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Regularization of triangular latticed shell panels for Bézier surfaces

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Abstract

A new method is proposed for regularization of triangular lattice shell panels whose design surface is a tensor product Bézier surface. By carrying out clustering and optimization alternately, better solutions can be achieved compared with the case of conducting both only once. A continuous relaxation method is applied to conduct clustering of panel shapes. Effectiveness of the proposed method is demonstrated through a numerical example.

Keywords: latticed shell, tensor product Bézier surface, regularization, structural optimization, clustering, panelization

1. Introduction

As complex shapes for architectural design have become possible in the framework of computational modeling, there is an increasing interest in obtaining rational shapes considering cost and constructability of free-form surfaces. Discretization to latticed shell is one of the solution for cost reduction, and uniformity of discretized structural elements is an important factor to reduce the number of types of members and joints. Moreover, regularization of members is expected to prevent buckling of extremely long members and difficulty in constructability due to existence of short members.

Ogawa *et al*. [1] proposed a shape optimization method of latticed shells for maximum linear buckling loads and uniform member lengths. Although the difference between the maximum and minimum lengths are used for the regularization of members, very simple models with quadrilateral grids are used in the numerical example.

As the extension of regularization of member lengths, we can consider the shapes of enclosed areas by the members. Winslow [2] formulated an optimization problem for rationalizing triangular lattices from a freeform surface with upper and lower limits for the edge lengths. This constraint is due to cladding requirements and another formulation to regularize panel shapes themselves is necessary. Singh and Schaefer [3] proposed a regularization method for triangulated surfaces, where initial triangles are classified by k-means clustering and the positions of the vertices are optimized to match the case if the triangles are substituted by canonical triangles of the clusters. However, this method inevitably alters the geometry of surface and may cause an unexpected change of structural behavior.

In this paper, we propose a 2-step regularization method of triangular latticed shell panels on a tensor product Bézier surface, where clustering and optimization are alternately conducted. In the process of clustering, continuous relaxation method is applied to express the degree of participation to each cluster for panel shapes. Applicability of the proposed method is demonstrated through a numerical example.

2. Optimization Problem

The outline of the formulation of optimization problem is described in this section.

In general, a tensor product Bézier surface of order $M \times N$ is expressed with parameters $u, v \in [0,1]$ as

$$\mathbf{S}(u,v) = \sum_{i=0}^{M} \sum_{j=0}^{N} B_i^M(u) B_j^N(v) \mathbf{R}_{ij} \tag{1}$$

where $B_i^M(u)$ and $B_j^N(v)$ are the Bernstein basis polynomials, and $\mathbf{R}_{ij} \in \mathbb{R}^3$ is a position vector of (i, j) control point. Note that the locations of control points are all fixed in this paper.

Once continuous parameters u and v are discretized, finite number of nodes are generated on the tensor product Bézier surface. By connecting the neighboring nodes, a triangular mesh is obtained. Therefore, nodal coordinates of mesh nodes are functions of u and v.

Next, these triangular meshes are classified into sevaral groups depending on their shapes. Let n_d denote the number of samples. To avoid proximity in cluster centroids, their initial locations are separately arranged as follows; one of the data is randomly chosen as the first centroid, and the other centroids are chosen from data $\mathbf{x}_i \left(i = 1, \cdots, n_d\right)$ based on the probability as

$$p(\mathbf{x}_i) = \frac{D(\mathbf{x}_i)^2}{\sum_{i=1}^{n_d} D(\mathbf{x}_i)^2} \tag{2}$$

where $D(\mathbf{x}_i)$ is an euclidian distance from $\mathbf{x}_i$ to the closest centroid.

After determining the initial positions of the centroids, clustering is conducted. Let n_c denotes the number of clusters. The degree of participation in cluster j for data i is computed as

$$U_{ij} = \left(\sum_{k=1}^{c} \left(\frac{\|\mathbf{x}_i - \mathbf{c}_j\|}{\|\mathbf{x}_i - \mathbf{c}_k\|} \right)^{\frac{2}{m-1}} \right)^{-1} \quad \left(i = 1, \cdots, n_d,\ j = 1, \cdots, n_c\right) \tag{3}$$

Then the centroids are updated using U_{ij} as

$$\mathbf{c}_j = \frac{\sum_{i=1}^{n_d} U_{ij}^m \mathbf{x}_i}{\sum_{i=1}^{n_d} U_{ij}^m} \tag{4}$$

Equation (3) and (4) are alternately and repeatedly computed until convergence.

To apply this clustering method to triangular panel classification, we use three edge lengths of each triangular mesh $\mathbf{x}_i = \{L_{i1}, L_{i2}, L_{i3}\}$ $(L_{i1} \le L_{i2} \le L_{i3})$ .Let the panel with data $\mathbf{x}_i$ belongs to the cluster with the largest value of U_{ij}. The optimization problem can be formulated as

$$\begin{aligned} &\text{minimize } F(u,v) = \max_{j} \left(\max_{i_1, i_2} \left\| \mathbf{x}_{i_1}^j - \mathbf{x}_{i_2}^j \right\| \right) \\ &\text{subject to } \begin{matrix} u_k \in \Omega_{u_k} \\ v_k \in \Omega_{v_k} \end{matrix} \quad \left(k \in \mathbf{K}\right) \end{aligned} \tag{5}$$

where $\mathbf{x}_{i_1}^j$ and $\mathbf{x}_{i_2}^j$ are the i_1 th and i_2 th data in cluster *j*, Ω_{u_k} and Ω_{v_k} are the domain within which the variables u_k and v_k can take, and $\mathbf{K}$ is the set of indices of mesh nodes. $\max_{i_1, i_2} \left\| \mathbf{x}_{i_1}^j - \mathbf{x}_{i_2}^j \right\|$ is the maximum value of the norm of the difference of three edge lengths within cluster *j*, and it is computed for every cluster to extract the maximum value, which becomes the objective function *F*.

The first step of the regularization is simple iterations of clustering and optimization that is alternately and repeatedly implemented until the objective function value cannot improve for 5 consecutive times.

The second step starts from the best result of the first step, then moves one or two members to different clusters based on $c_a \in \mathbb{N}$ defined by

$$c_a = \underset{j}{\operatorname{argmax}}\left(\max_{i_1,i_2}\left\|\mathbf{x}_{i_1}^j - \mathbf{x}_{i_2}^j\right\|\right) \tag{6}$$

Let $\tilde{i}_1$ and $\tilde{i}_2$ denote indices of two critical panels which are critical to the computation of the objective function defined as

$$\left\{\tilde{i}_1,\tilde{i}_2\right\} = \underset{\{i_1,i_2\}}{\operatorname{argmax}}\left\|\mathbf{x}_{i_1}^{c_a} - \mathbf{x}_{i_2}^{c_a}\right\| \tag{7}$$

If c_a is an index of cluster that has the largest or smallest norm of centroid among all clusters, a panel out of two panels with $\mathbf{x}_{\tilde{i}_1}^{c_a}$ and $\mathbf{x}_{\tilde{i}_2}^{c_a}$ whose edge lengths are more distant from its centroid than the other is re-assigned to the cluster whose centroid is the second closest. If c_a is in the other case, both of these two panels are assigned to the second closest cluster. This modification is also alternately conducted with solving optimization problem (5) until its objective function value cannot improve. Note that the second closest cluster is easily detected from the degree of participation U_{ij}.

3. Numerical example

We use Python 3.6.4 and SLSQP to conduct analysis and optimization. In application of SLSQP, sensitivity coefficients of objective functions with respect to u and v are analytically derived. The control polygon is composed of 25 control points to create a high-pitched quartic tensor product Bézier surface, as illustrated in the left Fig. 1. The number of equal mesh division is 10 in x- and y-directions. Connecting neighboring nodes generates an initial shape with 121 nodes and 200 triangular panels, as shown in the right Fig. 1.

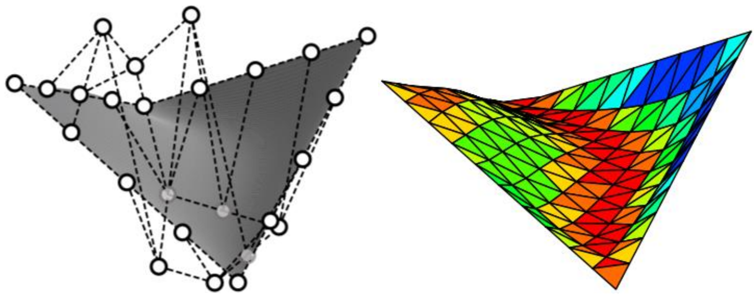

Figure 1: Control polygon of tensor product Bézier surface (left) and initial panelization (right).

The number of cluster is set to be 10 and clustering parameter m is 2.0. Ω_{u_k} is set such that the parameter u_k simultaneously satisfy $0 \le u_k \le 1$ and $\bar{u}_k - 0.1 \le u_k \le \bar{u}_k + 0.1$, where $\bar{u}_k$ is a initial value of u_k. Ω_{v_k} is set in the same way.

The regularization result is described in Table 1. N^t is the number of panels in the cluster, and $\left\|\mathbf{L}^{\mathrm{U}} - \mathbf{L}^{\mathrm{L}}\right\|$ is the maximum value of norm of difference in edge lengths of triangular panels within the same cluster. $\left\|\mathbf{L}^{\mathrm{U}} - \mathbf{L}^{\mathrm{L}}\right\|$ becomes less than 0.140 after the regularization, which means that more uniform triangular panels are obtained for every cluster. The cycle of clustering and optimization was carried out for 16 times in the first step; on the other hand, it was done only once in the second step, because the objective function value did not improve.

Table 1: Regularization result.

	initial panelization		optimal panelization		
cluster	N^t	$\left\|\mathbf{L}^{\mathrm{U}} - \mathbf{L}^{\mathrm{L}}\right\|$	N^t	$\mathbf{c}$	$\left\|\mathbf{L}^{\mathrm{U}} - \mathbf{L}^{\mathrm{L}}\right\|$
0	32	0.084	34	(0.98,1.02,1.45)	0.139
1	43	0.229	58	(1.02,1.10,1.46)	0.139
2	36	0.179	26	(1.06,1.14,1.68)	0.139

3	23	0.174	25	(1.00,1.27,1.67)	0.139
4	25	0.174	15	(1.03,1.29,1.75)	0.139
5	10	0.188	9	(1.18,1.22,1.94)	0.139
6	8	0.262	5	(0.98,1.49,1.99)	0.139
7	6	0.208	10	(1.12,1.53,2.22)	0.139
8	4	0.203	4	(1.30,1.53,2.38)	0.038
9	13	0.230	14	(1.19,1.65,2.39)	0.137

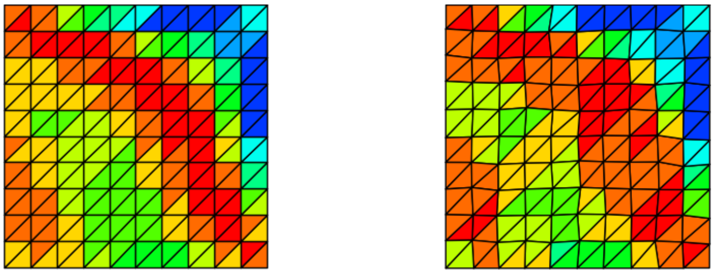

Figure 2: plan of panelized mesh of initial(left) and optimal(right) solutions.

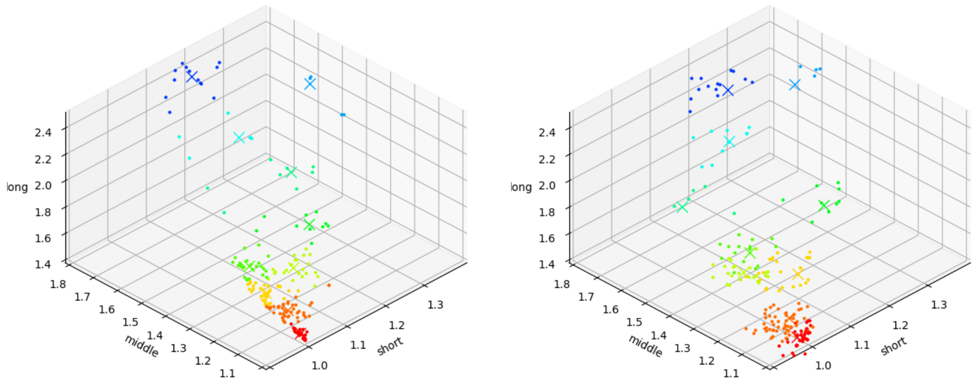

Figure 3: clustering of edge lengths for initial(left) and optimal(right) solutions.

4. Conclusion

Regularization of triangular latticed shell panels whose design surface is a tensor product Bézier surface can be fulfilled by alternate and repetitive implement of clustering and optimization. Continuous relaxation method is successfully incorporated into the clustering process. However, modification of clustering for 1 or 2 critical members does not contribute to improvement of objective function because of too strict stop condition. Future work is necessary to develop more effective modification method that positively introduces heuristics.

Acknowledgements

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