# A Continuous Topology Transition Model for Shape Optimization of Plane Trusses with Uniform Cross-Sectional Area

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**1. Abstract**

A continuous topology transition model (CTTM), which simulates continuous transition between trusses with different topologies, is developed for general plane trusses. A procedure using the CTTM and a simulated annealing method is presented for simultaneous optimization of topology and geometry of a plane truss with uniform cross-sectional area. Re-annealing procedures are introduced to improve the convergence property to global optima. In the examples, optimal solutions are found for cantilever-type and bridge-type plane trusses under displacement constraints.

**2. Keywords**

Topology, Optimization, Truss, Simulated annealing, Geometry optimization, Re-annealing

**3. Introduction**

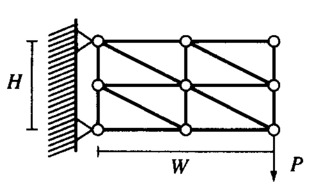
It is widely recognized that simultaneous optimization of topology and geometry of a truss is extremely difficult [1]. Recently, several optimization algorithms have been presented based on the heuristics such as genetic algorithms (GA) [2-5] and simulated annealing (SA) [6,7] and based on the concepts in which the optimal trusses are obtained from simple trusses by adding new nodes and members [1,6,8,9]. In those algorithms, however, the topological variables or the rules for addition and removal of nodes and members need to be introduced. Therefore computational effort seems to be very large if those algorithms are applied for optimizing large trusses. In this paper, a new CTTM is developed for general plane trusses. A procedure using the CTTM and SA is presented for simultaneous optimization of topology and geometry of a plane truss with uniform cross-sectional area.

**4. Optimization Problem**

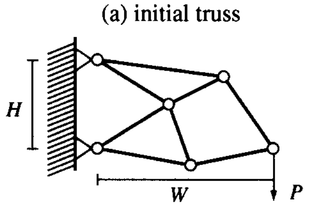
Problem Formulation

Consider a pin-jointed plane truss, in which the coordinates of the supports and the loaded joints are fixed. An illustrative example for optimization of a cantilever-type truss is as shown in Fig.1, where the distances of $H$ and $W$ are fixed. The optimal design problem of trusses considered in this study is stated as follows:

> Find the nodal coordinates, topology and the cross-sectional area that minimize the total structural volume subject to the displacement constraints and the requirement on the cross-sectional area such that all the members have the same value.

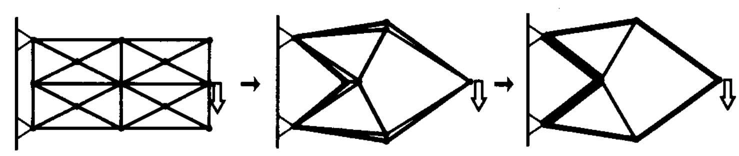



(a) initial truss

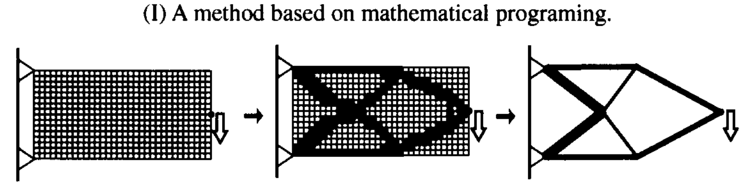



(b) optimal truss with uniform cross-sectional area

Figure 1.
Shape optimization of a plane truss.



(a) initial solution (truss) (b) optimal solution (c) equivalent truss
(I) A method based on mathematical programing.



(a) initial solution (plate) (b) optimal solution (c) equivalent truss
(II) A method based on homogenization method.

Figure 2.
Illustrations of simultaneous optimization of topology and geometry of a truss.

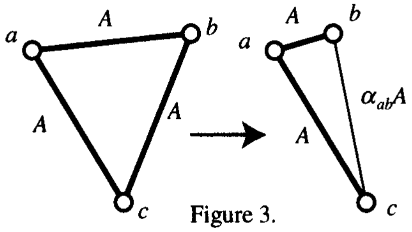



Figure 3.
Concept of CTTM ; (node $b$ approaches to node $a$)

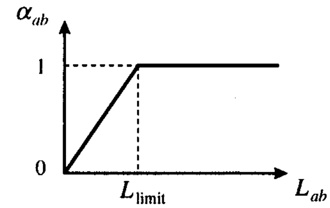
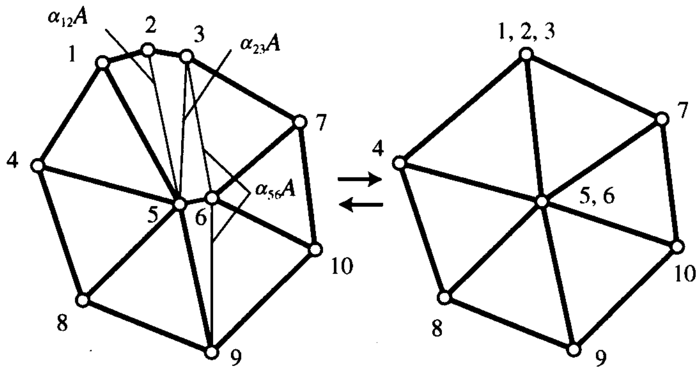



Figure 4.
Reduction factor for cross-sectional area.




(a) 19-bar truss (b) 12-bar truss
Figure 5.
Illustration of CTTM for the truss with multiple triangular units.

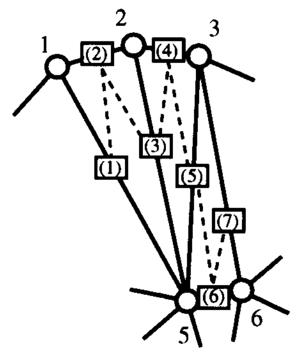


Figure 6.
A sequence of members for the CTTM in the truss of Fig.5(a).

Requirement on Member Cross-sectional Area

The requirement on the member cross-sectional area such that all the members have the same value is important in view of the practical design, because it is not practically admissible to allow the cross-sectional areas to have arbitrary different positive values. It should be noted here that due to this requirement the simultaneous optimization of topology and geometry of a truss turns out to be made more difficult. In the problem where this requirement is not considered, the optimal truss can be obtained as an equivalent truss to the optimal solution by a method based on the mathematical programing (See Fig.2(I)) or a method based on the homogenization method [10] (See Fig.2(II)), while these methods cannot be applied to the problem considered in this study.

**5. Continuous Topology Transition Model for General Plane Truss**

For simultaneous optimization of topology and geometry of regular plane trusses, the second author presented a continuous topology transition model (CTTM) which simulates continuous transition between trusses with different topologies [11]. The present authors developed an optimization algorithm using the CTTM and GA, and the effectiveness of the CTTM has been revealed though the examples [12]. However the CTTM presented in Ref.[11,12] is limited to the regular plane trusses. In this study, the CTTM is extended for general plane trusses.

Concept of Extended CTTM

The concept of the extended CTTM is illustrated by using a single triangular truss unit as shown in Fig.3. Let $L_{kl}$ and $A_{kl}$ denote respectively the length and the cross-sectional area of the member connecting node $k$ and $l$. In the left truss in Fig.3, in which $L_{ab}, L_{bc}, L_{ac}$ are greater than the sufficiently small length $L_{\text{limit}}$, all members have the same cross-sectional area; i.e. $A_{ab} = A_{bc} = A_{ac} = A$, while in the right truss, in which $L_{bc}, L_{ac} > L_{\text{limit}}$ and $L_{ab} < L_{\text{limit}}$, the cross-sectional area $A_{bc}$ is reduced as $A_{bc} = \alpha_{ab}A$, where $\alpha_{ab}$ is the reduction factor defined by the following equations as shown in Fig.4:

$$\begin{cases} \alpha_{ab} = \dfrac{L_{ab}}{L_{\text{limit}}} & (0 \le L_{ab} < L_{\text{limit}}) \\ \alpha_{ab} = 1 & (L_{\text{limit}} \le L_{ab}) \end{cases} \tag{1}$$

Note that $\alpha_{ab}$ can be multiplied to $A_{ac}$ instead of $A_{bc}$.

CTTM for General Plane Truss

An example of the CTTM for the truss with multiple triangular units is shown in Fig.5. In the 19-bar truss, $L_{12}, L_{23}$ and $L_{56}$ are shorter than $L_{\text{limit}}$ and hence $0 < \alpha_{12}, \alpha_{23}, \alpha_{56} < 1$. Since three triangular units exist in the region surrounded by nodes 1, 2, 3, 6 and 5, an algorithm must be introduced to select members in which the cross-sectional area is reduced according to the reduction factor. A sequence of member numbers is first defined so that long members and short members appear alternately as shown in Fig.6. In the sequence, the long member is assigned with odd number and the short one, even number. Then, the cross-sectional area of the first member in the sequence is set to the uniform value $A$. Finally, the cross-sectional area of the $(2j+1)$-th member is calculated by the reduction factor defined by the length of the $(2j)$-th member, where $j$=1, 2, 3. Although the optimal solution depends on the numbering process, the difference of the mechanical properties of the trusses, e.g., with $A_{15} = A$ and $A_{36} = A$ is sufficiently small because $L_{12}$, $L_{23}$ and $L_{56}$ are less than $L_{\text{limit}}$.

If the CTTM is applied to the optimization of a pin-jointed truss, existence of extremely thin or short members leads to singularity of the stiffness matrix. Therefore the truss is modeled as an equivalent rigidly-jointed frame with sufficiently small radius of gyration used $\gamma$, and the lower bound $D$ is given for the member length [11].

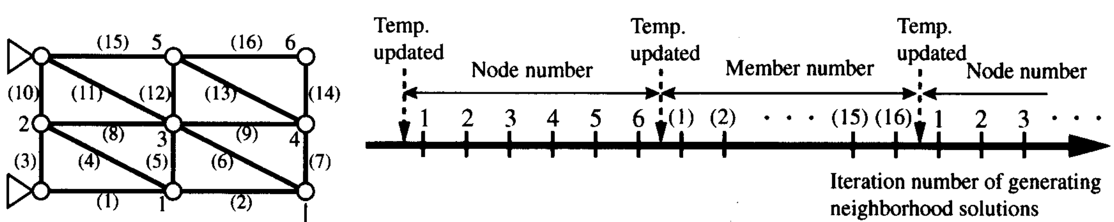
Fig. 5 shows that the 19-bar truss is reduced to the 12-bar truss when the nodes 1,2,3 and 5,6 coincide, respectively. It should be noted that in the CTTM the topology varies only in the appearance while in the computational sense the topology never vary. Therefore in the optimization process the 12-bar truss can return to the 19-bar truss when the nodes 1,2,3 and 5,6 separate, respectively. The advantage of using the CTTM is summarized as: (a) the difficulties due to the requirement on cross-sectional area, such that all the members must have the same value, is successfully avoided, and (b) the topological variables and the rules for addition and removal of nodes and members need not be introduced and therefore computational effort is dramatically reduced.

## 6. Optimization Algorithm Based on Simulated Annealing

Since the reduction factor in the CTTM is defined as a piecewise linear function of member length which has the discontinuity in its gradient as shown in Fig.4, the traditional gradient-based optimization techniques are difficult to be applied. In the present study, the simulated annealing procedures [13] are used as the optimization techniques.

Generating Neighborhood Solutions

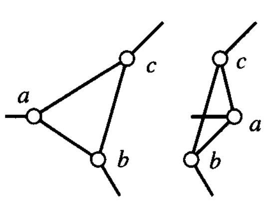
In order to generate neighborhood solutions, a nodal coordinate is varied randomly within the specified distance $R_{\max}$.




(a) node- and member-numbers

(b) variation of the node- and member-numbers updated for generating neighborhood solutions

Figure 7.
Example of order of design variables updated for generating neighborhood solutions and timing of temperature updated.




(a) feasible (b) infeasible

Figure 8.
Constraints of nodal location based on the signed area.

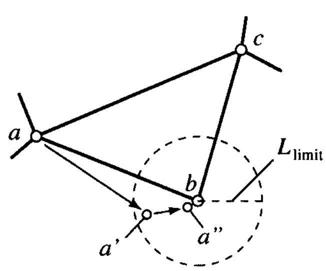



Figure 9.
Forced node movement.

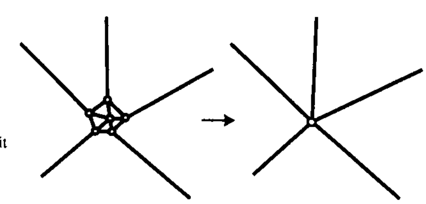


Figure 10.
Combination of nodes at the beginning of re-annealing.

Since the CTTM cannot consider a truss including a quadrilateral truss unit such as Fig.15(e), a variable that defines the existence of a member is also used only for the case where a quadrilateral truss unit is allowed to exist. In generating the neighborhood solution, the variables including nodal coordinates and the variables that define the existence of members are changed sequentially at each step; i.e. only one variable is updated (See Fig.7).

Probabilistic Replacement Criterion and Annealing Schedule

The neighborhood solution replaces the current solution with a probability,

$$Q = \min\left[\,1, \exp\left(-\Delta E / T\right)\right], \quad \Delta E = E' - E \tag{2a,b}$$

where $E$ and $E'$ denote the objective values of current and neighborhood solutions, respectively. $T$ is the strategy temperature, which decreases according to the functions as follows,

$$T_{k+1} = \eta\, T_k \tag{3}$$

where $T_k$ denotes the temperature at the $k$-th step and $\eta$ denotes the temperature reduction factor. The temperature is updated at the point that each cycle of updating nodal coordinates or the variables which define the existence of members is completed as shown in Fig.7.

Techniques for Improvement of Convergency to Global Optimal Solutions

(T-1) Constraints on nodal locations based on the signed area of the triangular unit are introduced as shown in Fig.8 to avoid the intersection of members.

(T-2) If a member in the converged solution has length between 0 and $L_{\text{limit}}$, the cross-sectional area of one of the adjoining members is between 0 and $A$, and hence the solution is infeasible. To avoid convergence to such a solution (See Fig.9), when node ($a$) is moved in the process of generating the neighborhood solution to the point ($a'$) in the region in which an adjoining member length is shorter than $L_{\text{limit}}$, the node is forced to move to the point ($a''$) where the member length has a sufficiently short value and the reduced value of the cross-sectional area is set to $A/10000$.

(T-3) Sufficient number of nodes and members must be given in the initial structure to obtain optimal trusses with various topologies. If the numbers of nodes and members are increased, however, the possibility also increases for

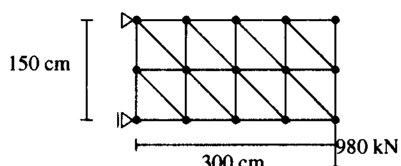



Figure 11.
Initial truss (Ex.1).

Table 1.
Annealing schedule (Ex.1).

| | Initial Temp. ; $T_0$ | Temp. reduction factor ; $\eta$ |
|---|---|---|
| Case A | 3000 | 0.97 |
| Case B | 1500 | 0.97 |
| Case C | 6000 | 0.97 |
| Case D | 3000 | 0.95 |

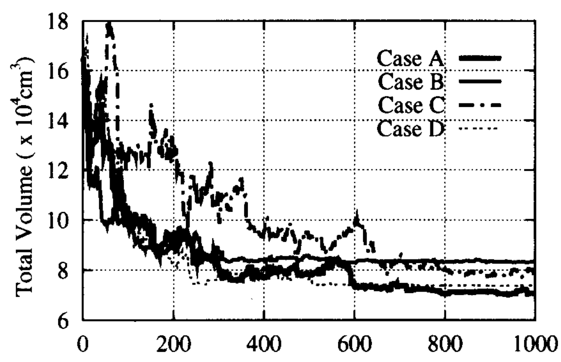



Figure 12.
Variations of the total structural volume (Ex.1).

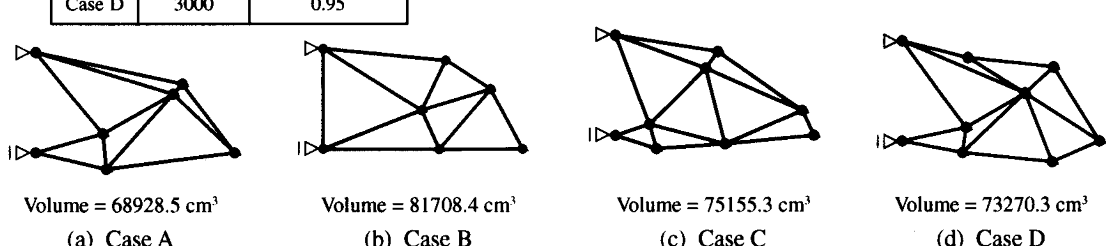



Figure 13.
Comparison of the shape and the total structural volume of the converged solutions (Ex.1).

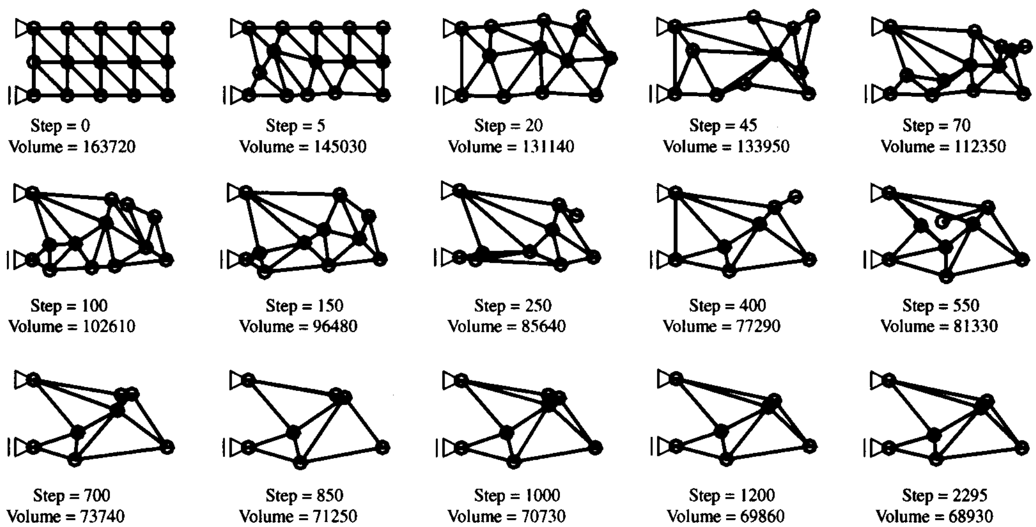



Figure 14.
Variation of topology in the annealing process for Case A (Ex.1).

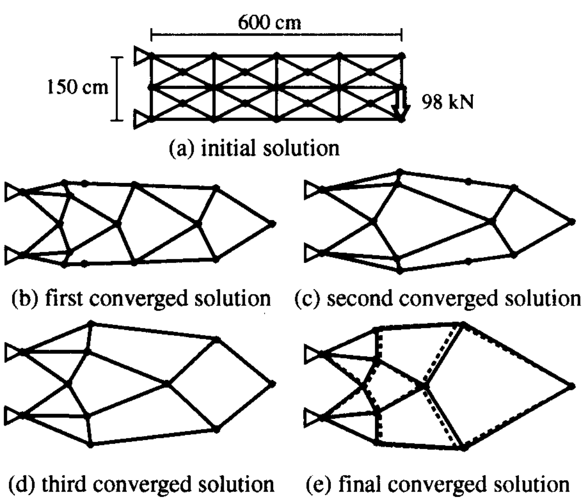



Figure 15.
Optimization of a symmetric cantilever-type truss (Ex.2).

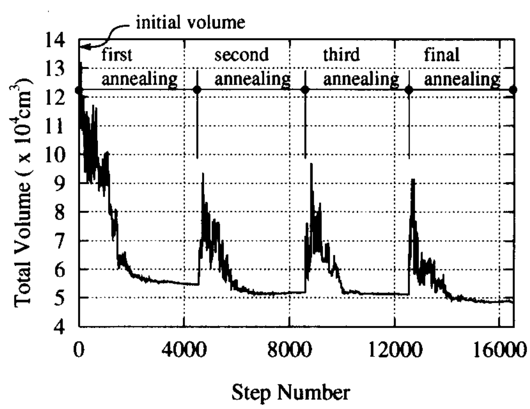



Figure 16.
Variation of total volume in the optimization process (Ex.2).

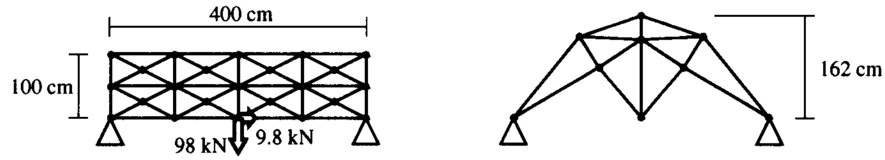
reaching the local optima with more nodes and members than the global optima. Therefore, re-annealing procedures are introduced, in which the initial solutions are generated by combining the closely spaced nodes of the final solution of the previous annealing stage (See Fig.10). The combined nodes never separate in the re-annealing process.

## 7. Examples

In the following examples, the allowable displacement of nodes is 1cm and elastic modulus is 205.8 GPa, $\gamma$= 3.16 cm, $D$=0.1cm, $L_{limit}$=20cm, $R_{max}$=80cm (for Ex.1) or 40cm (for Ex.2 and 3). A quadrilateral truss unit is allowed to exist only in the Ex.2. Re-annealing procedure is applied in the Ex.2 and 3.

### Example 1

An optimal solution is found for a cantilever-type truss as shown in Fig.11. Consider four cases listed in Table 1 where the initial temperature and the temperature reduction factor are different. Fig.12 and 13 show the variation of the total structural volume and the shape of the converged solution in each case. The step number in Fig.12 does not indicate the iteration number of generating neighborhood solutions but the number of replacement of solutions. The converged solution for Case A can be considered to be optimal. It is observed that the converged solution strongly depends on the annealing schedule. Note that for Case C and D there are possibilities of convergence to the optimal solution by applying the re-annealing procedure. Fig.14 shows the variation of the topology in the annealing process for Case A.




(a) initial solution
(Total Volume=4401 cm$^3$)

(b) optimal solution
(Total Volume=1597 cm$^3$)

Figure 17.
Optimization of a bridge-type truss (Ex.3).

Example 2

An optimal solution is found for a symmetric cantilever-type truss as shown in Fig.15. The four stages of annealing and re-annealing procedures have been carried out. Each converged solution is shown in Fig.15(b-e) and the variation of the total structural volume is plotted in Fig.16. In Fig.15(e), the broken lines indicate the optimal solution by the quasi-Newton method found from the truss with optimal topology by the proposed method. Since the good agreement between the two results is observed, the accuracy of the proposed method is confirmed.

Example 3

An optimal solution is found for a bridge-type plane truss as shown in Fig.17. In the optimal solution, the numbers of nodes and members as well as the total volume are dramatically reduced as compared with those of the initial solution.

**8. Conclusions**

The continuous topology transition model (CTTM) has been presented for general plane trusses. The procedure using the CTTM and the simulated annealing method has been developed for simultaneous optimization of topology and geometry of a plane truss with uniform cross-sectional area. The advantage of using the CTTM is summarized as: (a) the difficulties due to the requirement on cross-sectional area, such that all the members must have the same value, is successfully avoided, and (b) the topological variables and the rules for addition and removal of nodes and members need not be introduced and therefore computational effort is dramatically reduced. Re-annealing procedures have been introduced, in which the initial solutions are generated by combining the closely spaced nodes of the final solution of the previous annealing stage. In the examples, optimal trusses have been found for cantilever-type and bridge-type plane trusses under displacement constraints.

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