Machine Learning for Layout optimization of Braces of Building Frames

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Background

• Seismic retrofit of building frames.
• Add braces to upgrade stiffness.
• Increase of stress in existing beams and columns.
• Optimal locations of braces considering effect on existing members.
Difficulty in combinatorial problems

• Combinatorial problem:
  – Difficult to use mathematical programming.
  – Use heuristics.
  – Large computational cost for problem with many variables.

Use machine learning for reduction of computational cost.
Support vector machine (SVM)

- Classification to 2 classes
- Linear or nonlinear (Gauss) kernel
Binary decision tree

- Classification to multiple classes
- Regression tree for numeric attribute

\[
\begin{align*}
x_1 & \leq 1 \\
x_1 & > 1 \\
x_2 & \leq 0.5 \\
x_2 & > 0.5
\end{align*}
\]
Application of machine learning to structural optimization

- Neural network for prediction (approximation) of structural responses
- Optimal member grouping
- Optimal parameters for heuristics
- Shape optimization of periodic structures
- Learning features of feasible solutions
- Optimal search region/direction in heuristic approach
Optimization of brace locations

- Plane steel frame
- Algorithm: simulated annealing (SA)
- Five patterns of braces

- Consider seismic retrofit
  - Fix cross-sections of beams and columns
  - Optimize patterns and locations of braces
Design variables

- Five patterns denoted by $x = 1, 2, 3, 4, 5$
  $y = (y_1, y_2, y_3, \ldots, y_m), (y_i \in \{1, 2, 3, 4, 5\})$

- Number of locations = 15
Optimization problem

• Objective function:
  \[ \text{minimize maximum stress} \sigma_{max} \text{ of beams and columns} \]

• Constraints:
  \[ \text{interstory drift angle} \delta_{max} \leq 0.005 \]
  \[ \text{number of braces in each story} \leq 2 \]
Simulated annealing (SA)

1. Randomly generate initial solution.
2. Randomly generate neighborhood solutions and select their best solution. Select the best solution with probabilistically even it does not improve the objective value.
3. Reduce the temperature parameter if the termination condition is not satisfied and go to 2.

\[ P_r = \exp \left( - \frac{\Delta F}{cT} \right) \]
Outline of SA

Current solution

Neighborhood solutions

Analysis

Current solution

Neighborhood solutions
Preprocessing: binarization

• SVM handles only ordered variables → binarization using dummy variables

• Integer variable \( y_i \in \{1,2,3,4,5\} \) is converted to five dummy binary variables \( x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5} \in \{0,1\} \)

\[
\begin{array}{c}
\bar{y}_n \\
\hline
4 \\
\end{array} \quad \text{Binarization} \quad \begin{array}{cccc}
x_{n1} & x_{n2} & x_{n3} & x_{n4} & x_{n5} \\
0 & 0 & 0 & 1 & 0 \\
\end{array}
\]
Preprocessing: convolution

- Relation to neighboring brace is important
  → Filtering by convolution

- Convolution:
  Extract features applying filters to original data

Example of filter
Preprocessign: pooling

- Increase of number of variables by convolution
  → Reduce variables by pooling

Example: combine two same features in the same story.
Accuracy of learning

- Label +1: Approximate optimal
- Label −1: Non-optimal
- Small ratio of ‘False-Negative’ → High accuracy of learning

<table>
<thead>
<tr>
<th>Predicted Label</th>
<th>Actual label</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+1</td>
</tr>
<tr>
<td>+1</td>
<td>True Positive (TP)</td>
</tr>
<tr>
<td>−1</td>
<td>False Negative (FN)</td>
</tr>
</tbody>
</table>
Optimization result

- 5-story 3-span frame
- Story height: 4 m, Depth 6 m
- No. of steps: 1000
  No. of neighborhood solutions: 50
- Initial temperature: 1.0
- Temperature reduction ratio: 0.99
Analysis model

- Use OpenSees for analysis
- Beam-Column element for all members
- Pin-support at column base
- Apply only horizontal loads calculated from Japanese building regulation
- Model rigid floor by multiplying axial stiffness of beam by 10
- Assign large stiffness for base beam
Optimization problem

• Objective function:
  minimize maximum stress $\sigma_{max}$ of beams and columns

• Constraints:
  interstory drift angle $\delta_{max} \leq 0.005$
  number of braces in each story $\leq 2$

• Select the best solution among 20 trials of SA
Optimal solution

Table:

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\text{max}}$ (N/mm$^2$)</th>
<th>$\theta_{\text{max}}$</th>
<th>V (m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No brace</td>
<td>649.54</td>
<td>0.0220</td>
<td>0</td>
</tr>
<tr>
<td>Optimal</td>
<td>84.83</td>
<td>0.0019</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Comparison to standard layout

- Optimal solution is better than standard layout

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$\sigma_{\text{max}}$ (N/mm$^2$)</th>
<th>$\theta_{\text{max}}$</th>
<th>V (m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern A</td>
<td>169.32</td>
<td>0.0028</td>
<td>0.27</td>
</tr>
<tr>
<td>Pattern B</td>
<td>221.82</td>
<td>0.0034</td>
<td>0.18</td>
</tr>
<tr>
<td>Pattern C</td>
<td>118.57</td>
<td>0.0016</td>
<td>0.37</td>
</tr>
<tr>
<td>Optimal</td>
<td>84.83</td>
<td>0.0019</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Learning for 2 classes

• Randomly generate 10000 solutions
• Approximate optimal: top 10% solutions
  Non-optimal: worst 10% solutions
Five strategies for convolution and pooling

S1

S2

pooling

filter

S3

S4

S5
Learning for 2 classes

• Two types of filters

Filter 4 (exclude no-brace): $4 \times 4 \times 4 = 64$ patterns
Filter 5 (exclude no-brace): $5 \times 5 \times 4 = 100$ patterns
## Learning results by SVM

<table>
<thead>
<tr>
<th></th>
<th>Filter 4</th>
<th></th>
<th></th>
<th>Filter 5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error</td>
<td>FN</td>
<td>FP</td>
<td>Error</td>
<td>FN</td>
<td>FP</td>
</tr>
<tr>
<td>S1</td>
<td>0.0250</td>
<td>17/1000</td>
<td>35/1000</td>
<td>0.0250</td>
<td>17/1000</td>
<td>35/1000</td>
</tr>
<tr>
<td>S2</td>
<td>0.0410</td>
<td>41/1000</td>
<td>28/1000</td>
<td>0.0110</td>
<td>10/1000</td>
<td>10/1000</td>
</tr>
<tr>
<td>S3</td>
<td>0.0560</td>
<td>64/1000</td>
<td>28/1000</td>
<td>0.0120</td>
<td>15/1000</td>
<td>18/1000</td>
</tr>
<tr>
<td>S4</td>
<td>0.1495</td>
<td>133/1000</td>
<td>144/1000</td>
<td>0.0385</td>
<td>26/1000</td>
<td>57/1000</td>
</tr>
<tr>
<td>S5</td>
<td>0.2050</td>
<td>194/1000</td>
<td>221/1000</td>
<td>0.0965</td>
<td>107/1000</td>
<td>106/1000</td>
</tr>
</tbody>
</table>
Learning result by SVM

• Calculation of score

\[ S(x) = \frac{1}{a} \cdot \beta \cdot x + b \]

\[ \beta = (\beta_1, \ldots, \beta_m) : \text{coefficient vector} \]

• If \( \beta_i \) is large (positive), \( x_i = 1 \) contributes to approximate optimal solution.

• If \( \beta_i \) is small (negative), \( x_i = 1 \) contributes to non-optimal solution.
Filters characterizing approximate optimal solutions
Filters characterizing non-optimal solutions
# Learning results by BDT

<table>
<thead>
<tr>
<th></th>
<th>Filter 4</th>
<th></th>
<th>Filter 5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error</td>
<td>FN</td>
<td>FP</td>
<td>Error</td>
</tr>
<tr>
<td><strong>S1</strong></td>
<td>0.0390</td>
<td>38/1000</td>
<td>40/1000</td>
<td>0.0390</td>
</tr>
<tr>
<td><strong>S2</strong></td>
<td>0.0725</td>
<td>65/1000</td>
<td>63/1000</td>
<td>0.0290</td>
</tr>
<tr>
<td><strong>S3</strong></td>
<td>0.0920</td>
<td>76/1000</td>
<td>73/1000</td>
<td>0.0340</td>
</tr>
<tr>
<td><strong>S4</strong></td>
<td>0.2309</td>
<td>317/1000</td>
<td>107/1000</td>
<td>0.0930</td>
</tr>
<tr>
<td><strong>S5</strong></td>
<td>0.3413</td>
<td>407/1000</td>
<td>263/1000</td>
<td>0.2194</td>
</tr>
</tbody>
</table>
Feature tree by BDT
SA with machine learning

Current solution

Neighborhood solutions

Approximate optimal

Yes

Analysis

No

Neighborhood solutions
# Comparison of computational time

<table>
<thead>
<tr>
<th></th>
<th>SA</th>
<th>SVM</th>
<th>BDT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learning</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis (s)</td>
<td>---</td>
<td>2093</td>
<td>2093</td>
</tr>
<tr>
<td>Learning (s)</td>
<td>---</td>
<td>12.4</td>
<td>6.1</td>
</tr>
<tr>
<td><strong>Optimization</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prediction (s)</td>
<td>---</td>
<td>483.7</td>
<td>360.5</td>
</tr>
<tr>
<td>Analysis (s)</td>
<td>14314.3</td>
<td>7961.6</td>
<td>7162.6</td>
</tr>
<tr>
<td>Optimization incl. learning (s)</td>
<td>14314.3</td>
<td>10550.7</td>
<td>9622.2</td>
</tr>
<tr>
<td><strong>Number of analyses</strong></td>
<td>67368</td>
<td>35710</td>
<td>27819</td>
</tr>
<tr>
<td><strong>Optimal objective value</strong></td>
<td>84.83 N/mm²</td>
<td>87.08 N/mm²</td>
<td>87.51 N/mm²</td>
</tr>
</tbody>
</table>
Optimization result

SA

SA with SVM

SA with BDT
Conclusions

• A method based on SA for optimization of brace locations of building frames.
• Minimize maximum additional stress of beams and columns under horizontal static loads representing seismic loads.
• Distinct classification of approximate optimal and non-optimal solutions is effective to improve the accuracy of learning and prediction.
• Convolution using filters with ‘no brace’ generally improves the accuracy of prediction; however, it increases the computational cost.
• Pooling in each of lower stories is effective to reduce the number of variables, while maintaining the accuracy.
Conclusions

• Computational cost can be successfully reduced using BDT or SVM for detecting non-optimal solutions during optimization.

• Properties of approximate optimal and non-optimal solutions can be extracted from the feature trees as an output of BDT and the coefficients of the function for estimating the score of SVM.

• Braces should be continuously located to reduce the additional stresses in beams and columns due to horizontal seismic loads.