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Step-by-step unbalanced force iteration for asymmetric cable dome structures

Qing MA[ab], Makoto OHSAKI[b], Zhihua CHEN*, Xiangyu YAN[a]

*Tianjin University
No. 92 Weijin Road, Nankai District, Tianjin, 300072
zhchen@tju.edu.cn

a Tianjin University
b Kyoto University

Abstract

Cable dome is favored by engineers and architects for its high structural efficiency since it was invented. However, most of the established cable domes are in the symmetric configuration. To overcome the difficulties in design of asymmetric cable domes, Unbalanced Force Iteration (UFI) and Step-by-Step UFI are proposed in this paper. The UFI is based on equilibrium matrix and stiffness matrix. Feasibility of the geometry can be judged by the convergence property of UFI and the unbalanced force can be eliminated if the initial geometry is feasible. Step-by-Step UFI is utilized for geometry modification when the geometry is not feasible. In each iteration of Step-by-Step UFI geometry is modified by the displacement of finite element analysis and feasible geometry will be gradually found. Two examples of asymmetric cable domes are presented to illustrate the method.

Keywords: cable dome, feasible geometry, prestress design, form finding

1. Introduction

Cable dome was invented by D.H. Geiger for Seoul Olympics with the concept of tensegrity[1]. The structure is favored by engineers and architects for its high structural efficiency. More than 10 cable domes have been built for large span roof structures since then. However, most of the established cable domes are in the symmetric configuration. Symmetric geometry are usually feasible, which means the obtained prestress complies with the uniilateral condition of internal force of cables and struts[2]. The difficulties in designing asymmetric cable domes are: (1) the feasibility of the initial geometry should be confirmed before prestress design or form finding; (2) modification of geometry should comply with the architectural requirements[3].

To overcome the difficulties, Unbalanced Force Iteration (UFI), which is based on equilibrium matrix of cable-strut structures, is first proposed to judge of initial geometry. The equilibrium prestress can be obtained along with the process of judgement, if the initial geometry is feasible. Modification of geometry is necessary if the initial geometry is not feasible. The Step-by-Step UFI, which incorporate finite element analysis into UFI, is used for the modification of geometry. The coordinates is updated according to displacement derived by finite element analysis in the Step-by-Step UFI. Examples of cable domes with feasible and infeasible geometry are presented to illustrate the method.

2. Basic formulation

2.1. Equilibrium equation

Connectivity of the members can be described by the connectivity matrix $\boldsymbol{C} \in R^{n \times m}$, where m and n are the numbers of elements and nodes, respectively. If the kth member connects node i and j, the (k, e)

component $C_{k,e}$ of matrix $\boldsymbol{C}$ is defined as:

$$C_{k,e} = \begin{cases} +1 \; (e = i) \\ -1 \; (e = j) \\ 0 \quad (\text{others}) \end{cases} \tag{1}$$

Let $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ $(\in R^{n\times 1})$ denote the coordinate vectors. The diagonal matrices of coordinate differences $\boldsymbol{U}$, $\boldsymbol{V}$ and $\boldsymbol{W}$ $(\in R^{m\times m})$ are expressed by

$$\begin{cases} \boldsymbol{U} = \text{diag}(\boldsymbol{Cx}) \\ \boldsymbol{V} = \text{diag}(\boldsymbol{Cy}) \\ \boldsymbol{W} = \text{diag}(\boldsymbol{Cz}) \end{cases} \tag{2}$$

Let $\boldsymbol{L} \in R^{m\times m}$ denote the diagonal matrix of member length. The equilibrium matrix including all nodes is obtained as

$$\boldsymbol{D}^0 = \begin{bmatrix} \boldsymbol{C}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{L}^{-1} \\ \boldsymbol{C}^{\mathrm{T}}\boldsymbol{V}\boldsymbol{L}^{-1} \\ \boldsymbol{C}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{L}^{-1} \end{bmatrix} \tag{3}$$

Self-equilibrium state of the cable dome structure stiffened by prestress $\boldsymbol{T} \in R^{m\times 1}$ should be:

$$\boldsymbol{Dt} = \boldsymbol{0} \tag{4}$$

Where $\boldsymbol{D} \in R^{f\times m}$ is the equilibrium matrix considering the constraints. $\boldsymbol{D}$ is derived by removing the rows corresponding to the fixed displacement components of $\boldsymbol{D}^0$.

2.2. Linear stiffness matrix

When properly tensioned, the linear stiffness matrix of cable dome structure is the same as truss. The global linear stiffness matrix $\boldsymbol{K}_E \in R^{f\times f}$ considering constraints is obtained as

$$\boldsymbol{K}_E = \boldsymbol{D}\boldsymbol{K}_e\boldsymbol{D}^{\mathrm{T}} \tag{5}$$

Where $\boldsymbol{K}_e \in R^{m\times m}$ is diagonal matrix of the element stiffness. Let E denote Young's modulus and A_k denote the cross-sectional area of the kth element, the diagonal components of $\boldsymbol{K}_e$ can be calculated by

$$\boldsymbol{K}_e^{k,k} = \frac{EA_k}{l_k} \tag{6}$$

Let $\boldsymbol{d} \in R^{f\times 1}$ denote the nodal displacement vector of the structure subjected to the external load vector $\boldsymbol{F} \in R^{f\times 1}$. The nodal displacement vector $\boldsymbol{d}$ under nodal loads $\boldsymbol{F}$ can be obtained by:

$$\boldsymbol{K}_E\boldsymbol{d} = \boldsymbol{F} \tag{7}$$

The resulting internal force can be calculated by

$$\boldsymbol{t} = \boldsymbol{K}_e\boldsymbol{D}^{\mathrm{T}}\boldsymbol{d} \tag{8}$$

3. Step-by-Step Unbalanced Force Iteration Method

3.1. Estimation of feasibility

Equilibrium and unilateral condition that positive (tensile) force in cables and negative (compressive) force in struts will be satisfied if geometry of cable dome is feasible. Then the *prestress design* will be conducted. Otherwise, the geometry are to be modified, which is called *form finding*. Therefore, the geometry feasibility should be estimated first. The initial prestress vector $\boldsymbol{t}_0$ is randomly assigned. The unbalanced force $\boldsymbol{P}_0 \in R^{f\times 1}$ is calculated by

$$\boldsymbol{P}_0 = \boldsymbol{D}\boldsymbol{t}_0 \tag{9}$$

If $\boldsymbol{P}_0$ does not equals to zero, the prestress should be modified to compensate the nodal unbalanced force. The nodal unbalanced force is re-distributed to satisfy the condition, which is conducted by applying unbalanced force $\boldsymbol{F} = -\boldsymbol{P}_0$ in opposite direction assuming all elements as truss elements. Using Eq. (7), the internal force$\Delta\boldsymbol{t}_0$ induced by unbalanced force is obtained from

$$-\boldsymbol{P}_0 = \boldsymbol{K}_\mathrm{E}\boldsymbol{d}_0 = \boldsymbol{D}\boldsymbol{K}_\mathrm{e}\boldsymbol{D}^\mathrm{T}\boldsymbol{d}_0 = \boldsymbol{D}\Delta\boldsymbol{t}_0 \tag{10}$$

From Eqs. (9) and (10), we obtain

$$\boldsymbol{D}(\boldsymbol{t}_0 + \Delta\boldsymbol{t}_0) = \boldsymbol{0} \tag{11}$$

Thus, prestress is modified as follows to obtain a self-equilibrium prestress mode $\boldsymbol{t}_1$:

$$\boldsymbol{t}_1 = \boldsymbol{t}_0 + \Delta\boldsymbol{t}_0,\ \ \boldsymbol{D}\boldsymbol{t}_1 = \boldsymbol{0} \tag{12}$$

Let $\boldsymbol{T}^c = (T_1^c, \ldots, T_{m^c}^c)^\mathrm{T}$ and $\boldsymbol{T}^s = (T_1^s, \ldots, T_{m^s}^s)^\mathrm{T}$ denote the force vectors of cables and struts, where m_c and m_s are the numbers of cables and struts. If unilateral condition is not satisfied, the prestress $\boldsymbol{t}_1$ obtained by Eq. (12) should be revised as follows:

$$\begin{cases} T_i^c = \left|T_i^c\right|, & (i = 1, \ldots, m_c) \\ T_i^s = -\left|T_i^s\right|, & (i = 1, \ldots, m_s) \end{cases} \tag{13}$$

Then, the iterative process of UFI is carried out again according to Eqs. (9)-(13) and geometry feasibility is judged by whether the unbalanced force converge to zero.

3.2 Geometry modification

If the initial geometry is not feasible, modification of geometry is needed. Prestress should be revised in order to comply with the requirement of unilateral requirement. In order to avoid large unbalanced force incompatible prestresses are adjusted to a positive constant N_1 for cables and a negative constant N_2 for struts, respectively, as:

$$\begin{cases} T_i^c = N_1 & \text{for } T_i^c < 0 \\ T_i^s = N_2 & \text{for } T_i^s > 0 \end{cases} \tag{15}$$

Let $\boldsymbol{t}^*$ denote the prestress after modification, the unbalanced force vector $\boldsymbol{P}^*$ is should be

$$\boldsymbol{P}^* = \boldsymbol{D}\boldsymbol{t}^* \tag{16}$$

The displacement $\boldsymbol{d}$ due to the unbalanced load is obtained by:

$$\boldsymbol{K}\boldsymbol{d} = -\boldsymbol{P}^* \tag{17}$$

where $\boldsymbol{K}$ is the tangent stiffness matrix, which consists of linear and geometrical stiffness matrices. The tangent stiffness matrix $\boldsymbol{K}$ in nonlinear analysis process is not constant, and it is updated at every step of solving Eq. (17).

Let $\boldsymbol{x}^i$, $\boldsymbol{y}^i$ and $\boldsymbol{z}^i$ denote the coordinate vectors at the ith step of iteration and $\boldsymbol{d}_x^i$, $\boldsymbol{d}_y^i$ and $\boldsymbol{d}_z^i$ denote the displacement vectors. Thus, the nodal coordinates are updated as

$$\begin{cases} \boldsymbol{x}^{i+1} = \boldsymbol{x}^{i} + d_x^i \\ \boldsymbol{y}^{i+1} = \boldsymbol{y}^{i} + d_y^i \\ \boldsymbol{z}^{i+1} = \boldsymbol{z}^{i} + d_z^i \end{cases} \tag{18}$$

The geometry modification process is summarized in Fig. 2. The feasibility of initial geometry is first to be checked the by UFI. Then geometry and prestress will be modified. The feasible prestress will be obtained until equilibrium and unilateral conditions are both satisfied. The method for force and form finding is named Step-by-Step UFI.

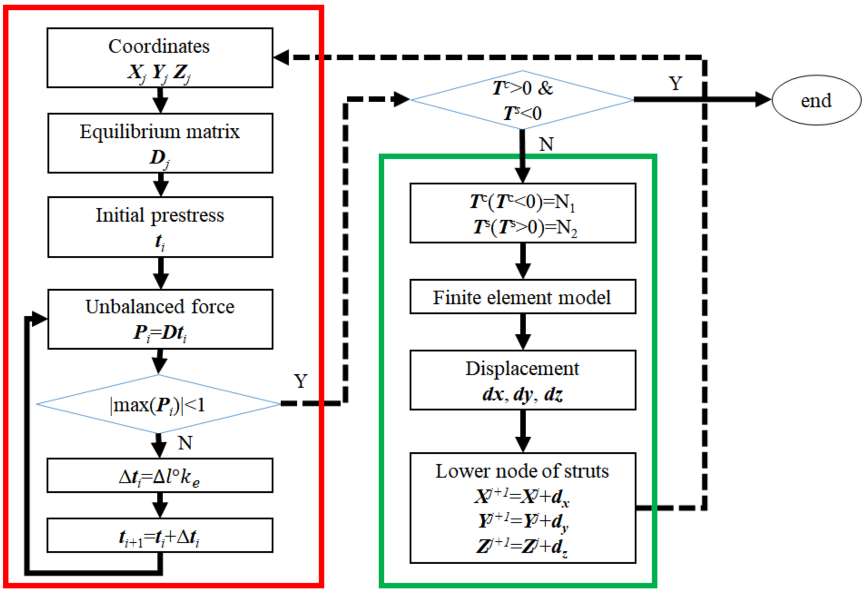


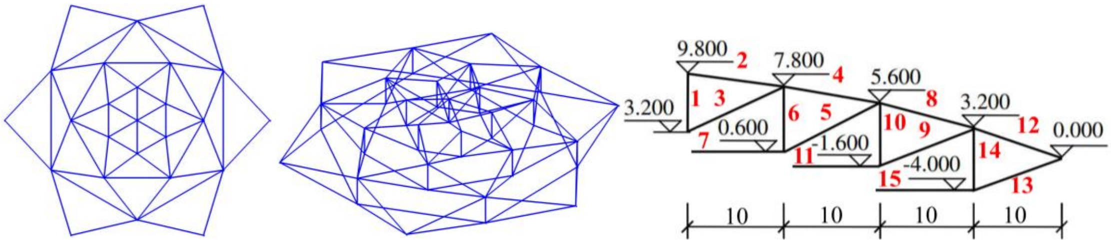
Figure 1: Flowchart of Step-by-Step UFI

4. Examples

Two numerical examples are presented for cable dome structures with symmetric and feasible geometry and asymmetric and infeasible initial geometry, respectively. The process is conducted with Matlab Ver. 2016 [4] and FEA by Ansys Ver. 18 [5]. Convergence tolerance of both examples are set to 1.0 N.

4.1. Example I: Circular Levy form cable dome

A circular Levy from cable dome is shown in Fig. 3. The detailed geometry is shown in Fig.3(c). There are 15 groups of elements, where groups 1, 6, 10 and 14 are struts, and others are cables. There are three rings of hoops in this example, and circumference of each hoop is evenly divided into six parts.



(a) top view　　(b) perspective view　　(c) geometry and member numbers

Figure 2: Circular Levy form cable dome

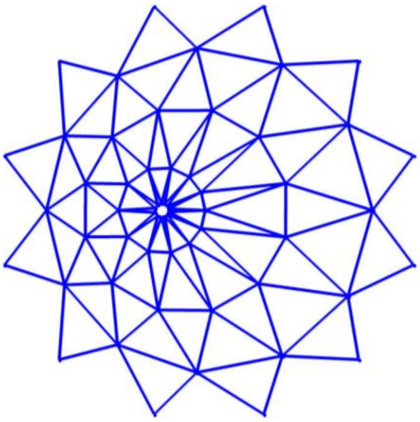
The unbalanced force converged to 4.63×10^{-5} N, which is negligibly small, with only one step of UFI. Table 1 shows the results of UFI and comparison between UFI and DSVD[7]. The UFI result is very close to DSVD result. It can be inferred from this example that the unbalanced force can be quickly and easily eliminated if the geometry is feasible.

Table 1 unbalanced force result of UFI

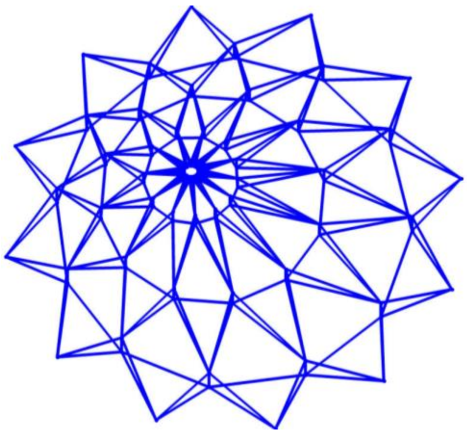
Ele. No.	UFI (N)	DSVD (N) [6]	Error (N)	Ele. No.	UFI (N)	DSVD (N) [6]	Error (N)
1	-2500000.0	-2500000.0	0	9	4695333.2	4695333.2	-1.38e-02
2	2124591.5	2124591.5	-5.83e-04	10	-2675618.4	-2675618.4	5.49e-03
3	997035.4	997035.4	-3.37e-04	11	3333799.4	3333799.4	-2.39e-03
4	2569757.5	2569757.5	-3.41e-03	12	47467213.6	47467213.6	-8.56e-02
5	1200479.4	1200479.4	-3.53e-03	13	38226219.8	38226219.8	-7.55e-02
6	-898310.2	-898310.2	1.37e-03	14	-14619850.8	-14619850.8	2.76e-02
7	1315217.4	1315217.4	-1.64e-03	15	16962740.9	16962740.9	-2.55e-02
8	9100297.5	9100297.5	-1.66e-02				

4.2. Example II: Asymmetric Levy form cable dome

This example is a Levy form cable dome with circular boundary but asymmetric geometry. The dome has three rings of hoops and an inner ring. The diameter of boundary is 100 m, and the diameter of three hoops are 75 m, 50 m and 20 m, respectively. Each hoop is divided into 12 parts. There are 252 cables and 48 struts.



(a) top view



(b) perspective view

Figure 3: Asymmetric Levy form cable dome

The history of UFI according to initial geometry is shown in Fig. 4. The initial prestress values of cables and struts are 200000 N and -50000 N, respectively. The unbalanced force varies around 1.36×10^{4} N after the 100th iteration, which verifies the infeasibility of initial geometry. Therefore, the Step-by-Step UFI is carried out. The flexible nodes are the lower nodes of struts, whose number is 48, i.e., the number of variable coordinates is $48\times3=144$.

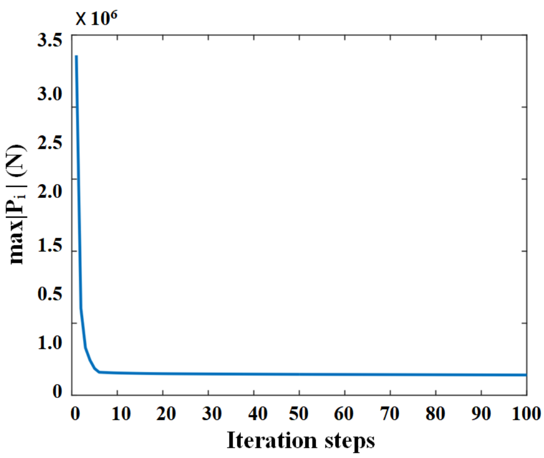


Figure 4: History of UFI

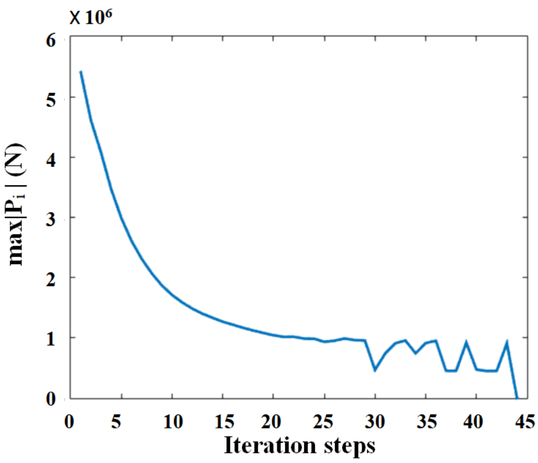
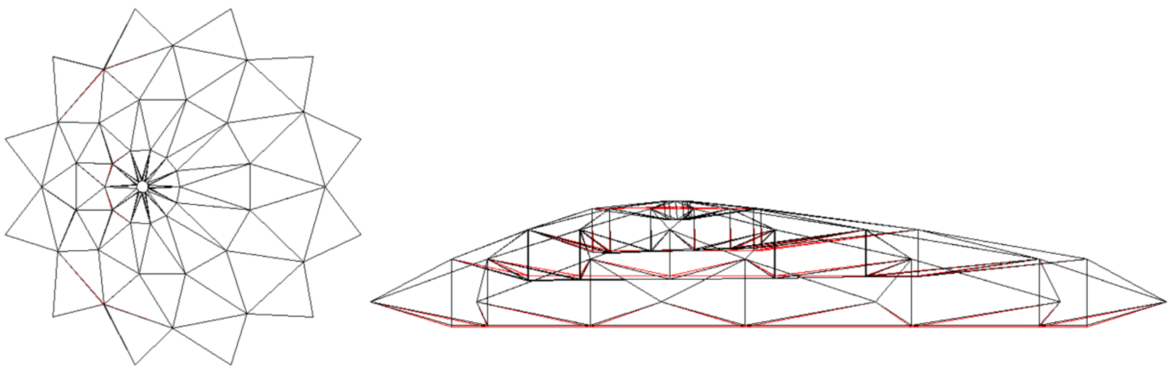


Figure 5: History of Step-by-Step UFI

The adjustment constant N_1 and N_2 are set to be 5000 N and -5000 N, respectively. The Step-by-Step UFI process converged at the 44th iteration with the final unbalanced force 0.19N. The history of iteration result is shown in Fig. 5.



(a) horizontal view (b) vertical view

Figure 6: Comparison of initial and final geometries (red: initial; black: final)

The initial and final geometries are compared in Fig. 6. The horizontal difference is shown in Fig. 6(a), it is obvious that the final and initial geomtry are almost coincide. The vertical difference is shown in Fig. 6(b). The most parts of the final structures, the vertical distance between intial and final coordinates are within 1 m, which has little influence on the clearence of roof. The average nodal distance between initial and final geometry is 0.45m, and the maximum distance is 1.13 m, which exists at the lower inner ring. Note that the difference in the hoops are sufficiently small compared with the size of the dome.

5. Conclusion

Methods to judge the geomtry feasiblity and form finding of cable dome structures have been presented. The results obtained in this paper are summarized as follows:

1. The geometry feasibility can be judge by the convegence property of Unbalanced force iteration(UFI).

2. For structures with infeasible geometry, feasible nodal coordinates can be found by Step-by-Step UFI. The final coordinates is close to the initial geometry.

4. Cable dome with asymmetric geometry presented in this paper gives alternative choices for design of cable domes with a variety of architectural assignment.

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