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Approximate shape design of gridshells using spatial discrete elastica

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Abstract

Gridshell is one of the bending-active structures generated by elastically bending and connecting beams. Shape of each curved beam is crucial for reducing the interaction forces at connections. The authors developed a method based on 'planar discrete elastica' for designing a curved surface. This study proposes a new method using 'spatial discrete elastica' which utilizes biaxial bending. An approximate algorithm is developed for designing surfaces of gridshells using spatial discrete elastica, where the interaction forces between primary and secondary beams are reduced by gradually adjusting the external moments at the both ends of primary beams.

Keywords: gridshell, bending-active structures, discrete elastica, shape design

1. Introduction

Gridshell is defined as a curved surface structure composed of flexible members [1]. It is a very efficient roof structure in view of construction period of time and cost. Shape of gridshell is generated from straight beams, which are first mutually connected by hinge joints on a plane. Forced displacements and external moments are given at the boundary to create the target shape by bending members. Target shapes of primary members which define the shape of overall shape of surface should be generated in view of mechanical efficiency, because the shapes of curved members directly affects the structural behavior of a gridshell.

Elastica is a buckled shape of beam-column under point loads at both ends. The interaction forces at joints can be reduced, if the curved beams are designed as elastica, because no forces except those at both ends are needed to maintain the shape of elastica. Discrete elastica is a discretized piecewise linear curve with the same segment length. A planar elastica model is obtained by solving an optimization problem. The objective function is the total potential energy composed of the penalized strain energy and external work by external moments, which control the shapes of elastica. Span and height of the support are specified as constraints. In our previous research [2], we generated simple surfaces that have square or equilateral triangle plans, and confirmed applicability of planar discrete elastica by large deformation analysis. However, it was difficult to generate more complex shapes, because discrete elastica cannot deform in the out-plane direction. Therefore, in this study, we propose a method based on the spatial discrete elastica model to generate more complex shapes of gridshells.

2. Spatial discrete elastica

The planar discrete elastica with uniaxial bending is extended to the spatial discrete elastica utilizing biaxial bending. Spatial discrete elastica has N+1 segments, which have the same length l. The ith segment connects nodes i (= 0,…, N) and i+1. The deflection angle of segment i from xy-plane and xz-

plane are denoted by $\mathbf{\Psi}\ (=\Psi_0,\dots,\Psi_N)$ and $\mathbf{\Phi}\ (=\Phi_0,\dots,\Phi_N)$, respectively. Figure 1(a) shows an example of spatial discrete elastica. We assign two rotational springs at all nodes representing biaxial bending stiffness of members, which are assumed to have the same value *EI*. Deformation of a segment of the continuous elastica with constant curvatures κ_y and κ_z around *y*-axis and *z*-axis, is equivalent to the discrete elastica with two kinds of spring rotations $\Psi_{i-1}-\Psi_i=\kappa_y l$ and $\Phi_{i-1}-\Phi_i=\kappa_z l$ assuming uniform curvature in the continuous elastica. Then, the strain energy *S* is defined as

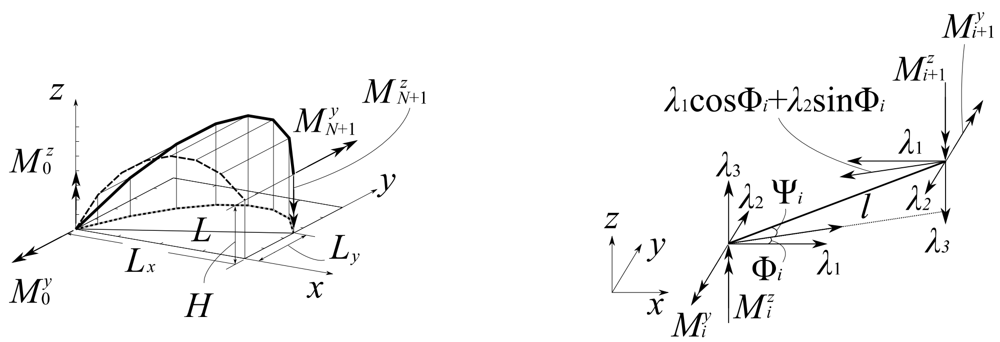
$$S=\frac{1}{2}EI\left(\kappa_y{}^2+\kappa_z{}^2\right)l=\frac{EI}{2l}\left\{\left(\Psi_{i-1}-\Psi_i\right)^2+\left(\Phi_{i-1}-\Phi_i\right)^2\right\} \tag{1}$$

Thus, the stiffness of rotational spring between two segments is obtained as *EI*/*l*.

The objective function is the total potential energy consisting of the penalized strain energy at every rotational spring and external work corresponding to external moments. The external moments M_0^y and M_{N+1}^y around *y*-axis, and M_0^z and M_{N+1}^z around *z*-axis are given at the both ends to control a shape of elastica. Span L_x in *x*-direction, L_y in *y*-direction and height difference *H* between the left and right supports are constrained. The optimization problem of minimizing the total potential energy Π with respect to 2*N*+3 variables $\mathbf{\Psi}=(\Psi_0,\dots,\Psi_N)$, $\mathbf{\Phi}=(\Phi_0,\dots,\Phi_N)$ and *l* is formulated as

$$\begin{aligned}
\min_{\mathbf{\Psi},\mathbf{\Phi},l}.\ \ & \Pi\left(\mathbf{\Psi},\mathbf{\Phi},l\right)=\sum_{i=1}^{N}\left[\frac{EI}{2l}\left\{\left(\Psi_i-\Psi_{i-1}\right)^2+\left(\Phi_i-\Phi_{i-1}\right)^2\right\}+\beta l\right] \\
& \qquad -M_0^y\Psi_0-M_{N+1}^y\Psi_N-M_0^z\Phi_0-M_{N+1}^z\Phi_N \\
\text{subject to}\ \ & \sum_{i=0}^{N}l\cos\Psi_i\cos\Phi_i=L_x,\quad \sum_{i=0}^{N}l\cos\Psi_i\sin\Phi_i=L_y,\quad \sum_{i=0}^{N}l\sin\Psi_i=H
\end{aligned} \tag{2}$$

where β is the penalty parameter for the beam length. Although the detail is omitted, it is seen from the optimality conditions of problem (2) that the Lagrange multipliers λ_1, λ_2 and λ_3 for the constraints represent the support reaction forces in *x*-, *y*- and *z*-directions, respectively, as illustrated in Fig. 1(b).



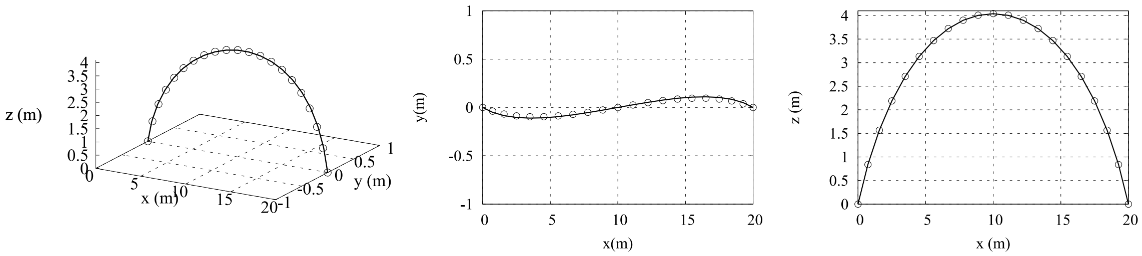
(a) (b)

Figure 1: Spatial discrete elastica and equilibrium of moments at segment *i*; (a) spatial discrete elastica (solid line); dotted and dashed lines: projections onto *xy*- and *xz*-planes, (b) equilibrium of moments of segment *i*.

3. Comparison between discrete and continuous elasticas

The shape and reaction forces of discrete elastica obtained by solving problem (2) are compared with those of the continuous elastica obtained by large deformation analysis. The energy minimization problem is solved using sequential quadratic programming (SQP) available in the library SNOPT Ver. 7. Large deformation analysis is carried out by using Abaqus Ver. 6.16. The straight beam on a plane is deflected by giving forced displacements and external moments with the same values as discrete elastica at the both ends. Material of the beam is elastic glass fiber reinforced polymer (GFRP), which has Young's modulus 25 GPa, Poisson's ratio 0.221, and the yield stress 200 MPa. Circular section with radius 0.05 m and thickness 0.005 m is assigned.

We generate a curve as shown in Fig. 2(a). The external moments (kNm) are $M_0^y = -6.00$, $M_{N+1}^y = 6.00$, $M_0^z = -0.70$, $M_{N+1}^z = -0.70$, and the span lengths and height (m) are $L_x = 20.0$, $L_y = H = 0$. Figure 2 shows the shapes of discrete elastica (circle) and continuous elastica (solid line), which are very close. It is confirmed that the reaction forces at the both ends also have almost the same values. Therefore, spatial discrete elsatica may be useful for generating target shape of 3-dimensional curved members. Reaction forces are shown in Table 1, which shows that almost the same values are obtained by discrete and continuous elasticas.



(a) (b) (c)

Figure 2: Shapes of curve; line: continuous elastica, circle: spatial discrete elastica; (a) diagonal view, (b) plan view on *xy*-plane, (c) plan view on *xz*-plane

Table 1: Reaction forces in *x*-, *y*-, and *z*-directions of discrete and continuous elasticas.

	Spatial discrete elastica	Continuous elastica
λ_1 [kN] (*x*-dir.)	–0.9028	–0.9067
λ_2 [kN] (*y*-dir.)	–0.0700	–0.0701
λ_3 [kN] (*z*-dir.)	0.0000	–0.0009

4. Approximate shape design of gridshells

We propose an algorithm for generating curved surfaces of gridshells composed of flexible members in self-equilibrium state. Target shapes of all members are designed as elastica for reducing the interaction forces at hinge joints. Spatial discrete elasticas obtained by solving the problem (2) are utilized for generating equilibrium shapes of beams. We classify the members into primary beams, which mainly define a shape of surface of gridshell, and secondary beams that connect primary beams. The algorithm for approximate design of target surface shape of gridshell is summarized as follows:

Step 1: Generate target shapes of primary beams by solving the optimization problem (2). Go to Step 2 if the first cycle of this flow, otherwise go to Step 3.

Step 2: Generate the target equilibrium shape of secondary beams by solving the problem (2) without the external work.

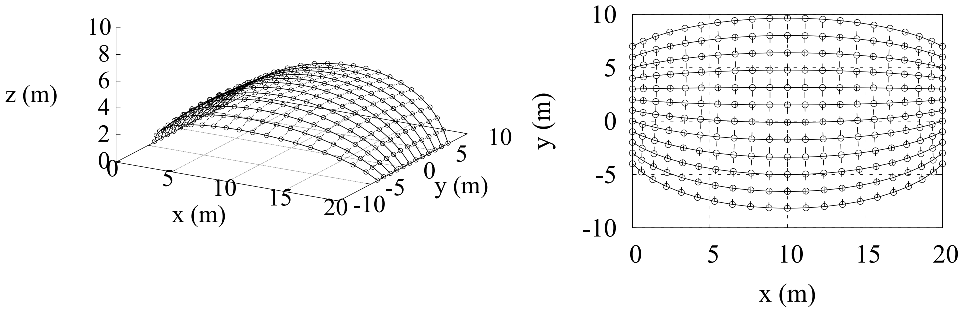
Step 3: Compute the quadratic error of coordinates between the nodes on the target secondary beam and the nodes on the primary beams to be connected to the secondary beam. Go to Step 5 if the convergence criteria are satisfied; otherwise, go to Step 4.

Step 4: Modify the external moments M_0^y, M_{N+1}^y, M_0^z and M_{N+1}^z to adjust the shapes of primary beams.

Step 5: Obtain the target surface of gridshell in approximate equilibrium state.

Target equilibrium shapes of secondary beams are obtained by solving the reformulated problem (2), where external work are eliminated from the objective function, and specifying the deflection angles at both ends defined by the two nodal coordinates of the primary beams at each end of the secondary beam. The convergence criteria are satisfied, if the largest norm of quadratic error is less than 1.0% of the rises of primary beams. Otherwise the external moments are modified by $\pm\Delta M$, which is 3 Nm in the following example, to update the shapes of primary beams so that the quadratic error between the nodes on the primary and secondary beams are reduced.

We generate a simple target surface using the proposed algorithm. The surface is composed of 12 primary beams consisting of 21 nodes, whose both ends are parallelly located along the *y*-axis with 1 m interval. The values of L_x and L_y of all primary beams are 20 m and 0, respectively. The both ends of all primary beams are located on the ground. 21 secondary beams connects primary beams with hinge joints that can rotate along the normal axis of the surface. The largest values of the quadratic errors in *y*- and *z*-directions are 0.0214 m and 0.0386 m, respectively, which are very small. Target surface obtained by this example is shown in Fig. 3, where the solid lines are the primary beams, and the blank circles are the nodes on the secondary beams. The nodes of primary beams are connected by dotted lines. It is seen from the figure that the nodes of the secondary members are very closely located to the nodes on the primary beams, which leads to very small interaction forces at the joints between the primary and secondary beams.



(a) (b)

Figure 3: Target surface of gridshell obtained by the proposed algorithm; solid line: primary beam, circle: node on target equilibrium shape of secondary beam, (a) diagonal view, (b) plan of target surface.

Conclusions

We presented a new method for shape design of gridshells using spatial discrete elastica. The conclusions obtained from this study are summarized as follows:

1. Spatial discrete elastica extended form of the plane discrete elastica can generate self-equilibrium shapes of beams with large deformation in 3-dimensional space. Spatial discrete elastica is obtained by solving the penalized energy minimization problem assigning biaxial bending. We confirmed that the deformed shapes of spatial discrete elastic are very close to those of continuous beams obtained by large deformation analysis. It can generate more complex shapes of curved beams for design of target shapes of flexural members.
2. The proposed algorithm can generate various shapes of gridshells using spatial discrete elasticas as primary beams for modeling the target shape of the surface. The errors between the locations of nodes on the secondary beams and the corresponding nodes on the primary beams are minimized to generate an approximate self-equilibrium shape of the gridshell.

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References

[1] M. Ohsaki, K. Seki and Y. Miyazu, Optimization of locations of slot connections of gridshells modeled using elastica, *Proceedings of the IASS Annual Symposium 2016, Tokyo, Japan, 2016,* Paper No. CS5A-1012, 2016.

[2] Y. Sakai and M. Ohsaki, Discrete elastica model for shape design of grid shells, *Proceedings of the IASS Annual Symposium 2017, Hamburg, Germany, 2017,* Paper No. 9557, 2017.