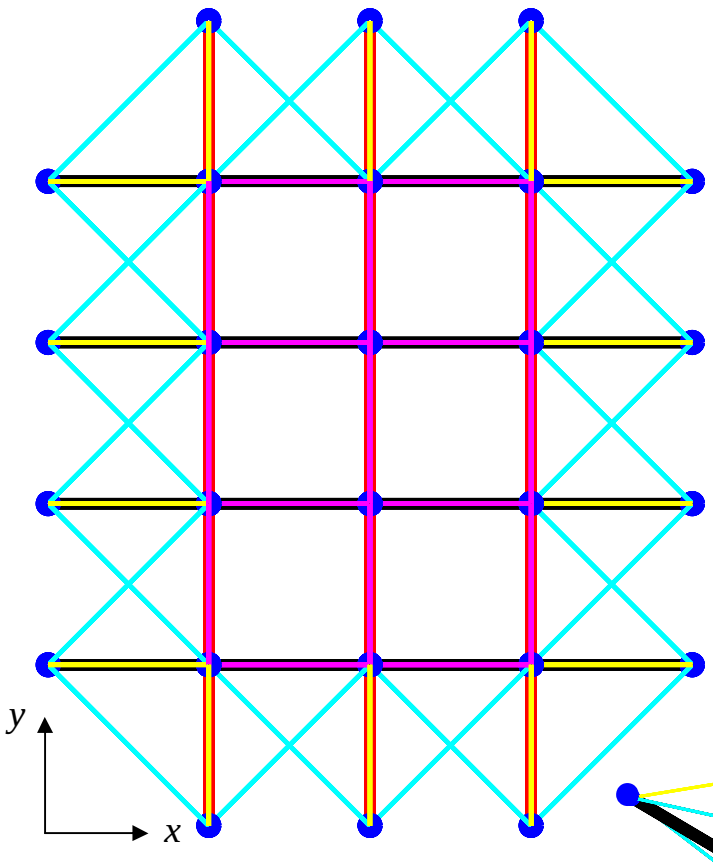
Given strain energy

Upper bound of force deviation

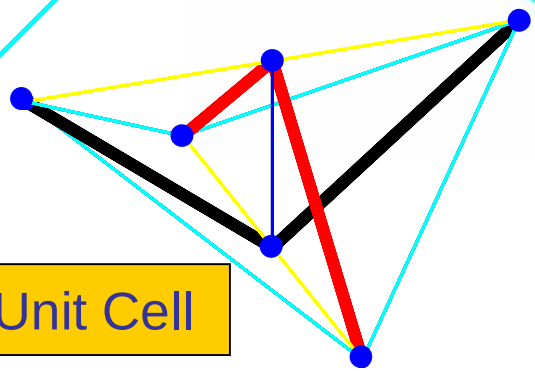
Constraint Approach

Tensegrity Grid

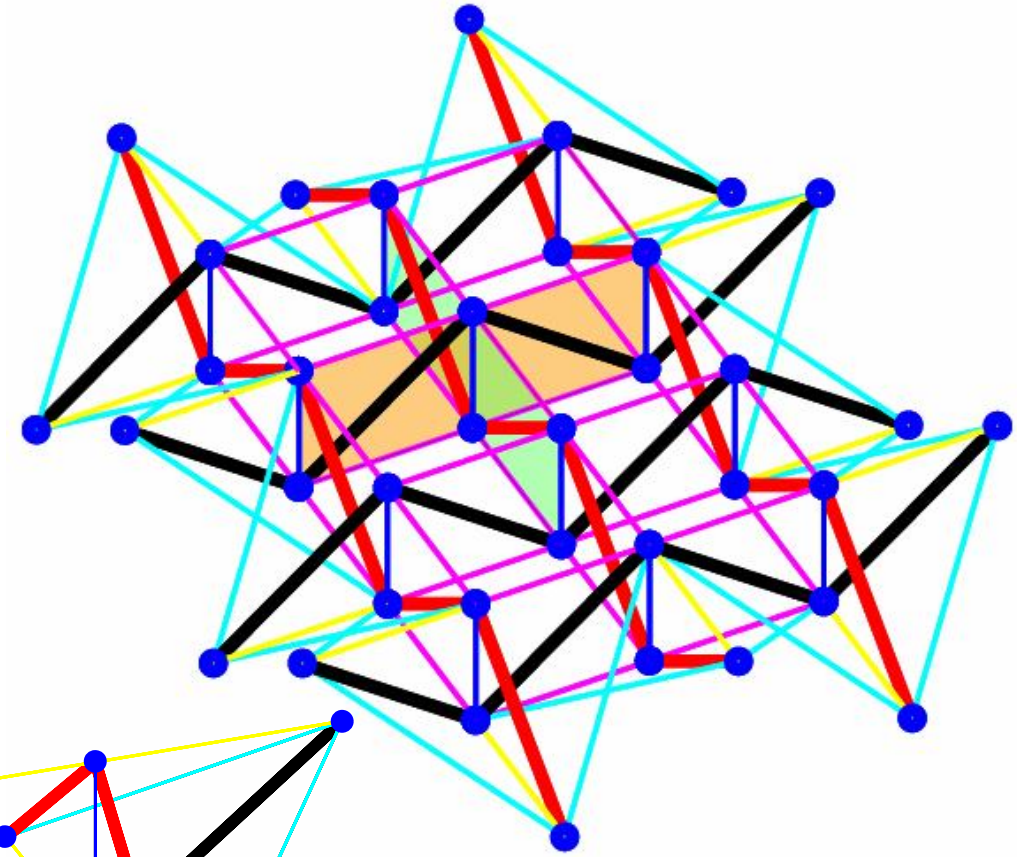
38 Nodes | 115 Members | 8 Force Modes | 1 Mechanism



Top View

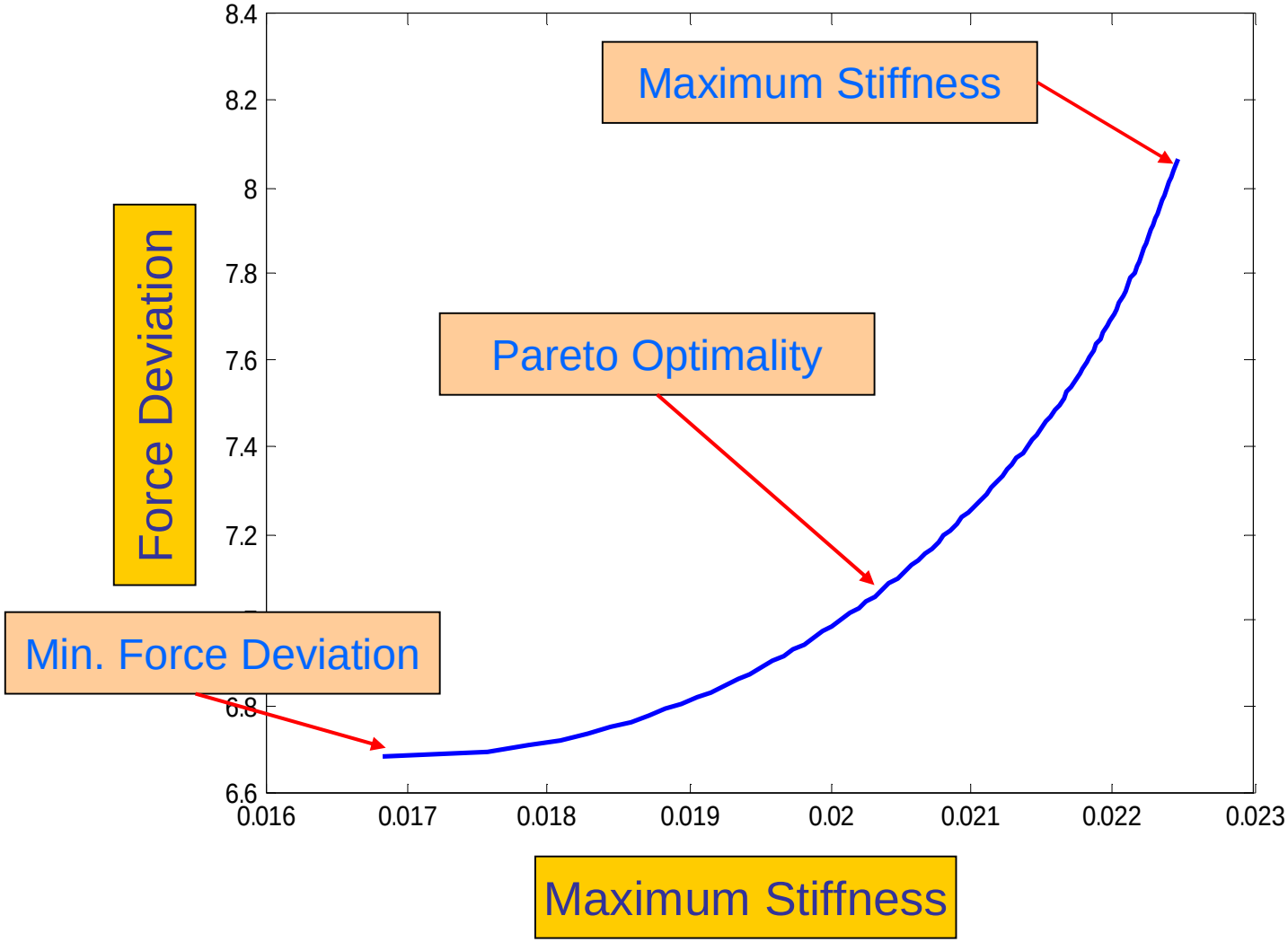


Unit Cell

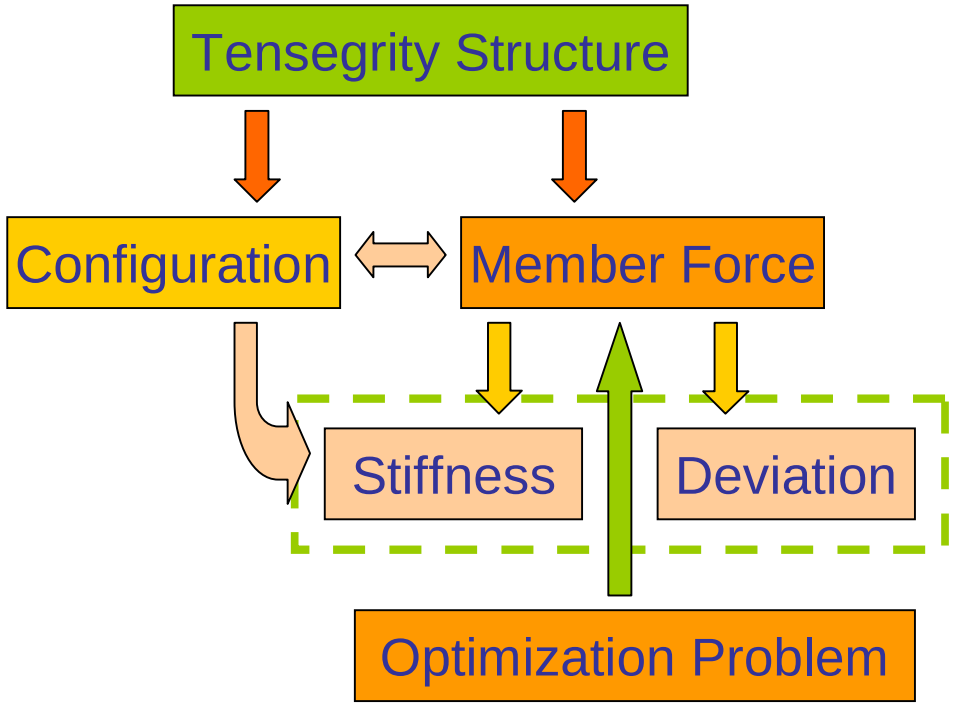


Side View

Optimal Solutions



Summary


Tensegrity Structure
Configuration
Member Force
Stiffness
Deviation
Optimization Problem