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FDMOPT: A new tool for simultaneous optimization of geometry and topology of truss structures

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Abstract

A new efficient tool is developed for Grasshopper users to simultaneously optimize topology and geometry of truss structures. Force density method is applied to formulate a minimization problem of compliance under volume constraint. The optimizer generates a variety of optimal solutions, because constraints on nodal locations are unnecessary to avoid the difficulties due to existence of melting nodes. In this study, an integrated workflow of FORTRAN and C# is presented to improve the optimization speed. Furthermore, a Bézier surface is introduced as a design surface to control nodal locations. The efficiency and accuracy of the proposed method are demonstrated through a numerical example.

Keywords: truss, simultaneous optimization of topology and geometry, force density method, latticed shell, tensor product Bézier surface, Grasshopper, interactive design

1. Introduction

There is an increasing number of optimization components available with Rhinoceros, which is one of the most popular modeling software in the field of architectural design, and Grasshopper, a graphical algorithmic editor of Rhinoceros. However, there is no component that simultaneously optimize topology and geometry of truss structures within acceptable computational cost.

Ohsaki and Hayashi [1] formulated a new optimization problem for simultaneous optimization of geometry and topology of trusses based on force density method (FDM) [2]. By using force densities as design variables, numerical difficulties caused by melting (overlapping) nodes [3] can be easily avoided. Moreover, the number of variables is equal to only the number of members; thus we need shorter computation time to obtain optimal solutions with a variety of geometry and topology.

In this study, to utilize the benefit of our method to the maximum extent, we develop a Grasshopper component to optimize topology and geometry of trusses. By packaging FDM in the component, interactive and integrated design process is successfully constructed.

2. Optimization problem

In this section, we explain the outline of the formulation of optimization problem. See Ref. [1] for details.

Consider a truss with m members and n nodes. Force density q_i of member i is defined with respect to the axial force N_i and the length L_i as $q_i = N_i / L_i$. Using the connectivity matrix $\mathbf{C} \in \mathbb{R}^{m \times n}$ and the force density vector $\mathbf{q} \in \mathbb{R}^m$, the force density matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$ can be defined as $\mathbf{Q} = \mathbf{C}^{\mathrm{T}} \mathrm{diag}(\mathbf{q}) \mathbf{C}$. The same matrix $\mathbf{Q}$ is used for formulating equilibrium equations in x-, y-, and z-directions. Hence, $\mathbf{Q}$ for three directions are assembled to $\tilde{\mathbf{Q}} \in \mathbb{R}^{3n \times 3n}$, which is re-assembled to classify free and fixed displacement components and their link components as

$$\tilde{\mathbf{Q}} = \begin{bmatrix} \tilde{\mathbf{Q}}_{\text{free}} & \tilde{\mathbf{Q}}_{\text{link}} \\ \tilde{\mathbf{Q}}_{\text{link}}^{\text{T}} & \tilde{\mathbf{Q}}_{\text{fix}} \end{bmatrix} \tag{1}$$

where the loaded nodes are also treated as fixed, because they cannot move in the process of geometry optimization. By specifying the fixed nodal coordinates $\mathbf{X}_{\text{fix}} \in \mathbb{R}^{n_{\text{fix}}}$, the free nodal coordinates $\mathbf{X}_{\text{free}} \in \mathbb{R}^{n_{\text{free}}}$ are obtained as a function of $\mathbf{q}$ from the following system of linear equations:

$$\tilde{\mathbf{Q}}_{\text{free}}\mathbf{X}_{\text{free}} = -\tilde{\mathbf{Q}}_{\text{link}}\mathbf{X}_{\text{fix}} \tag{2}$$

Using the fact that the solution to the compliance minimization problem is a statically determinate truss with the same absolute value of stress $\bar{\sigma}$ for all members, the cross-sectional area of member i is expressed as $A_i = |q_i| L_i / \bar{\sigma}$. Let E and c denote Young's modulus and a very small positive smoothing parameter, respectively. Then, the compliance, which is twice of the strain energy, can be expressed as

$$F(\mathbf{q}) = \sum_{i=1}^{m} \frac{\bar{\sigma} L_i^2 \sqrt{q_i^2 + c}}{E} \tag{3}$$

Since the product of the total structural volume V and the compliance of the optimal solution is independent of $\bar{\sigma}$, V can be calculated after minimizing the compliance with arbitrary positive value of $\bar{\sigma}$ [1]. The loading condition is incorporated by prescribing reaction forces at loaded nodes as

$$\sum_{i \in \mathbf{I}_R} \left(R_i - \bar{R}_i \right)^2 = 0 \tag{4}$$

where $\mathbf{I}_R$ is a set of indices of reaction forces to be specified, and $\bar{R}_i$ is the specified load value.

We further add constraints on nodal locations using a design surface such that the z-coordinate of point k on the surface is expressed as an explicit function of x- and y-coordinates as

$$z_k = f(x_k, y_k) \ , \ \ (k = 1, \cdots, n) \tag{5}$$

Thus, the optimization problem can be formulated as

$$\begin{aligned} &\text{minimize} \ \ F(\mathbf{q}) \\ &\text{subject to} \ \ (4), (5), \mathbf{x}(\mathbf{q}) \in \Omega_x, \mathbf{y}(\mathbf{q}) \in \Omega_y, \mathbf{q} \in \Omega_q \end{aligned} \tag{6}$$

where Ω_x, Ω_y, and Ω_q are the feasible regions of **x**, **y**, and **q**, respectively.

We use a tensor product Bézier surface of order $M \times N$ as the design surface. If the control point $\mathbf{P}_{ij}$ is described as $\mathbf{P}_{ij} = (i/M, j/N, b_{ij})^{\text{T}} \quad (b_{ij} \in \mathbb{R})$, and the plan of the surface is scaled so that $x, y \in [0,1]$ can be used as parameters of the surface, then z-coordinate of point k on the tensor product Bézier surface can be expressed as an explicit function of its x- and y- coordinates as

$$z_k = f(x_k, y_k) = \sum_{i=0}^{M} \sum_{j=0}^{N} B_i^M(x_k) B_j^N(y_k) b_{ij} \tag{7}$$

where $B_i^M(x)$ and $B_j^N(y)$ are the Bernstein basis polynomials.

3. Compilation of FDMOPT

In this section, the process of developing the optimizer is described using Grasshopper. In the optimization process, initial ground structure (GS), support and loading conditions, and additional constraints must be assigned as numerical data for the sequential quadratic programming (SQP) solver. To assist this complex process, a Grasshopper component is developed for automatic extraction of the information. The Grasshopper component compiled in C# is shown in Fig. 1. The top left 5 parameters P, C, S, LP, LV are the required inputs, and the bottom left 6 parameters L, x, y, z, S, I are optional. Once the component is double-clicked, it starts calling another optimization program compiled in

FORTRAN, where an SQP library SNOPT Ver. 7.2 is incorporated. The FORTRAN program randomizes the initial force densities to generate initial variables based on a prescribed random seed, conducts the optimization, returns an optimal solution back to the Grasshopper component, which displays the optimal shape on the Rhinoceros window. The whole optimization workflow is illustrated in Fig. 2. This way, a user-friendly interface is developed for the proposed optimization method.

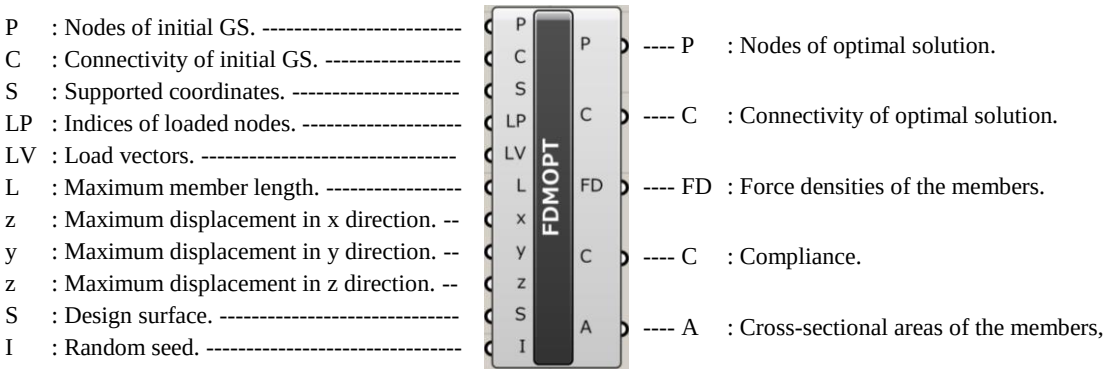

Figure 1: Compiled grasshopper component and its input and output parameters.

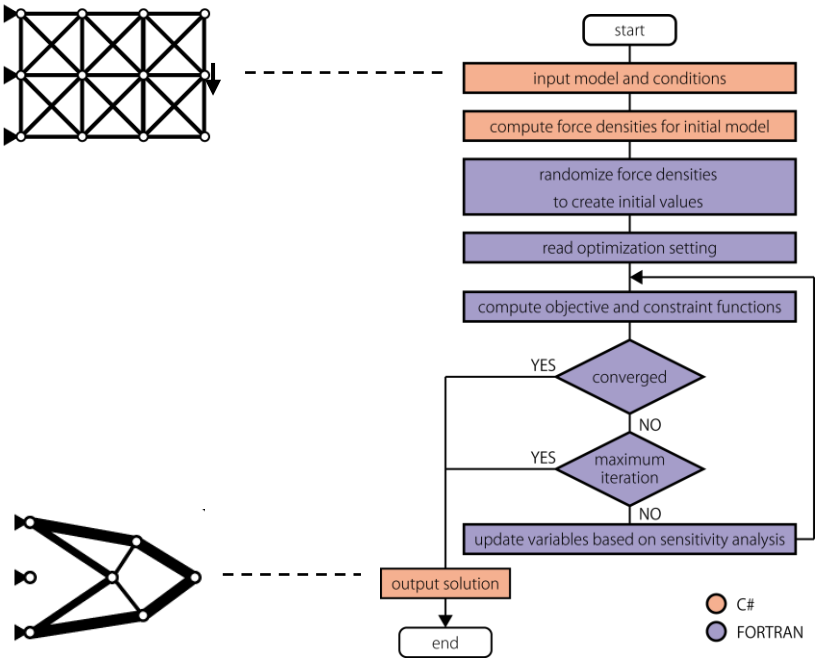

Figure 2: Integrated workflow of C# and FORTRAN.

4. Numerical example

In this section, we present an example for the demonstration of the proposed method. Units are omitted because they are not important in this research. The parameter values are $\bar{\sigma}=1.0$, $E=1.0$, $c=1.0\times10^{-6}$, and $V=10$. The initial value $\bar{q}_k$ of force density of member k is randomly provided within the range $[\,\bar{q}_k^{\text{init}}-1.0, \bar{q}_k^{\text{init}}+1.0\,]$, where $\bar{q}_k^{\text{init}}$ is the force density of member k of the initial GS with uniform cross-sectional areas 1.0. The upper and lower bounds for q_k are $\bar{q}_k+1.0\times10^2$ and $\bar{q}_k-1.0\times10^2$, respectively. We choose the best solution after obtaining 100 solutions for 100 different random seeds.

The left figure of Fig. 3 shows the control polygon. The number of equal mesh division is four in x- and y-directions. By connecting neighboring nodes, an initial ground structure with 72 members is generated as shown in the right figure of Fig. 3. Four corner nodes {1,5,21,25} on the surface are pin-supported and four center nodes {8,12,14,18} are subjected to unit loads in the positive direction of x-axis.

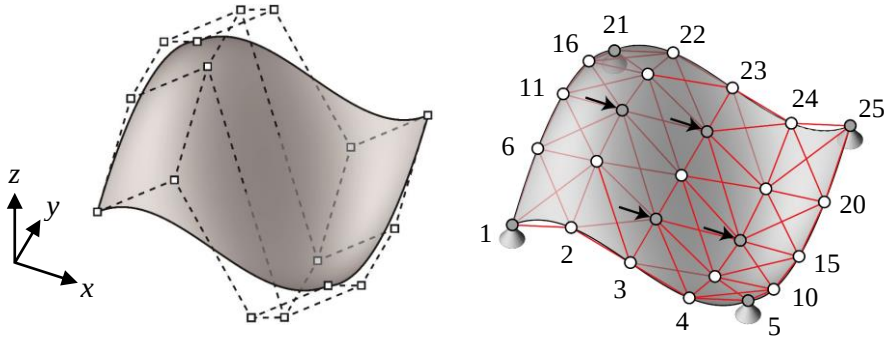

Figure 3: Control polygon of tensor product Bézier surface (left) and initial GS (right).

Among the 100 trials, the maximum, median, minimum, average values, and standard deviation of F for the 80 valid solutions are 3.209×10^{4}, 58.605, 12.767, 2.209×10^{3}, and 6.226×10^{3}, respectively. It takes 231 seconds for each trial on average on a PC with Intel Core i5 processor. The optimal solution with the least objective value $F=12.767$ is shown in Fig. 4. Red members are compressive and blue are tensile. The sets of nodes $\{1,2\},\{4,9,10,13,14,15,19\},\{7,8\},\{11,17,18\},\{20,24\},\{21,22,23\}$ are coalescent within the range of 0.05 to generate the simpler shepe with 12 nodes.

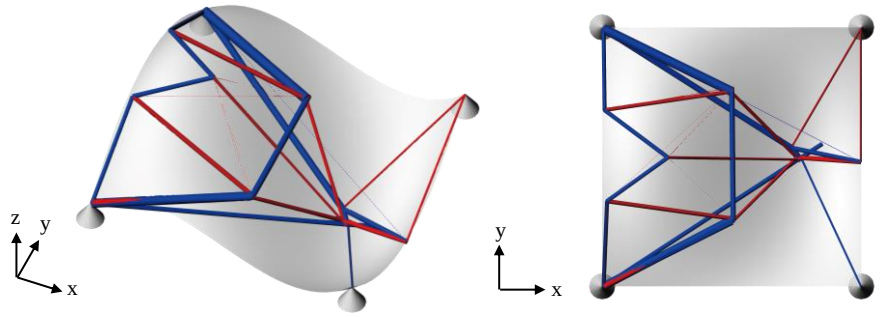

Figure 4: The best optimal solultion ($F=12.767$).

5. Conclusion

We proposed an interactive and integrative approach to truss design by developing a Grasshopper component for simultaneous optimization of geometry and topology of trusses. Numerical difficulty due to melting nodes can be successfully avoided using force density as design variable, which contributes to generate solutions with coalescent nodes. Moreover, a tensor product Bézier surface is successfully incorporated as a design surface of the optimizer, which enables controlling nodal locations.

Although the design surface is restricted to a Bézier surface with uniformly spaced control points in *x*- and *y*-directions, this restriction is expected to be relieved by using geometric transformation in future research.

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