Structural performance of triangular latticed shells with regularized panels for Bézier design surfaces

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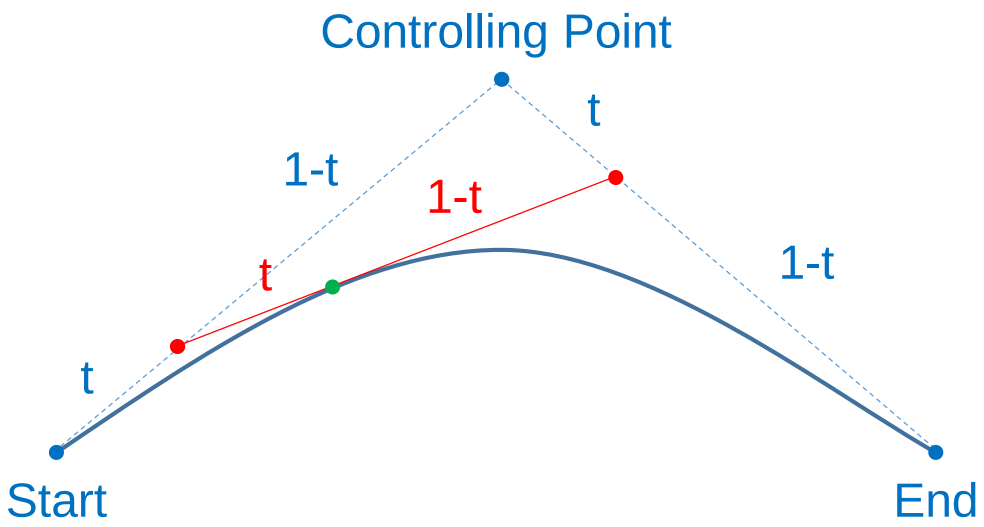
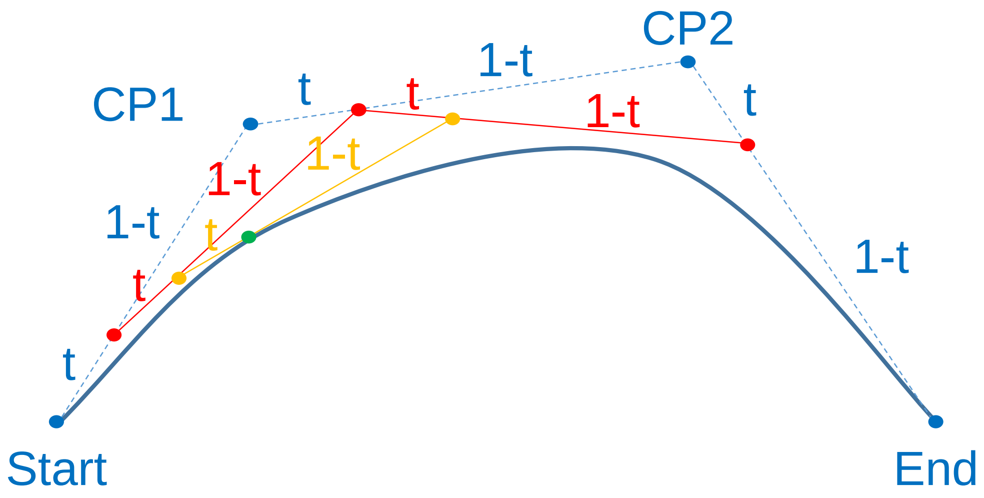
Bézier Curve

- 1 Controlling Point
  ⇒Quadratic Bézier Curve



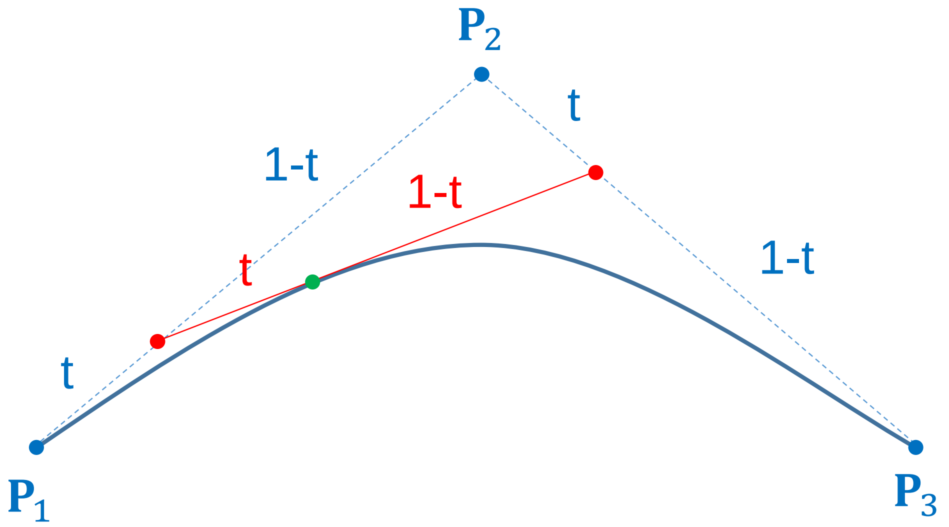
- 2 Controlling Points
  ⇒Cubic Bézier Curve



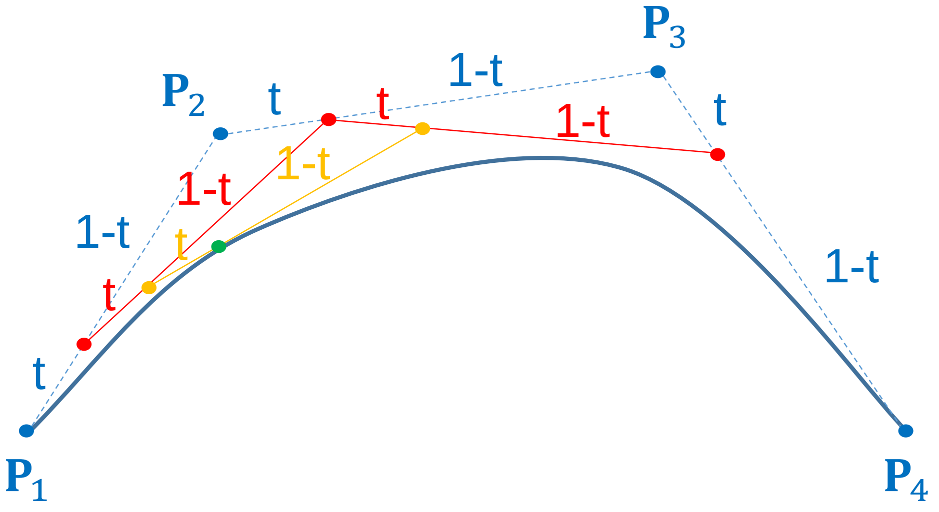
Equation of Bézier Curve

Bernstein Polynomials

- Quad: $\mathbf{P}(u) = (1-u)^2 \cdot \mathbf{P}_1 + 2\,u\,(1-u) \cdot \mathbf{P}_2 + u^2 \cdot \mathbf{P}_3$



- Cubic: $\mathbf{P}(t) = (1-u)^3 \cdot \mathbf{P}_1 + 3u\,(1-u)^2 \cdot \mathbf{P}_2 + 3u^2(1-u) \cdot \mathbf{P}_3 + u^3 \cdot \mathbf{P}_4$



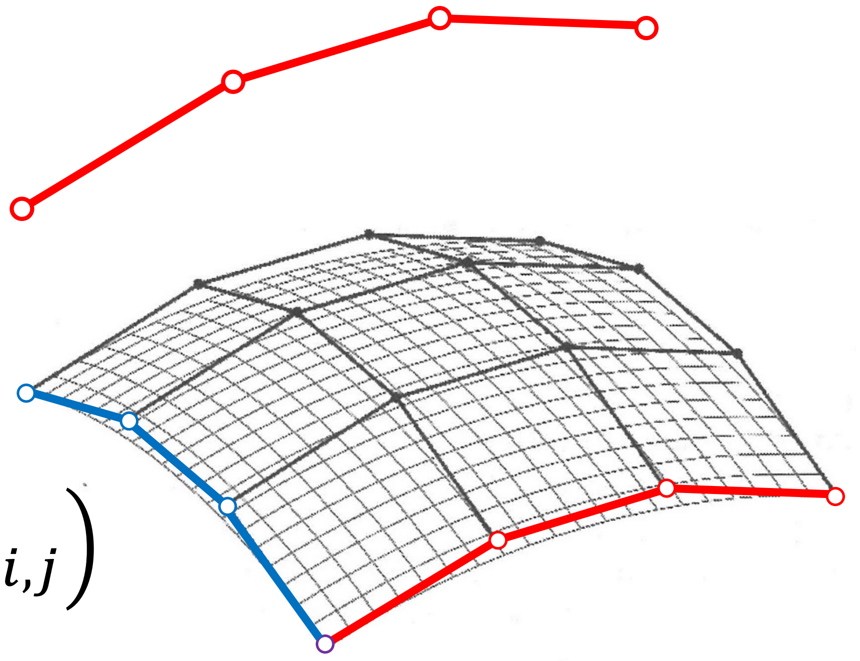
Bézier Curve → Bézier Surface

- Bézier Curve

$$S_b^n(u) = \sum_{i=0}^{n} \left(B_i^n(u) \cdot \boldsymbol{P}_i \right)$$

- Bézier Surface

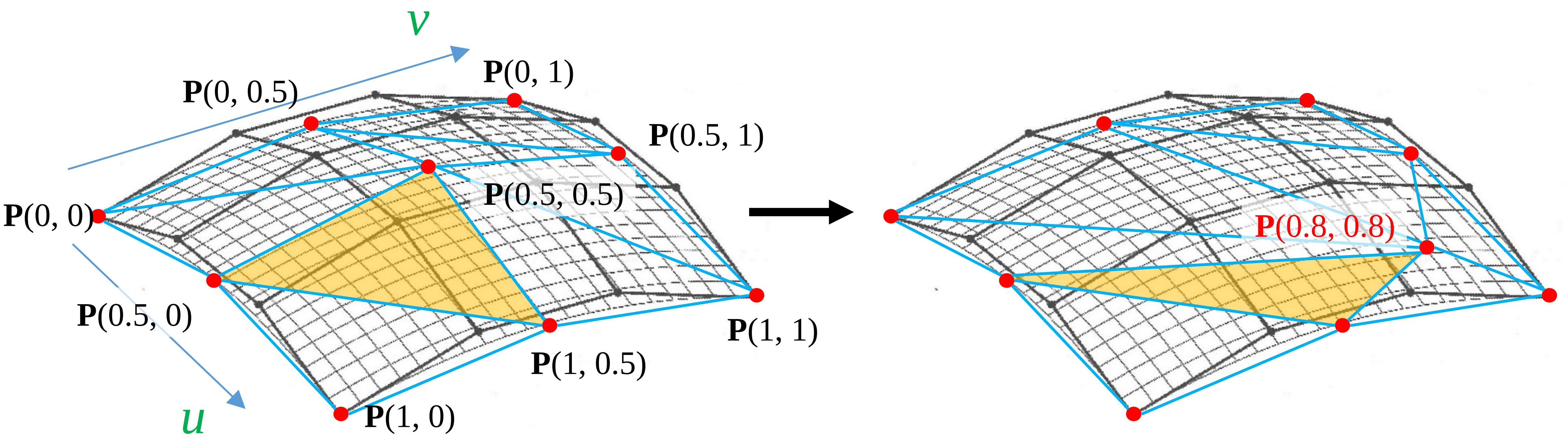
$$S_b^{n,m}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \left(B_i^n(u) \cdot B_j^m(v) \cdot \boldsymbol{P}_{i,j} \right)$$



Discretization of Bézier surface

- Any point on Bézier surface can be uniquely specified as $\mathbf{P}(u,v)$
- Once the connectivity is fixed, mesh shapes can be changed using u,v

→ Panel shape is a function of u,v

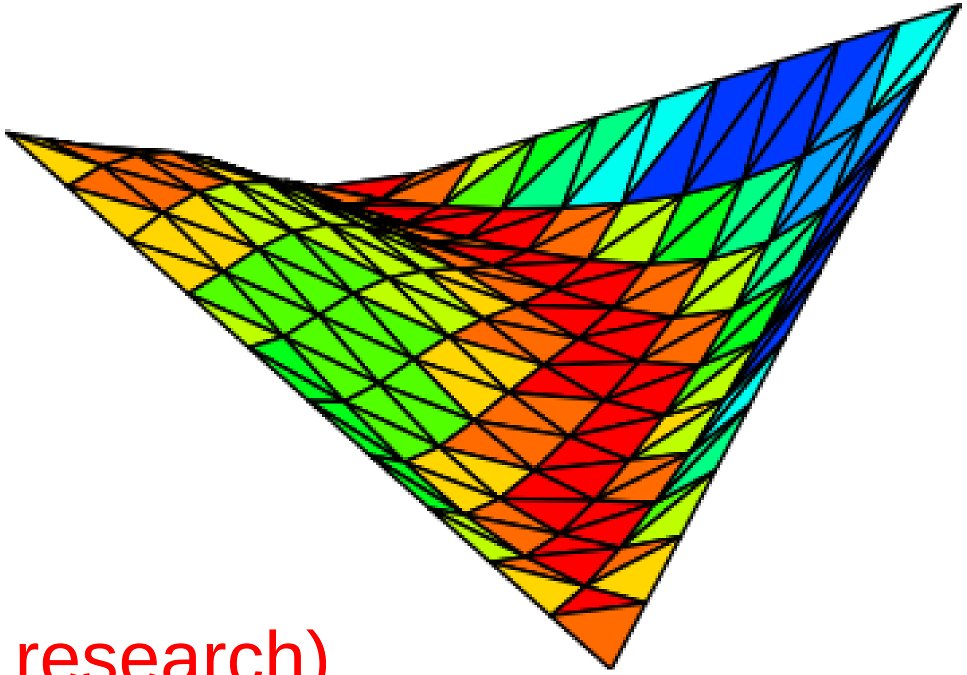


Purpose

If every panel shape is different:

- Expensive
- Difficult to construct

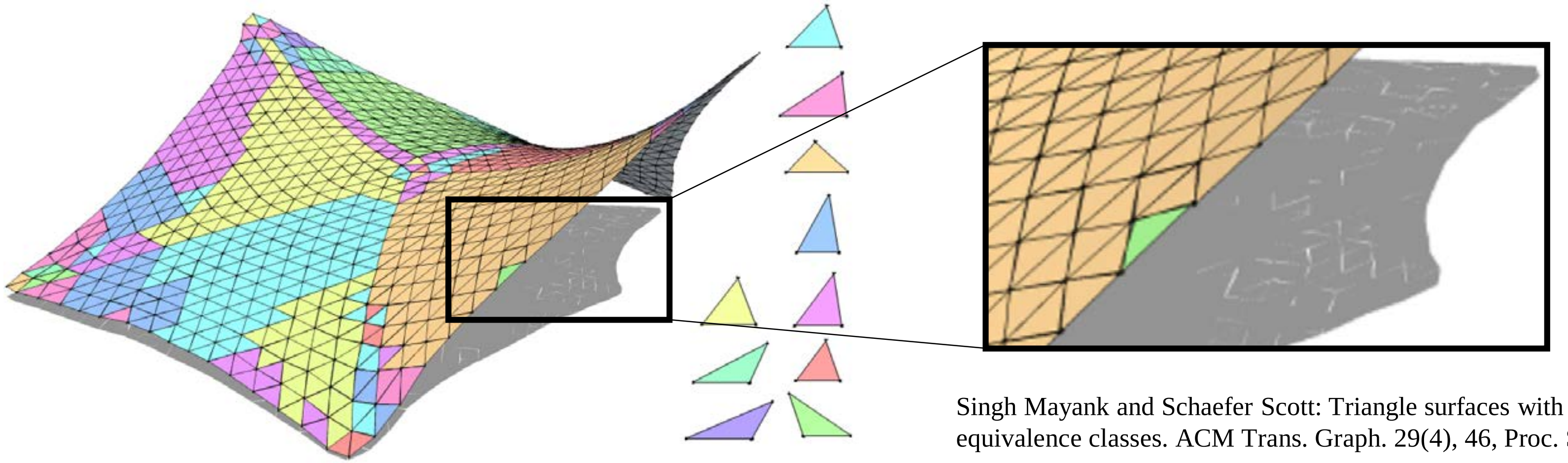
↓



- Classify panels (10 groups in this research)
- Obtain uniform panel shapes within each group

Previous study(Singh and Schaefer, 2010):

Classify panels into some groups, and optimize nodal locations so that the surface polygons match the canonical polygons as close as possible



Singh Mayank and Schaefer Scott: Triangle surfaces with discrete equivalence classes. ACM Trans. Graph. 29(4), 46, Proc. SIGGRAPH, 2010.

Problem:

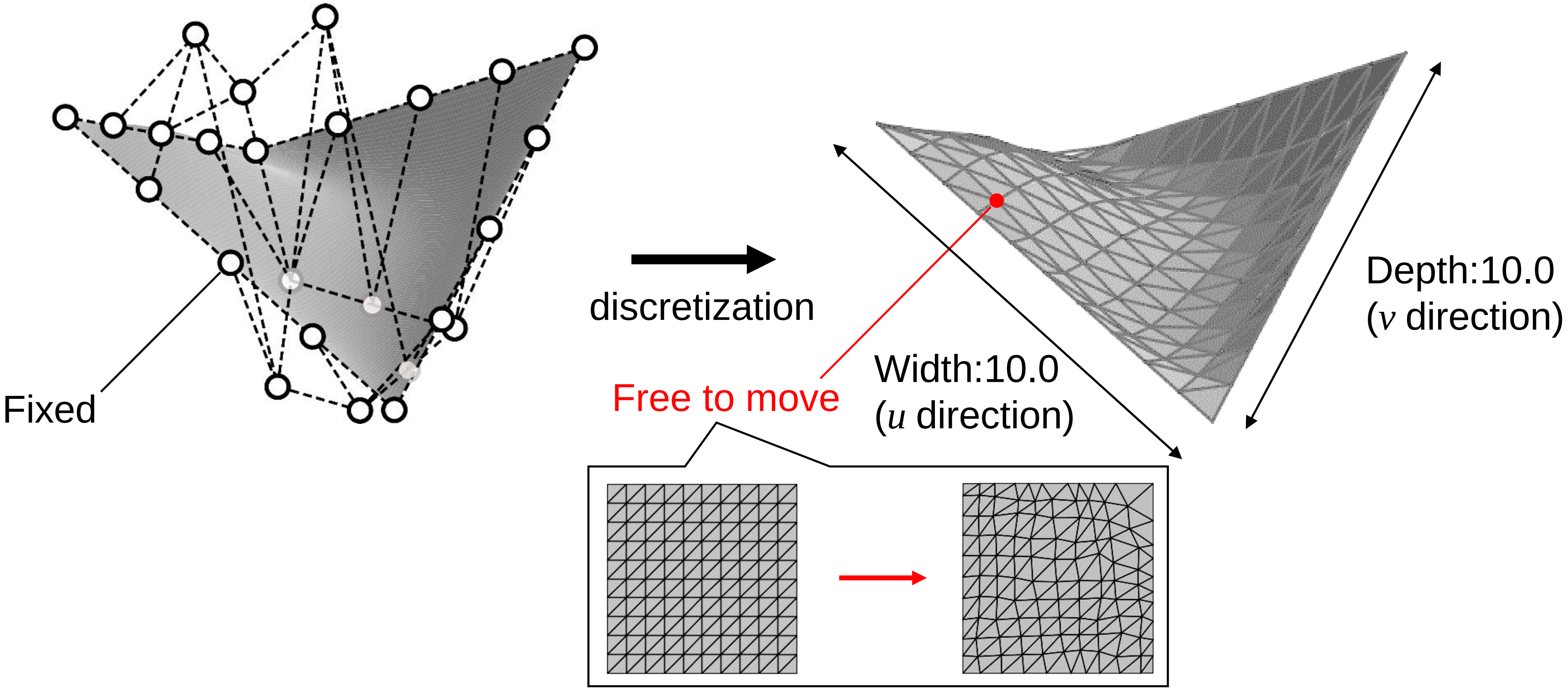
1. Surface geometry varies   2. Gaps between panels

Our approach:

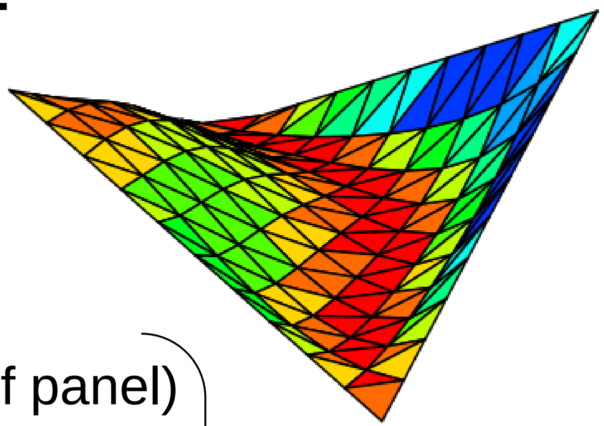
1. Surface geometry is fixed   2. No gap between panels

Bézier design surface and its discretization

- 5×5 control points constitute a Bézier surface
- No. of equal mesh divisions is 10 in u,v direction



Clustering using continuous variables



1. Randomly choose data as cluster centroids
2. Degree of participation in cluster j for data $\mathbf{x}_i$

Continuous within (0,1]

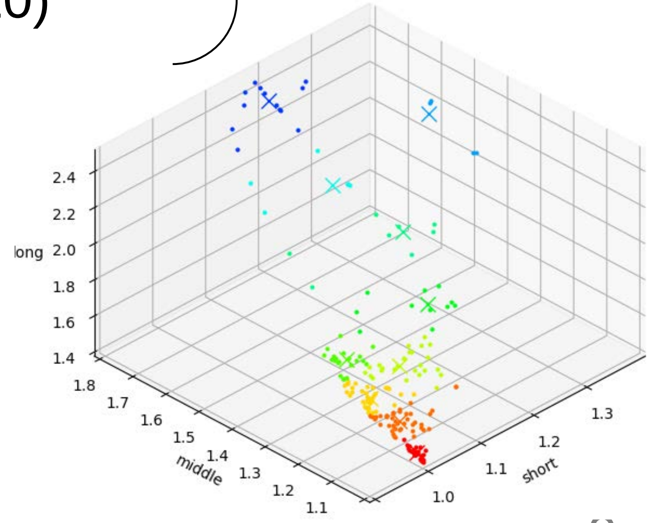
$$U_{ij} = \left(\sum_{k=1}^{n_c} \left(\frac{\left\| \mathbf{x}_i - \mathbf{c}_j \right\|}{\left\| \mathbf{x}_i - \mathbf{c}_k \right\|} \right)^2 \right)^{-1}$$

- $\mathbf{x}_i$: data(=3 edge lengths of panel)
- $\mathbf{c}_j$: cluster centroid
- U_{ij} : degree of participation
- n_d : number of data(=200)
- n_c : number of clusters(=10)

3. Cluster centroids

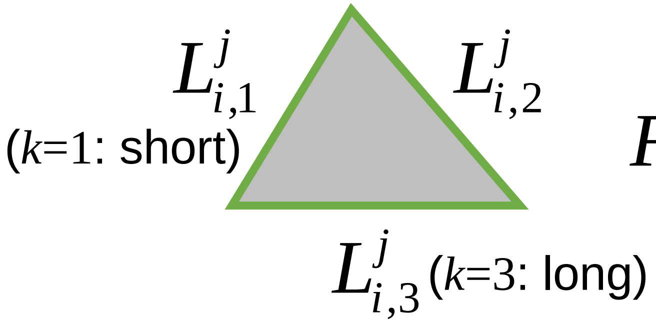
$$\mathbf{c}_j = \frac{\sum_{i=1}^{n_d} U_{ij}^2 \mathbf{x}_i}{\sum_{i=1}^{n_d} U_{ij}^2}$$

4. Compute 2 and 3 repeatedly until convergence



Optimization Problem

- We want to minimize maximum difference of edge length, but…

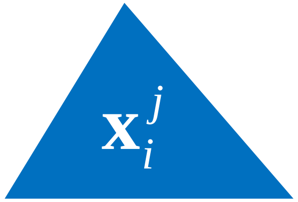


among clusters — within cluster — triangle edges

$$F(\mathbf{u},\mathbf{v}) = \max_{j}\left(\max_{i_1,i_2}\left(\max_{k=1,2,3}\left\|L_{i_1,k}^{j} - L_{i_2,k}^{j}\right\|\right)\right)$$

improve

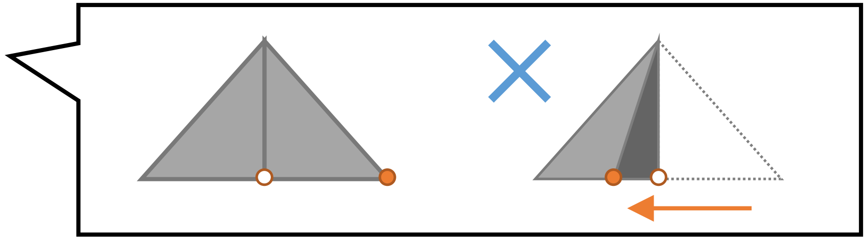
- Minimize maximum difference of p10 norm of edge lengths



among clusters — within cluster

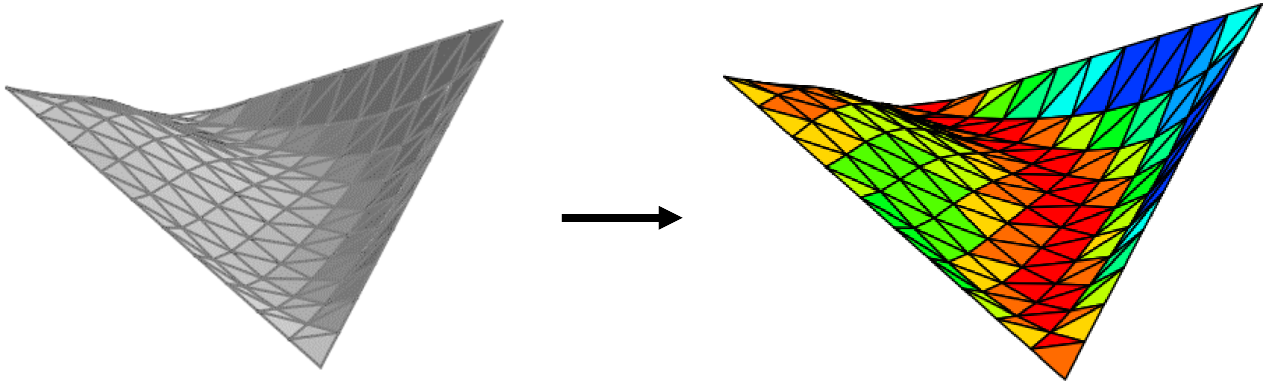
$$\text{minimize } \tilde{F}(\mathbf{u},\mathbf{v}) = \max_{j}\left(\max_{i_1,i_2}\left\|\mathbf{x}_{i_1}^{j} - \mathbf{x}_{i_2}^{j}\right\|_{10}\right)$$

$$\text{subject to } \mathbf{u}\in\Omega_{\mathbf{u}},\ \mathbf{v}\in\Omega_{\mathbf{v}}$$



Regularization workflow

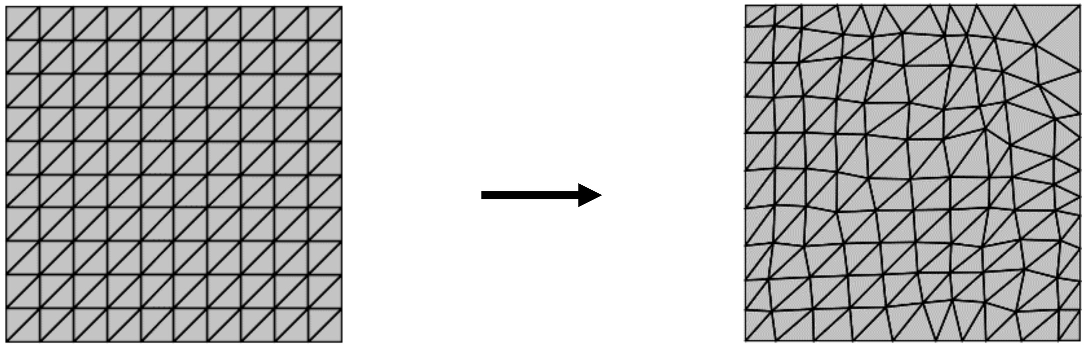
- Clustering



100 times

- Optimization

(with SLSQP)

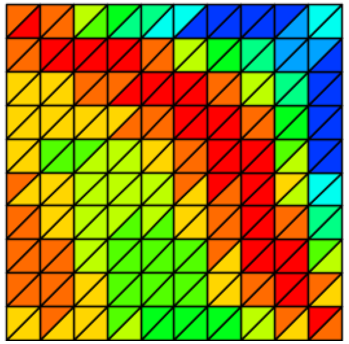


After 100 iterations

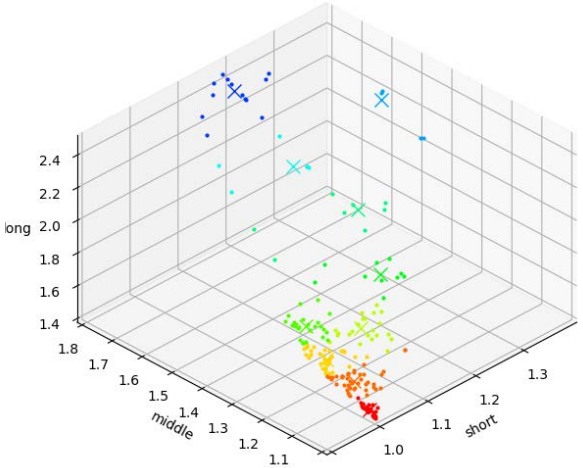
- Choose the best solution from 100 optimal results

Result

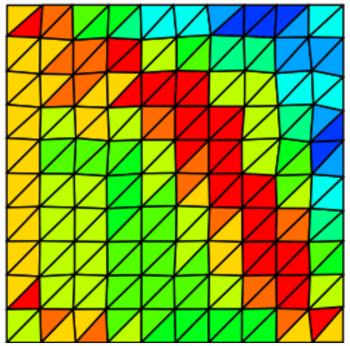
- Obtained more uniform panel shapes



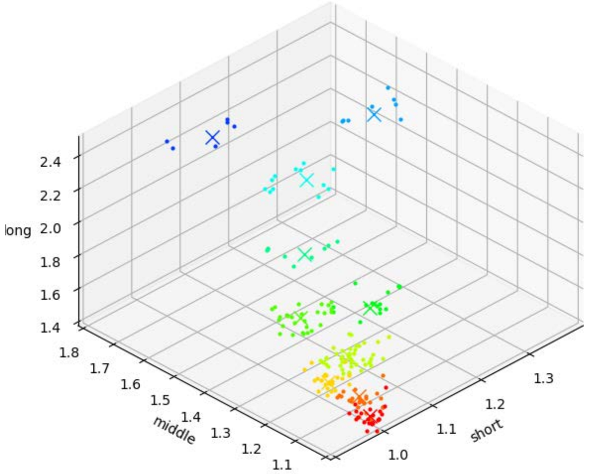
cluster	0	1	2	3	4	5	6	7	8	9
N_c	32	43	36	26	21	11	8	6	4	13
ave. short	1.005	1.017	1.010	1.059	1.018	1.139	1.178	1.151	1.311	1.164
ave. med	1.027	1.098	1.175	1.164	1.259	1.209	1.346	1.516	1.478	1.729
ave. long	1.432	1.521	1.583	1.692	1.700	1.866	2.074	2.212	2.394	2.453
$\max(L_{diff})$	0.080	0.176	0.156	0.149	0.145	0.146	0.212	0.181	0.152	0.193



$F = 0.212$



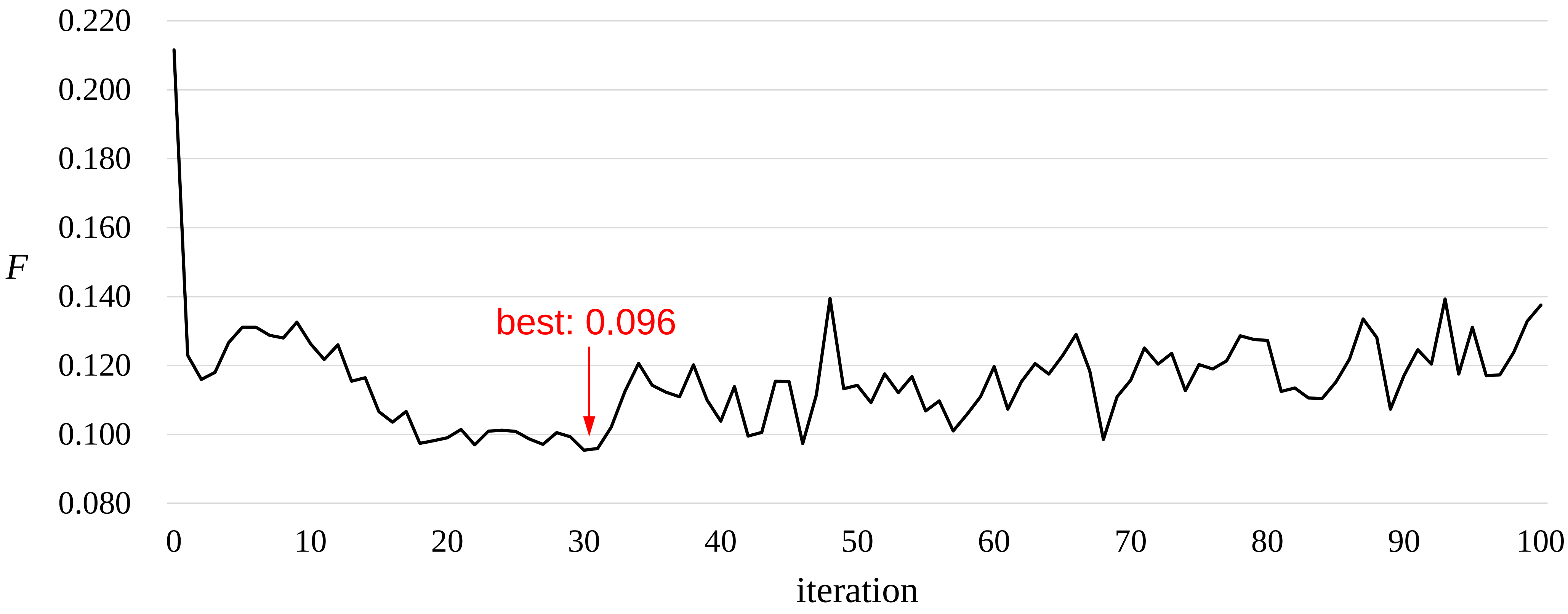
cluster	0	1	2	3	4	5	6	7	8	9
N_c	24	24	30	44	27	16	9	12	8	6
ave. short	0.999	1.001	1.009	1.027	1.018	1.094	1.086	1.140	1.287	1.099
ave. med	1.029	1.078	1.169	1.146	1.291	1.190	1.376	1.505	1.507	1.684
ave. long	1.430	1.506	1.494	1.611	1.735	1.794	1.972	2.175	2.329	2.352
$\max(L_{diff})$	0.096	0.092	0.096	0.096	0.096	0.096	0.096	0.095	0.096	0.095



$F = 0.096$

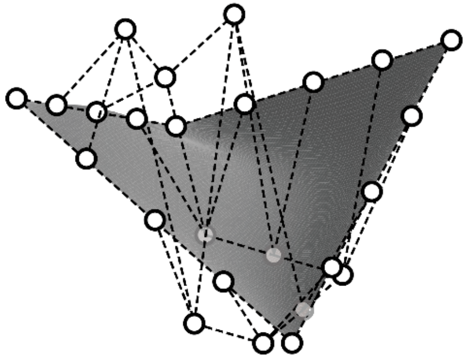
Iteration History

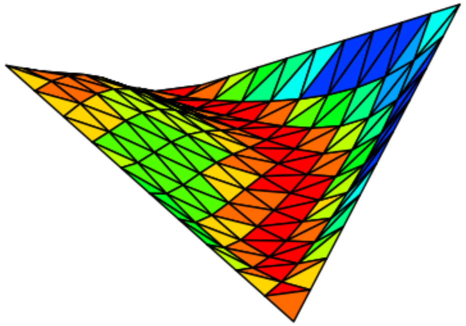
- Evaluation function is reduced especially in the early stage of iterations



Halve the rise of design surface

Easier to obtain uniform panel shapes

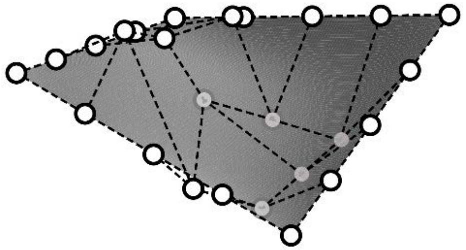


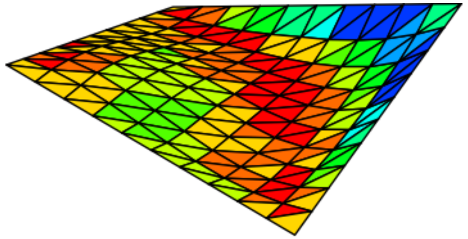


（initial solution）　（regularized solution）

$F = 0.212 \longrightarrow F = 0.096$

Halve z coordinates of control points



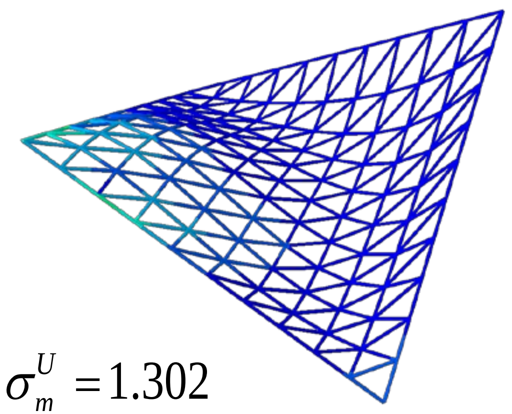
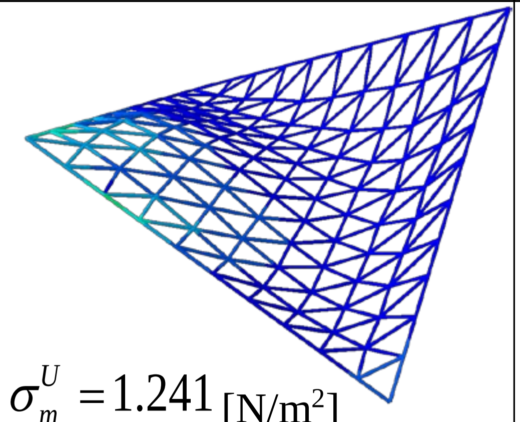
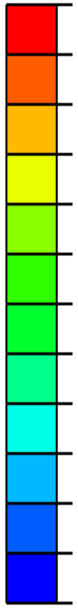
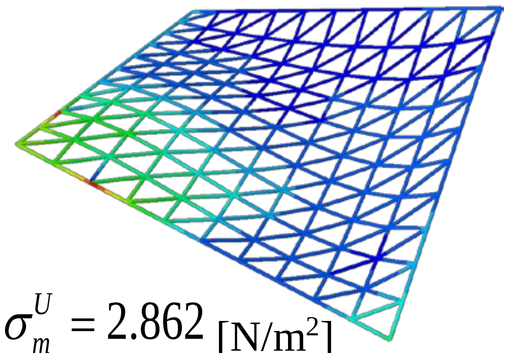
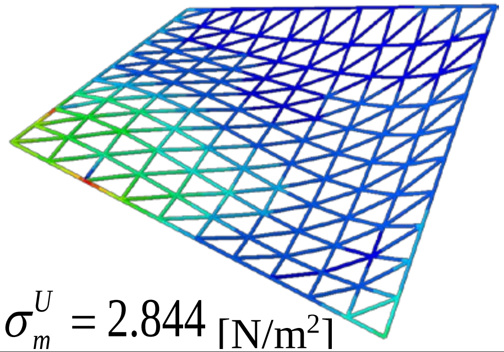


（initial solution）　（regularized solution）

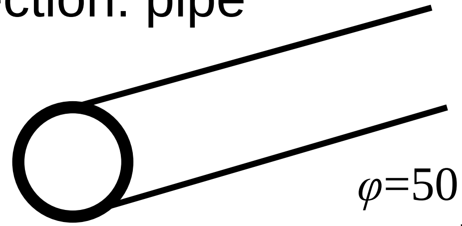
$F = 0.067 \longrightarrow F = 0.034$

Structural Analysis

Less rise causes more von Mises stresses on the members

	Initial solution	Regularized solution	Color
Original rise	 $\sigma_m^U = 1.302$	 $\sigma_m^U = 1.241$ [N/m²]	
Halved rise	 $\sigma_m^U = 2.862$ [N/m²]	 $\sigma_m^U = 2.844$ [N/m²]	

Section: pipe

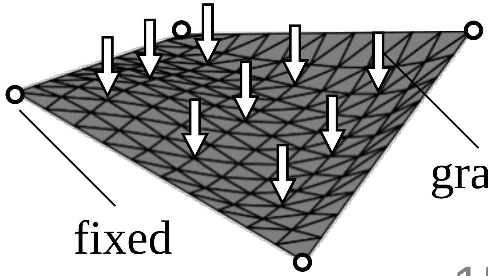


φ=50[mm]
t =5 [mm]

Material: steel

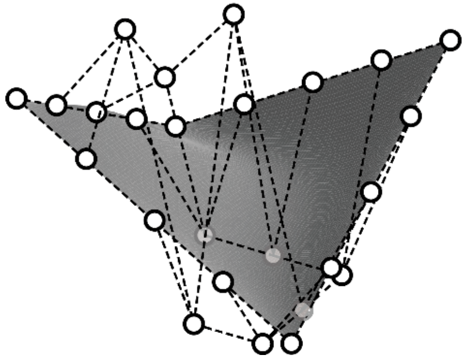
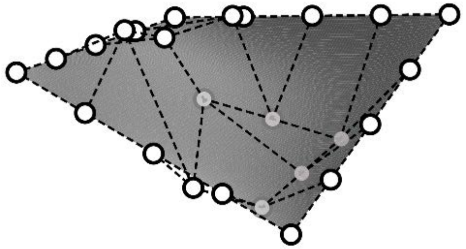
elastic modulus : 2.05×10^5[N/mm²]
Poisson's ratio : 0.3
density : 7870[kg/m³]

Load & Support condition



Uniformity of panel shapes V.S. Structural stability

- There is a trade-off between uniformity and stability

	Regularization of panel shapes	Structural stability against gravity
	✕ Difficult $F = 0.096$	○ Strong $\sigma_m^U = 1.241$
	○ Easy $F = 0.034$	✕ Weak $\sigma_m^U = 2.844$

Conclusion

New:

- A regularization method is proposed to obtain uniform panel shapes for a latticed shell whose design surface is a tensor product design surface
- Degree of participation to a cluster is expressed with continuous variables

Advantages:

✓Geometry of design surface is fixed

✓There is no gap between panels after the regularization

✓Quantify tradeoff between difficulty in regularization and structural stability