Multi-objective Optimization for Force Design of Tensegrity Structures

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Content

- Concept & Applications
- Force Design Problem
- Optimization Formulation
- Numerical Example
- Discussions & Conclusions

Tensegrity Grid
What is Tensegrity?

Tensegrity = Tension + Integrity

(R.B. Fuller 1975)

- Self-equilibrated
- Member Force
- Configuration

Member Type:
- Strut in Compression
- Cable in tension
- Bar without force

The struts in compression are floating in the sea of tension constructed by cables.
Self-equilibrium Equation

\[ \mathbf{D} \mathbf{s} = 0 \]

\( \mathbf{D} \) ? Equilibrium Matrix

\( \mathbf{s} \) ? Member Force

Configuration

\[ \mathbf{s} = \alpha_1 \mathbf{f}_1 + \alpha_2 \mathbf{f}_2 + \ldots + \alpha_i \mathbf{f}_i + \ldots + \alpha_{m-R} \mathbf{f}_{m-R} \]

Unknown Coefficient

Unknown Force

\( \mathbf{f}_i \) ? Force Mode

Force Deviation

Stiffness

Optimization Problem
Stiffness

\[ K = K^G + K^E > 0 \]

Member Force

Configuration

Member Stiffness
## Prestress & Stiffness

<table>
<thead>
<tr>
<th>No prestress</th>
<th>Neutrally Stable</th>
<th>( K^G = 0 )</th>
<th>( K^E \geq 0 )</th>
<th>( K \geq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>Stable</td>
<td>( K^G &gt; 0 )</td>
<td>( K^E \geq 0 )</td>
<td>( K &gt; 0 )</td>
</tr>
<tr>
<td>Compression</td>
<td>Unstable</td>
<td>( K^G &lt; 0 )</td>
<td>( K^E \geq 0 )</td>
<td>( K &lt; 0 )</td>
</tr>
</tbody>
</table>
Optimization 1

Maximize the Minimum Eigenvalue of \( Q \)

Structure collapses in the weakest direction!

Objective 1

Strengthen the Weakest

\[
\mathbf{K} = \mathbf{K}^G + \mathbf{K}^E
\]

\( \mathbf{K}^E \rightarrow 0 \) or \( +\infty \)

\[ +\infty \text{ stiffness} \]

\[
\mathbf{Q} = \mathbf{M}^T \mathbf{K}^G \mathbf{M} > 0
\]
Optimization 2 & Constraints

Objective 2

Minimize deviation of member forces

Constraints

Positive force for cable
Negative force for strut
Given strain energy
Optimization Problem

Objectives

Maximize the Minimum Eigenvalue of Q

Minimize deviation of member forces

Constraints

Positive force for cable

Negative force for strut

Given strain energy

Upper bound of force deviation

Constraint Approach
Tensegrity Grid

- 38 Nodes
- 115 Members
- 8 Force Modes
- 1 Mechanism

Top View

Unit Cell

Side View
Summary

Tensegrity Structure

Configuration \leftrightarrow \text{Member Force}

Stiffness \uparrow \downarrow \text{Deviation}

Optimization Problem