SHAPE OPTIMIZATION OF RULED SURFACE CONSIDERING STATIC STIFFNESS

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Shape Optimization of Free-form Shell

Previous works

Mechanical performance
Strain energy

Minimization → Shape with high stiffness

Non-mechanical performance

Constructability,
aesthetic view, cost

← Desirable structural shape

This study

・Define surface as ruled surface

+

・Maximize strain energy
→ Minimize static response

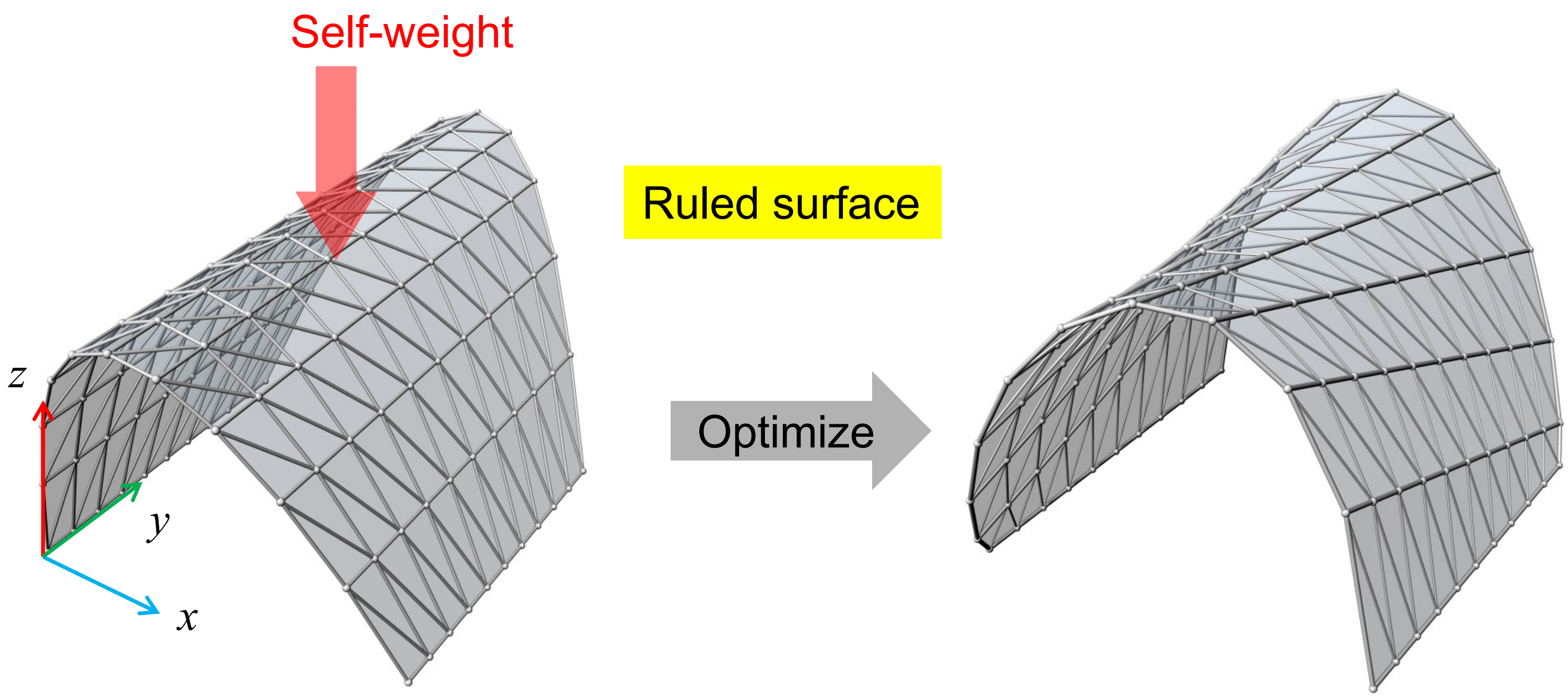
→ Shape optimization of shells considering static stiffness and constructability

Outline of research

◇Objective function: strain energy

$$f = \frac{1}{2} d^{\mathrm{T}} P$$

$\boldsymbol{d}$: Nodal displacement vector
$\boldsymbol{P}$: Nodal load vector

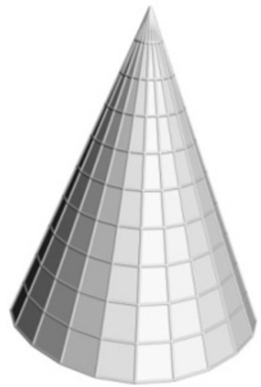


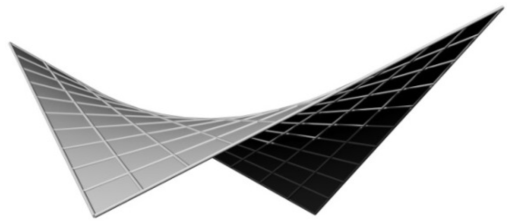
・Maximize strain energy
→ Minimize static response

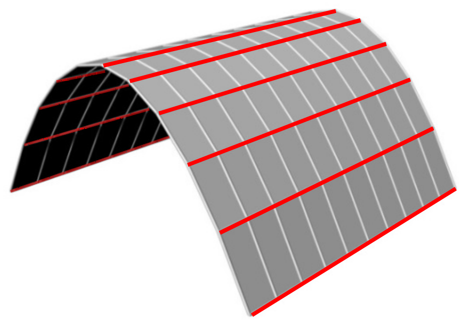
Ruled Surface

Surface that has straight generation line

- Ruled surface
  - Developable surface
    - Cylindrical surface
    - Conical surface
    - Tangential surface
  - Skew surface
    - Hyperbolic paraboloid
    - Herical surface
    - Conical surface
    - Cylindrical surface
    - Others



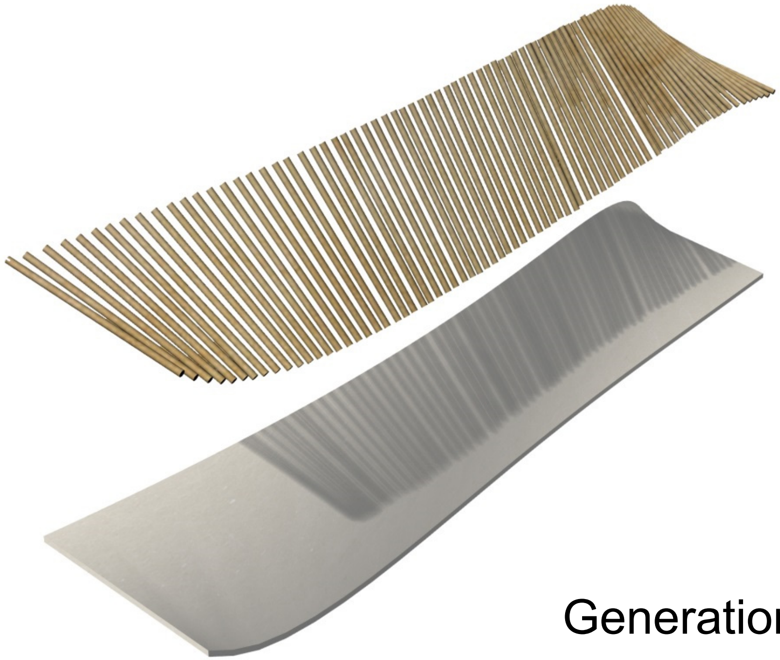




Ruled Surface

Surface that has straight generating line

→ Easy to construct



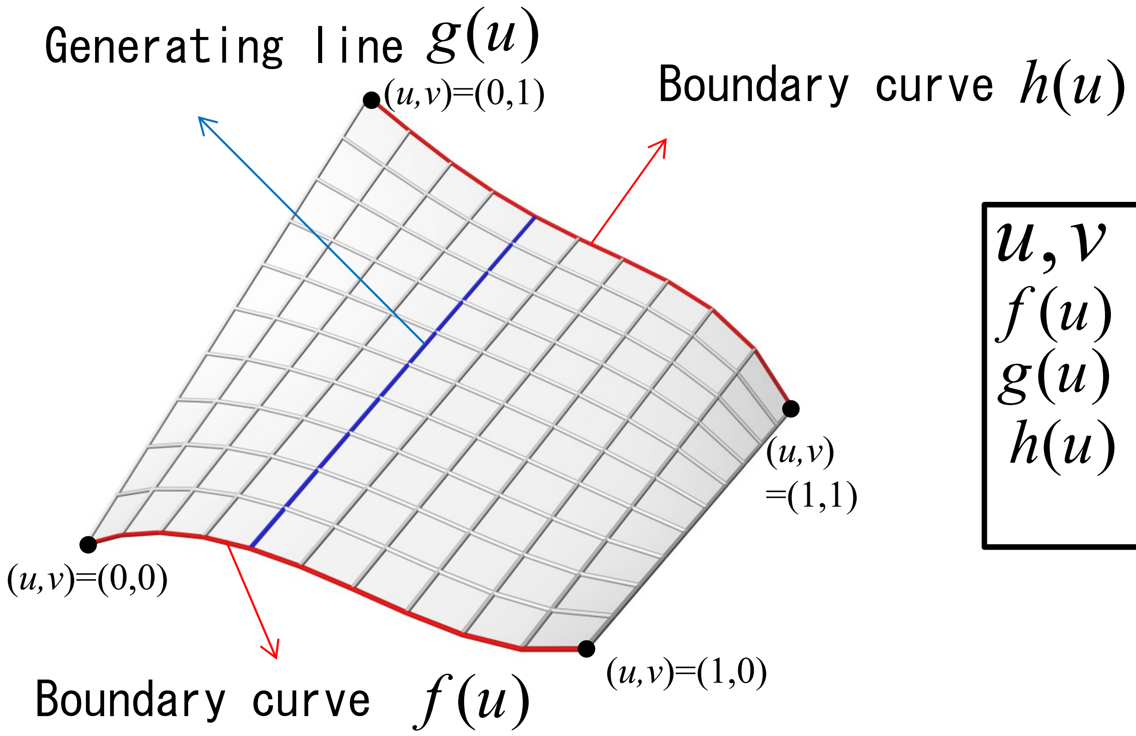
Generation of surface by straight lines

Parametric Definition of Ruled Surface

Definition of surface $S(u,v) = (1-v) f(u) + v\{h(u)\}$

$f(u), h(u)$: Boundary curves with same parameter

Straight lines between boundary $g(u) = h(u) - f(u)$



u, v : parameters
$f(u)$: boundary curve
$g(u)$: generating line
$h(u)$: boundary curve

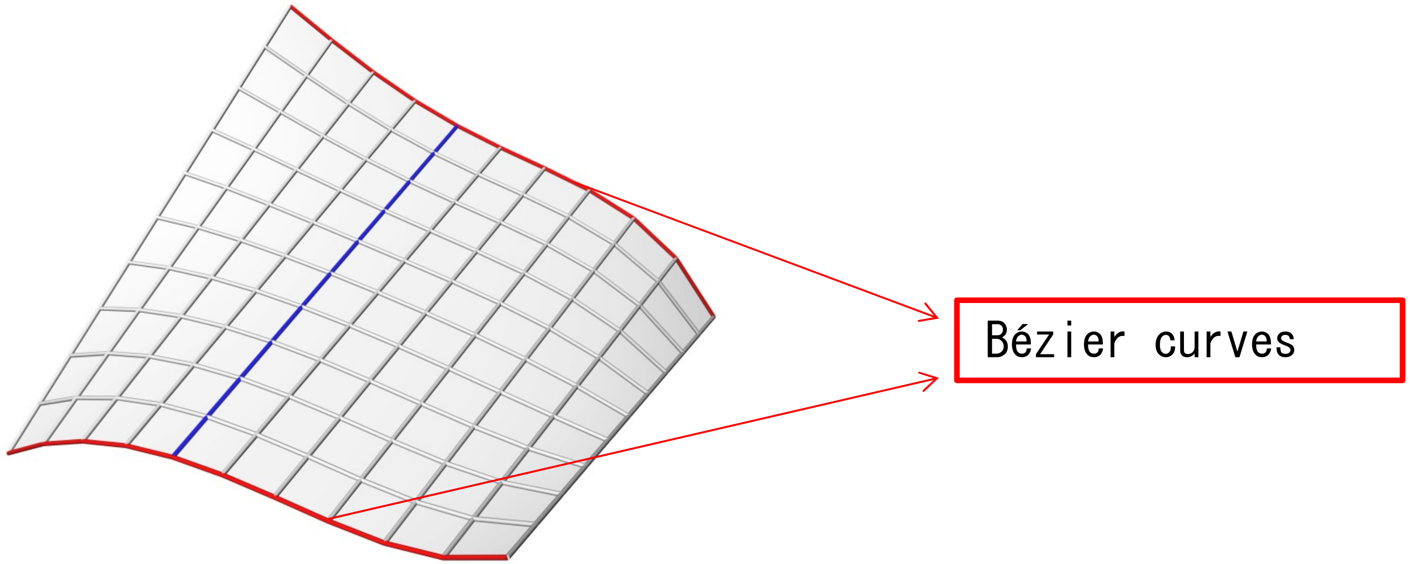
Boundary Bézier Curve

$f(u), h(u)$ Bézier curves

Connect Bézier curves by straight lines

Bézier curves

- Define various shapes using small number of variables



Optimization problem

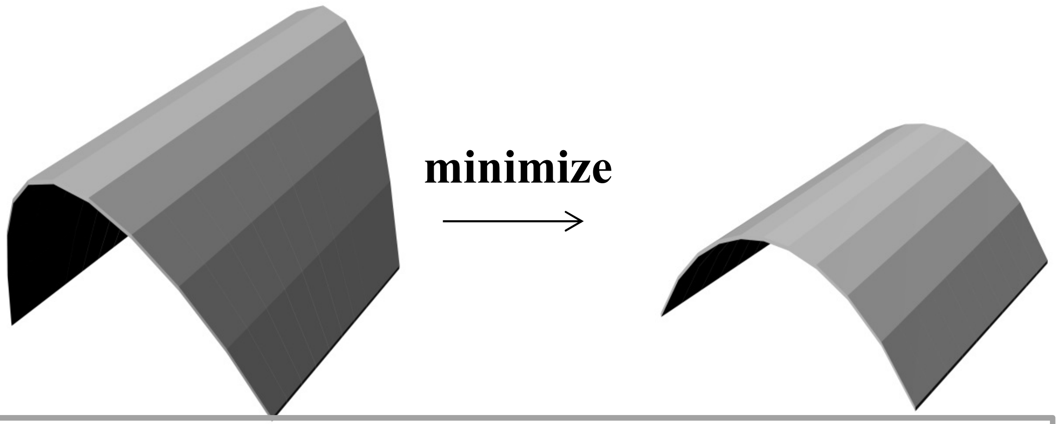
Mechanical performance measure
(Strain energy)

$$f = \frac{1}{2} d^{\mathrm{T}} P$$

d	:	displacement vector
P	:	load vector
S_0	:	initial surface area
S	:	surface area

⇨ Minimize

Nonlinear programming
Sequential quadratic programming



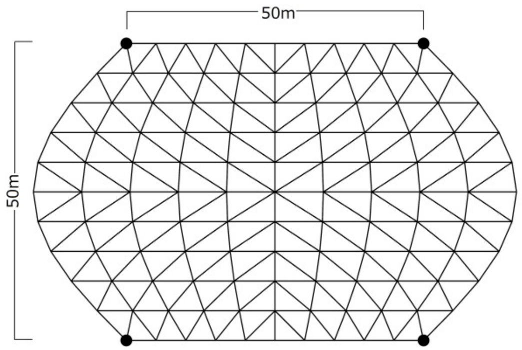
Reduce surface area to improve stiffness

Surface area constraint: $S - S_0 \geqq 0$

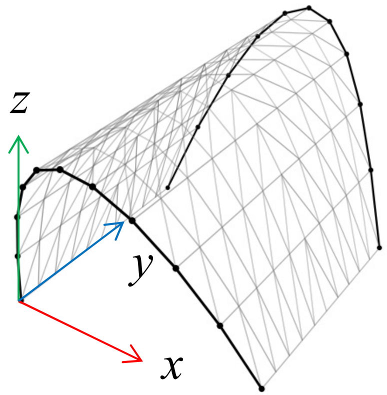
Prevent shrinkage

Case 1

Arch-shaped boundary curves

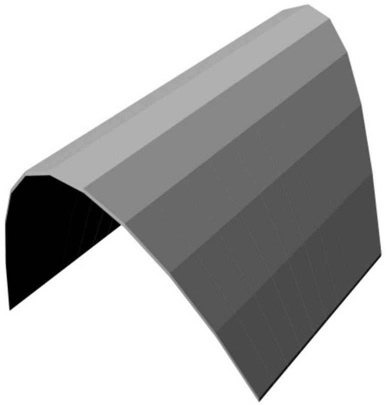


FEM Mesh

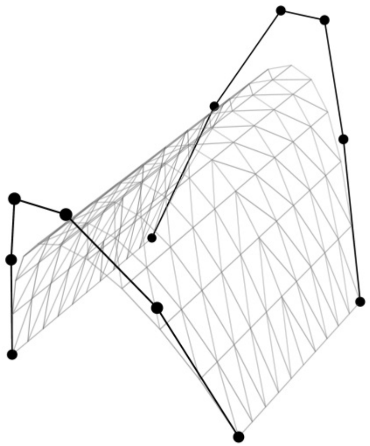


Bezier Curve

Young's modulus $[GPa]$	206
Poisson's ratio	0.2
Weight density $[kN/m^3]$	24.5
Thickness $[m]$	0.2



Initial Shape

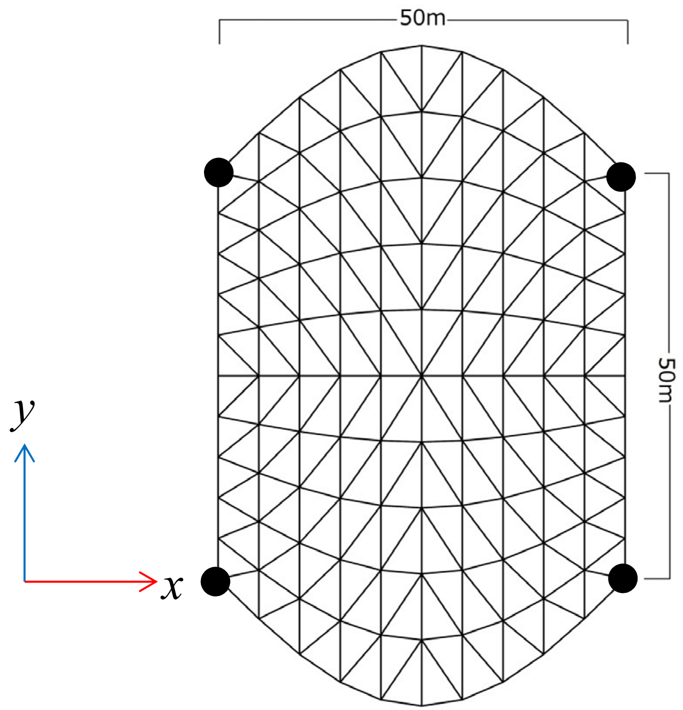


Control Curve

Control points of Bézier curves

$$\boldsymbol{q}_x, \boldsymbol{q}_y, \boldsymbol{q}_z$$

Case 1



Variables: x, y, z coordinates of control points except four corners

minimize $f(q_x, q_y, q_z)$

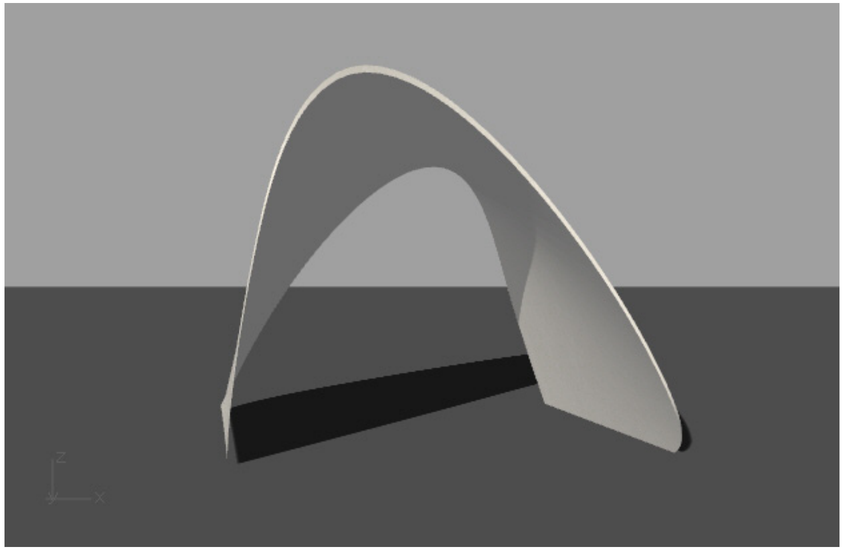
subject to $S - S_0 \geqq 0$

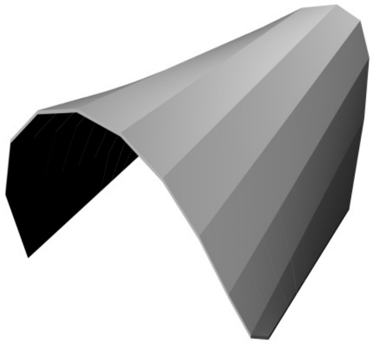
Boundary condition: fix four corners

Obj. func. : $f(x) = 171.7\ [kNm]$
Surface area: $S_0 = 6597\ [m^2]$
Vertical disp. : $d_z = 50.1\ [mm]$

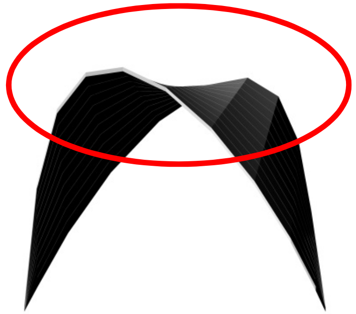
Case 1

- Twisted ridge of roof.
- Strain energy is drastically reduced.
- Surface area satisfies constraint.





Perspective view



Elevation 1

Elevation 2

Plan

Initial

Obj. func. : $f(x)=171.7\ [kNm]$

Surface area: $S_0 = 6597\ [m^2]$

Max. disp. : $d_z = 50.1\ [mm]$

Optimal

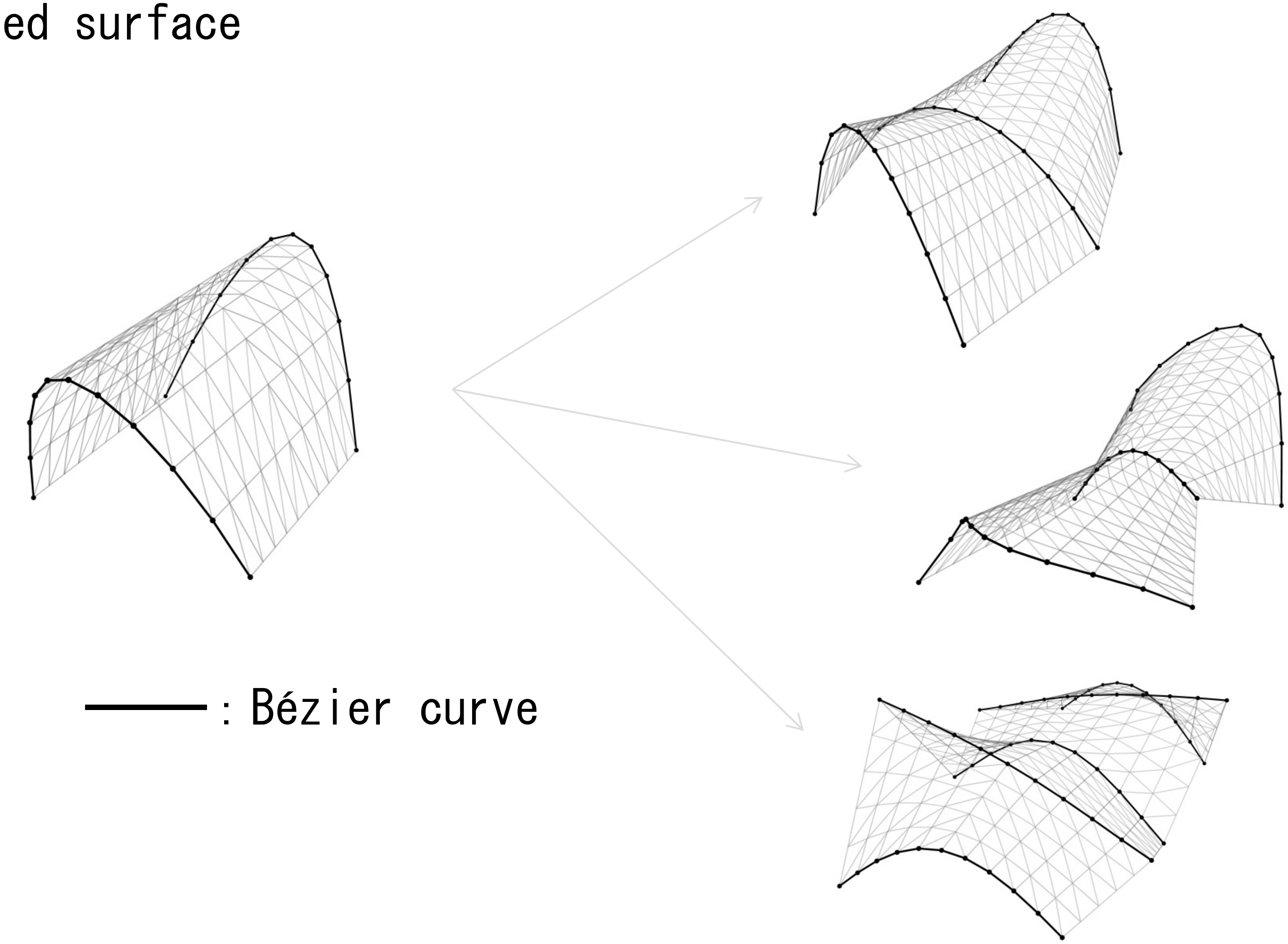
Obj. fun : $f(x)=11.7\ [kNm]$

Surface area: $S_0 = 6597\ [m^2]$

Max. disp. : $d_z = 3.5\ [mm]$

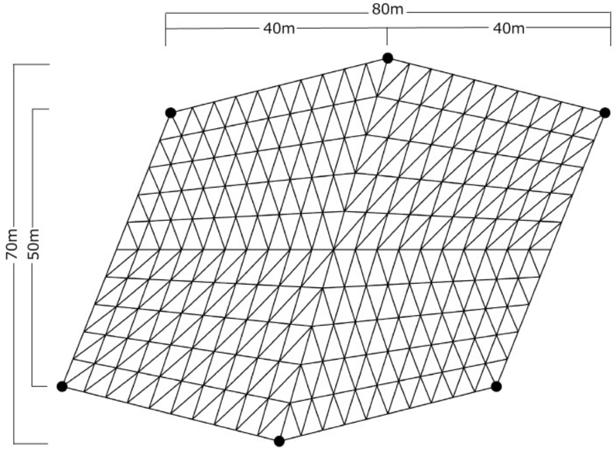
Case 2

Three boundary curves for ruled surface ⇨ Various surface shapes

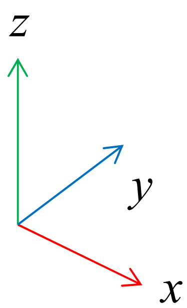


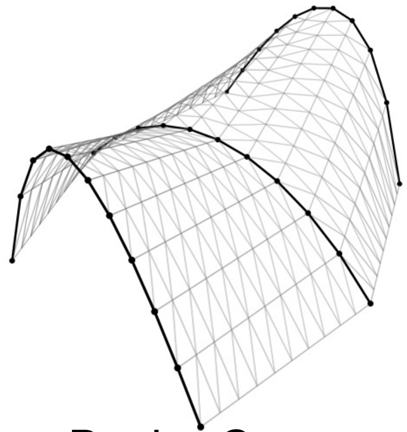
Case 2

Three boundary curves → Two ruled surfaces

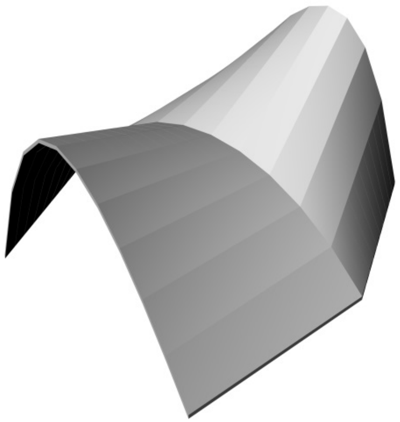


FEM Mesh

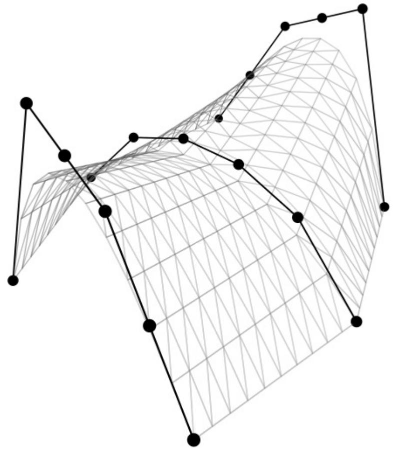




Bezier Curve

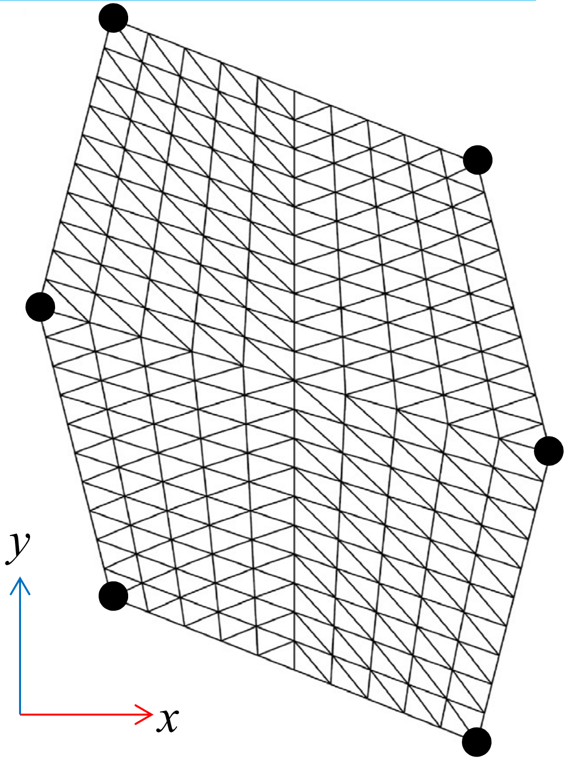


Initial Shape



Control Curve

Case 2



●： Fix displacements

Variables: x, y, z coordinates of control points except six ends

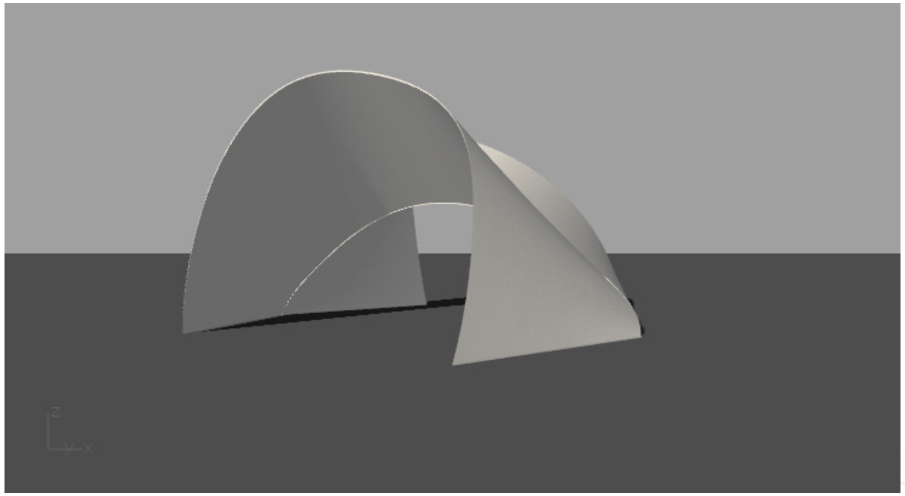
minimize $f(q_x, q_y, q_z)$

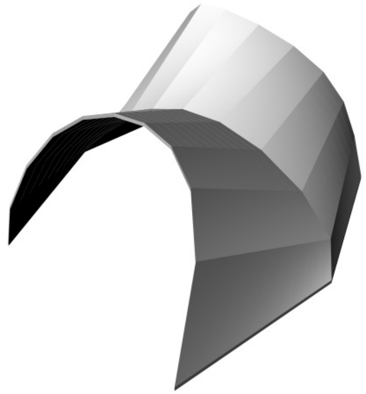
subject to $S - S_0 \geqq 0$

Obj. func.: $f(x) = 171.7\ [kNm]$
Surface area: $S_0 = 6597\ [m^2]$
Max. disp.: $d_z = 50.1\ [mm]$

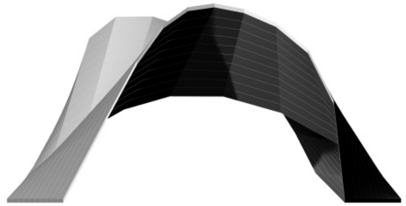
Case 2

- Roof becomes steeper
- Strain energy is also reduced

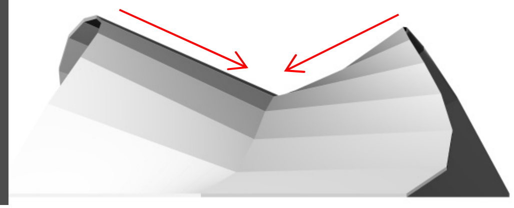




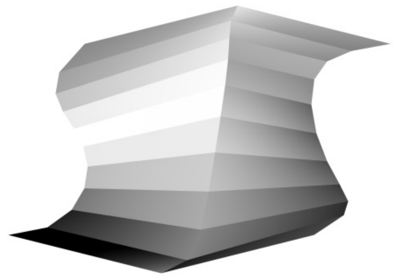
Perspective view



Elevation 1



Elevation 2



Plan

Initial

Obj. func. : $f(x) = 22.5\ [kNm]$

Surface area: $S_0 = 6515\ [m^2]$

Max. disp. : $d_z = 13.3\ [mm]$

Optimal

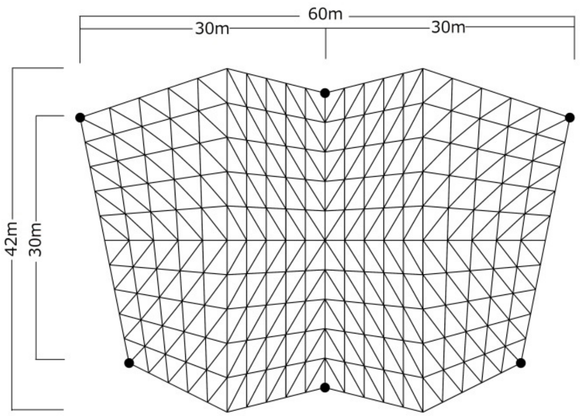
Obj. func. : $f(x) = 9.5\ [kNm]$

Surface area: $S_0 = 6516[m^2]$

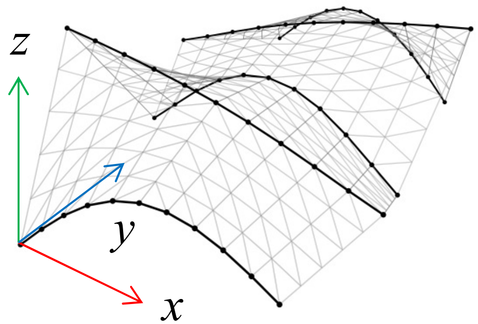
Max. disp. : $d_z = 3.2[mm]$

Case 3

Five boundary curves



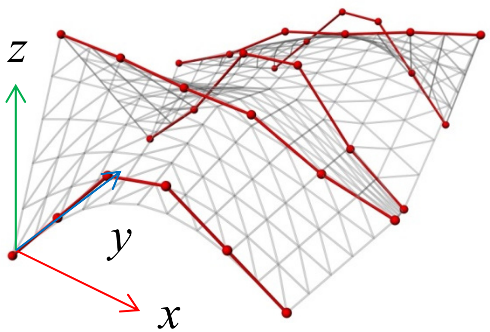
FEM Mesh



Bezier Curve

Increase of number of curves
＝ increase of number of design variables
⇒ Loss of convergence performance

Initial Shape

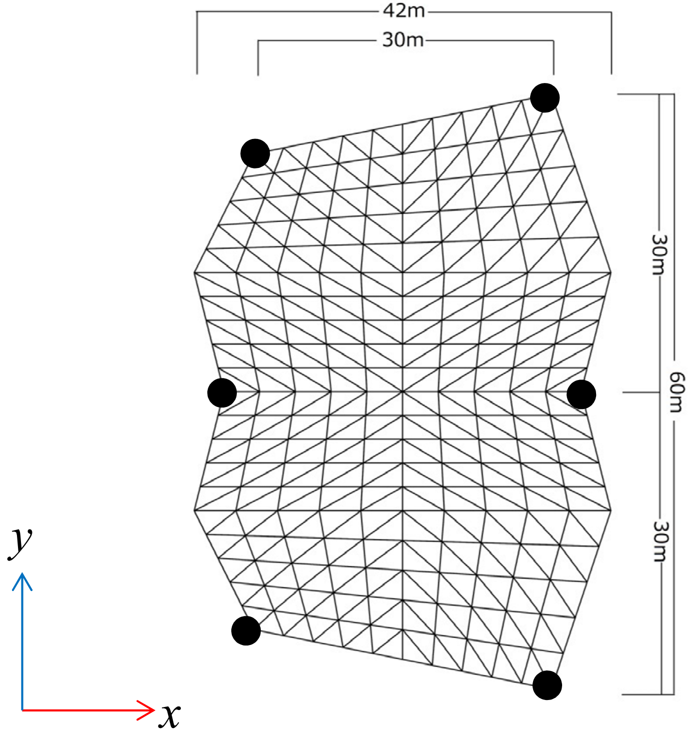


Control Curve

Fix x-coordinates of control points
⇒ Variables are y and z coordinates

$$\boldsymbol{q}_y, \boldsymbol{q}_z$$

Case 3



●： Fix displacements

Obj. func.： $f(x) = 1.80\ [kNm]$
Surface area： $S_0 = 2534\ [m^2]$
Max. disp.： $d_z = 2.0\ [mm]$

Variables： y and z coordinates of control points except ends of boundary curves.

minimize $f(q_y, q_z)$

subject to $S - S_0 \geqq 0$

Case 3

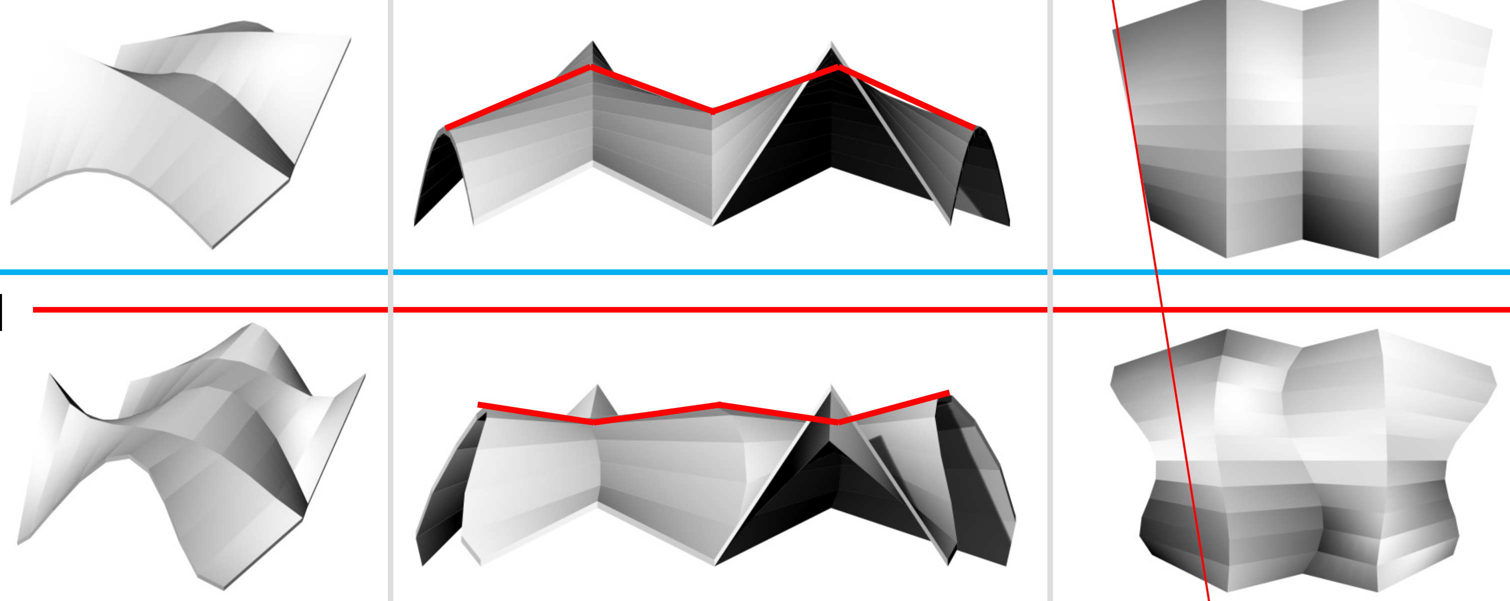
Strain energy is reduced

Convexity of center is reversed

Perspective | Elevation | Plan

Initial

Optimal



initial

Obj. func. : $f(x)=1.80\,[kNm]$

Surface area: $S_0 = 2534\,[m^2]$

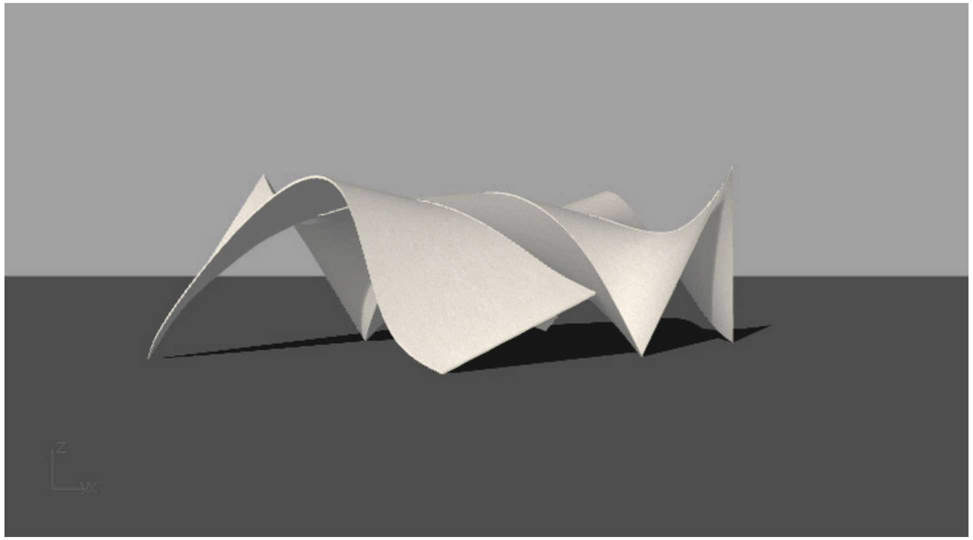
Max. disp. : $d_z = 2.0\,[mm]$

Optimal

Obj. func. : $f(x)=0.98\,[kNm]$

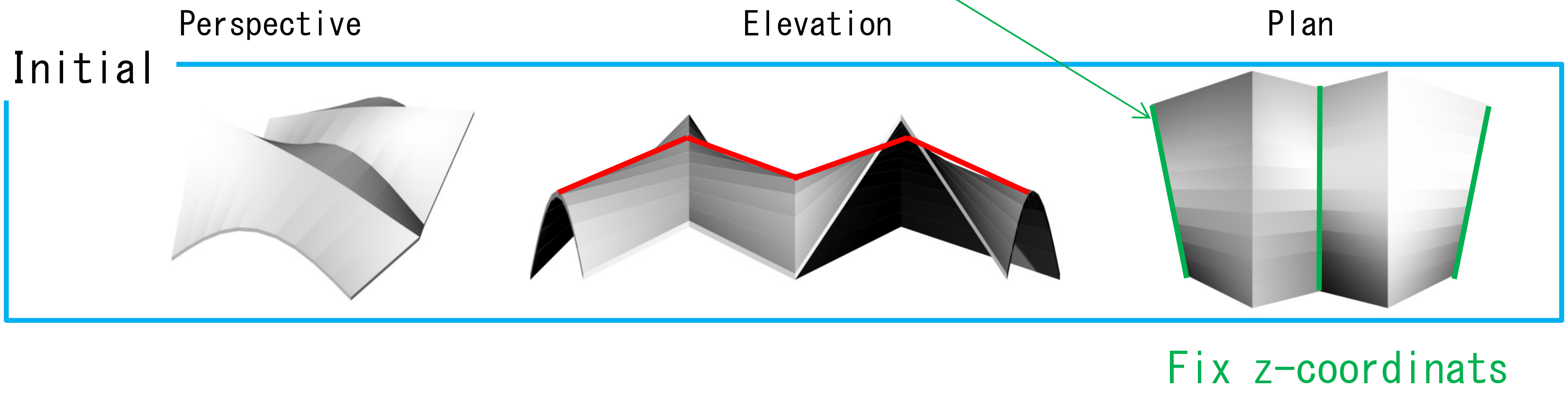
Surface area: $S_0 = 2534\,[m^2]$

Max. disp. : $d_z = 0.5\,[mm]$



Case 3

Fix z-coordinates of curves at sides and center



Shape deviation

Y-dir. $$D_y = \sqrt{\sum_{i=1}^{n=5}\sum_{j=1}^{m=11}\left(d_{y_{ij}} - d_{yo_{ij}}\right)^2}$$

Z-dir. $$D_z = \sqrt{\sum_{i=1}^{n=5}\sum_{j=1}^{m=11}\left(d_{z_{ij}} - d_{zo_{ij}}\right)^2}$$

d_y : optimal y−coordinate

d_{yo} : initial y−coordinate

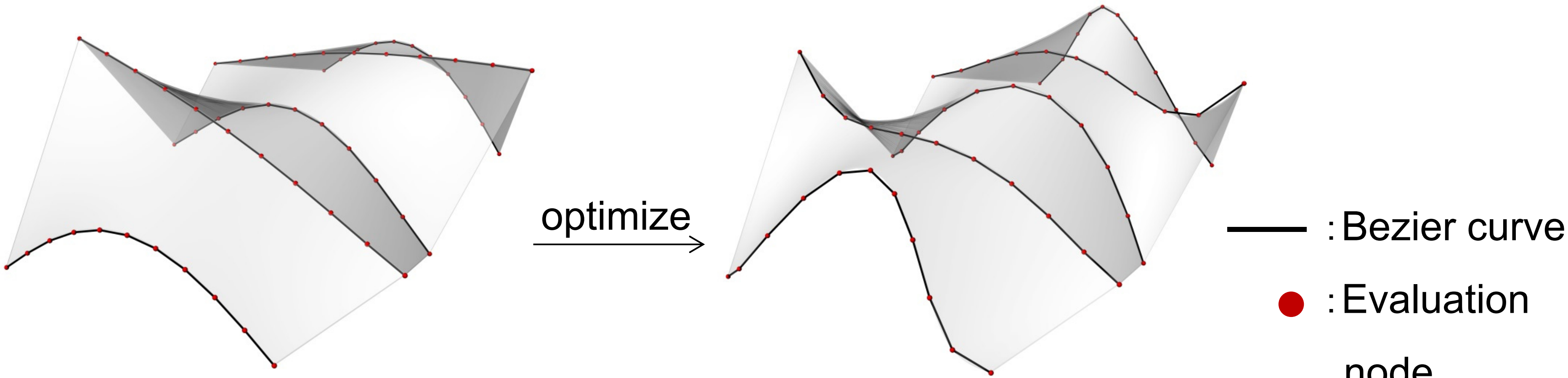
d_z : optimal z−coordinate

d_{zo} : initial z−coordinate

n : number of curves

m : number of nodes on a curve (11))

Model 3



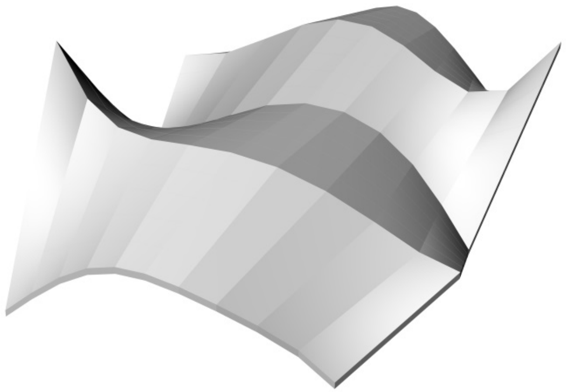
$D_y = 13.3[m], D_z = 23.3[m]$

Initial shape

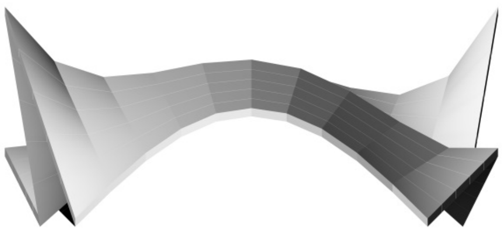
Optimal shape

Case 3

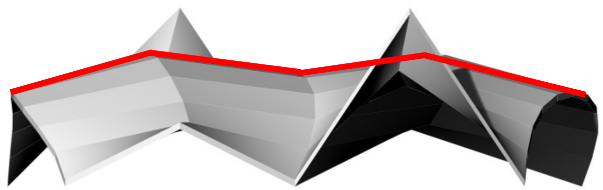
Reduce strain energy maintaining geometrical property.



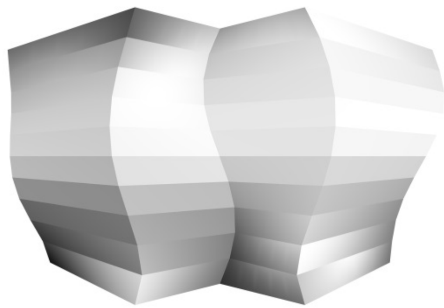
Perspective view



Elevation 1



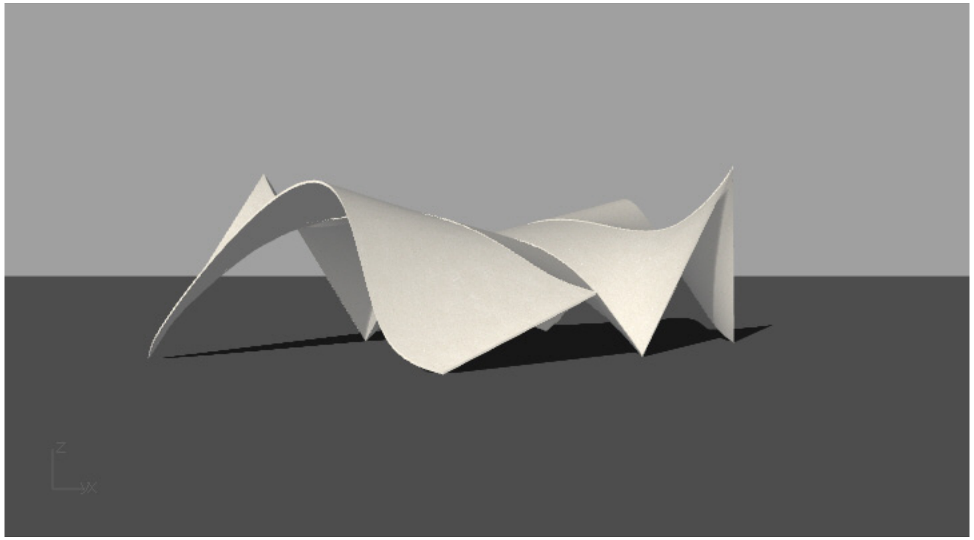
Elevation 2



Plan

Obj. func.: $f(x) = 1.24\ [kNm]$
Surface area: $S_0 = 2534\ [m^2]$
Max. disp.: $d_z = 0.6\ [mm]$

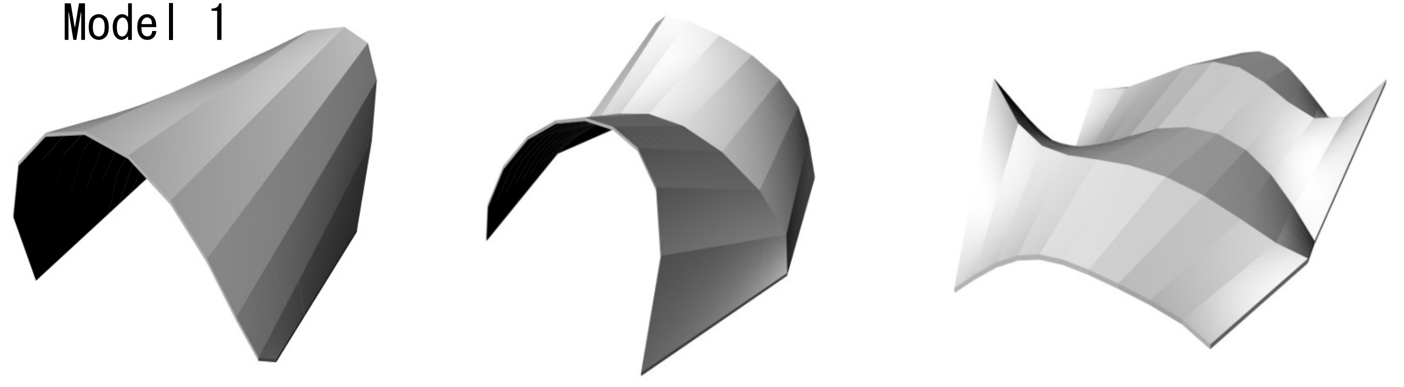
Non-restricted: $D_y = 13.3[m], D_z = 23.3[m]$
Restructed: $D_y = 14.9[m], D_z = 19.8[m]$



Seismic load

Horizontal load of 0.2G

Optimal shapes



	Model 1	Model 2	Model 3
Maximum horizontal displacement	88.8 mm	94.9mm	9.8 mm
Max.disp./ Max. Span	1/563	1/738	1/3673

Large horizontal stiffness

Conclusions

- Bézier curves for modeling ruled surface.
  → Various shapes can be easily defined by small number of parameters (coordinates of control points).
- Ruled surface has a straight line in one direction at any point on the surface.
  → Manufacturing cost of a free-form shell surface may be reduced.
- Optimal shape tends to have small surface area to increase static stiffness.
  → Lower-bound constraint should be given for the surface area.

Conclusions

- Optimal shapes can be found using NLP. However, the solution strongly depends on the initial shape.
- Shell roofs with internal rib arches can be generated by connecting several ruled surfaces.
- Convergence property of optimization process can be enhanced and desired property of the surface can be retained by restricting the variables for coordinates of the control points.