

Optimization of large-scale transmission tower using simulated annealing

Jingyao ZHANG*, Makoto OHSAKI^a, Zhengliang LI^b

*Nagoya City University 2-1-10 Kita-chikusa, Nagoya 464-0083, Japan zhang@sda.nagoya-cu.ac.jp

> ^aKyoto University, Japan ^bChongqing University, P. R. China

Abstract

Transmission towers are of great importance in the system of power transmission. Optimal design can greatly improve their safety as well as economy aspects, because they are constructed by mass manufacture. In this study, a size optimization problem is formulated for structural design of large-scale transmission towers, subjected to typical gravitational loads as well as wind loads. The list of available cross-sections is given *a priori*, and the optimal combination of sections is found by using simulated annealing (SA). It is shown by the numerical examples that the total volume is successfully reduced subjected to stress constraints.

Keywords: Transmission tower, optimization, simulated annealing, stress constraint

1. Introduction

Transmission lines are important lifeline infrastructures for daily life as well as industrial activities. Therefore, safety and serviceability of the whole transmission system have to be guaranteed even in hazards. In a transmission system, transmission towers are the most important structural components, supporting the wires and conductors. Hence, we present an optimal design methodology for large-scale transmission towers in this study.

The source of energy and power plants are usually far away from cities where energy is concentratedly consumed. Subsequently, the transmission systems nowadays are of longer distance as well as higher capacity. Furthermore, scale of the transmission towers becomes much larger.

Many transmission towers, especially large-scale towers, are built in highly complex environments, where the (wind and hydraulic) power sources are located. Hence, they are usually subjected to severe external loads, such as winds, earthquakes, sleet loads. Many transmission towers were damaged and even collapsed, which resulted in significant economic loss (Ishikawa [1]).

Transmission towers are constructed by mass manufacture: the number of designs is limited while a large number of towers are of the same design. This makes it important to reduce the construction costs for each single design. Furthermore, it leads to a great benefit in making use of optimization techniques in their design (Guo and Li [2]). However, there are little existing studies on optimal design of large-scale transmission towers in three dimensions, which motivates the current study.

In this study, an optimization method is presented for design of a large-scale transmission tower, subjected to typical gravitational loads as well as wind loads. Under the stress constraints, the optimal combination of cross-sections is found by using simulated annealing (SA).



Figure 1: The transmission tower and its 3D analysis model considered in this study.

2. Design Settings

Details of the transmission tower in the numerical examples are described firstly. The loading conditions, especially wind loads, are then given.

2.1. Transmission tower

In this study, we consider the optimal design of the transmission tower as shown in Figure 1. It is used for the 500kV transmission lines in China [3]. Its four sides are of the same shape, with the base width of 14.28m. Height of the tower is 74.9m. There are three crossarms located in different levels supporting the insulators.

The transmission tower consists of 2,246 members and 950 nodes. The three-dimensional analysis model is shown in Figure 1(b). Abaqus is used for structural analysis [4]. The members are modelled by beam element B31, with six degrees of freedom at each node. The tower is pin supported. Therefore, the total number of degrees of freedom for the analysis model is 5,388.

To reduce the number of design variables, the members are classified into 24 groups. Groups 1~10 are the short auxiliary members, and the other groups are classified as follows:

- Groups 11, 12: Horizontal members of the main legs
- Groups 13, 20: Diagonal members between the lowest crossarm and the cage
- Group 14: Suspension members of the upper two crossarms, and the diagonal members between the cage joint pannels and these two crossarms
- Group 15: Suspension members of the the lowest crossarm
- Group 16: Diagonal members of the cage located between the crossarms
- Group 17: Other diagonal members of the cage except for those in Group 16

- Group 18: Diagonal members of the tower body
- Group 19: Main members of the three crossarms
- Groups 21, 22: Vertical members of the cage
- Groups 23, 24: Members of the main leg

2.2. Loadings

Interruption of power supply is usually caused by natural events, and near half of them are due to high winds (Ishikawa [1]). Hence, in the design of transmission towers, the wind loads are of the most significance, besides the gravitational loads [5]. The loading conditions for the transmission tower are listed in Table 1, and the corresponding loads applied at the nodes of the tower are given in Table 2 [3]. There are three types of external loads in the design of a transmission tower: the vertical (gravitational) loads, the transverse loads normal to the direction of transmission line, and the longitudinal loads along the transmission line.

3. Optimal Design

In this section, we present the optimization problem for minimization of construction costs of the transmission tower, in terms of total volume. Moreover, simulated annealing method used to solve the problem is described in detail.

3.1. Optimization problem

Material costs, transportation, construction, and maintenance expenses of transmission towers are directly proportional to structural mass (material volume) (Shea and Smith [6]). To reduce construction costs of transmission towers, we consider the optimization problem of minimizing the total volume of the members, denoted by the objective function $f(\mathbf{x})$. The design variables in \mathbf{x} refer to the cross-section types of the members.

As a structural safety index, stress constraints under the three types of loadings are prescribed. The allowable stress for the short-term horizontal (transverse and longitudinal) loads is set as 295Mpa. The allowable stress for the long-term vertical (gravitational) loads is set as 2/3 of the allowable stress for the short-term loads.

Unit weight of conductor	1.523kg/m							
Unit weight of overhead ground wire	0.5	70kg/m						
	On tower	86kg/m ²						
Design wind pressure	On wires	52 kg/m ²						
	On insulator	60 kg/m ²						
Design span	Wind span	550m						
Design span	Weight span	1170m						
Overload factor	2.00							
Wind direction	90°							
Deviation angle	0°							

Table	1.	Loading	conditions
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Labal	Vertical	Horizontal load(N)								
Laber	load (N)	Transverse	Longitudina							
170S	13516	7960	5589							
170X	13516	7960	5589							
V1	168437	73915	49865							
V2	168437	73915	49865							
V3	168437	73915	49865							
V4	168437	73915	49865							
V5	168437	73915	49865							
V6	168437	73915	49865							
610P	0	1191.5	0							
610X	0	1191.5	0							
610Y	0	1191.5	0							
610XY	0	1191.5	0							
1000S	0	2696	0							
1000X	0	2696	0							
1000Y	0	2696	0							
1000XY	0	2696	0							
1250S	0	4124	0							
1250X	0	4124	0							
1250Y	0	4124	0							
1250XY	0	4124	0							
1500P	0	4590.75	0							
1500X	0	4590.75	0							
1500Y	0	4590.75	0							
1500XY	0	4590.75	0							
1700S	0	2072.75	0							
1700X	0	2072.75	0							
1700Y	0	2072.75	0							
1700XY	0	2072.75	0							
1900S	0	1401.25	0							
1900X	0	1401.25	0							
1900Y	0	1401.25	0							
1900XY	0	1401.25	0							
2100S	0	1532.75	0							
2100X	0	1532.75	0							
2100Y	0	1532.75	0							
2100XY	0	1532.75	0							
8000S	0	4996.5	0							
8000X	0	4996.5	0							
8000Y	0	4996.5	0							
8000XY	0	4996.5	0							

Table 2: External loads applied at the nodes of the tower

The members are modelled as beam elements with rigid joints at their ends. However, it is noticeable that influence of bending moments in the members is insignificant, since the slenderness ratio is very large. Hence, stresses of the members are calculated from axial forces only. The allowable stress for the members in compression is derived from Euler buckling load, for which the member lengths are taken as their buckling lengths.

Let $\sigma_i^j(\mathbf{x})$ denote the absolute value of the stress of member *i* subjected to load type *j*, and let $\overline{\sigma}_i^j(\mathbf{x})$ denote its upper bound. Therefore, the constraint condition on being lower than the upper bound of the stress can be written as follows:

$$G_i^j(\mathbf{x}) = \frac{\sigma_i^j(\mathbf{x})}{\bar{\sigma}_i^j(\mathbf{x})} \le 1.$$
(1)

In the case that the stress constraints are not satisfied, the maxinum ratio

$$G^{\max} = \max_{i,j} G_i^j(\mathbf{x}) \tag{2}$$

is used to define the penalty function as

$$P(\mathbf{x}) = 1000(G^{\max})^2.$$
(3)

This way, the candidate solution generated in the local search is unlikely to be selected as the new solution for the next iteration, when it does not satisfy the constraint condition. And furthermore, the constrained optimization becomes an unconstrained optimization problem defined as

$$\text{Minimize } F(\mathbf{x}) = f(\mathbf{x}) + P(\mathbf{x}). \tag{4}$$

3.2. Simulated annealing (SA)

Simulated annealing (SA) exploits an analogy between the metal annealing process and the search process of the best solution in a general combinatorial optimization problem (Kirkpatrick *et al.* [7]). Gradients of the objective function, which are not available for discrete problems, are not needed in the process of searching for the optimal solution. It also has the great advantage in searching for global optimal solution instead of being trapped in local optimum.

The algorithm of SA, searching for the solution of minimization of $F(\mathbf{x})$, is summarized as follows.

Algorithm of SA:

- <u>Step 1:</u> Specify the initial temperature *T*, scaling parameter *s*, and initial value for the searching range Δ . Determine the initial solution \mathbf{x}_0 randomly or deterministically. Set the initial counter for iterations as k = 0.
- <u>Step 2</u>: Generate a specified number of candidate solutions \mathbf{x}_{k}^{i} in the neighborhood of the current solution \mathbf{x}_{k} . Let \mathbf{x}_{k}^{*} denote the solution with the minimum objective function within the current solutions.

For the case of $F(\mathbf{x}_k^*) < F(\mathbf{x}_k)$, we update the new solution for the next iteration as $\mathbf{x}_{k+1} = \mathbf{x}_k^*$. Moreover, for the case of $F(\mathbf{x}_k^*) \ge F(\mathbf{x}_k)$, we adopt \mathbf{x}_k^* as the new solution only when a uniform random value $0 \le r \le 1$ is less than the acceptance probability *p* defined by

$$p = \exp\left(-\frac{\left|F\left(\mathbf{x}_{k}^{*}\right) - F\left(\mathbf{x}_{k}\right)\right|}{Ts}\right).$$
(5)

- <u>Step 3:</u> Decrease the temperature *T* by multiplication of the scaling parameter *s*, and reduce the searching range Δ for new solutions.
- <u>Step 4:</u> Terminate the algorithm if the termination condition is satisfied; otherwise, return to Step 2 with the updated counter k := k+1.

Label	Section number	Width \times Width \times Thickness(mm) $A \times B \times t$						
L40×4	1	$40 \times 40 \times 4$						
$L45 \times 4$	2	$45 \times 45 \times 4$						
$L50 \times 4$	3	$50 \times 50 \times 4$						
$L50 \times 5$	4	$50 \times 50 \times 5$						
$L56 \times 4$	5	$56 \times 56 \times 4$						
L56×5	6	$56 \times 56 \times 5$						
L63×5	7	$63 \times 63 \times 5$						
$L63 \times 5H$	8	$63 \times 63 \times 5$						
$L70 \times 5H$	9	$70 \times 70 \times 5$						
$L70 \times 6H$	10	$70 \times 70 \times 6$						
L75×5	11	$75 \times 75 \times 5$						
$L75 \times 5H$	12	$75 \times 75 \times 5$						
$L75 \times 6H$	13	$75 \times 75 \times 6$						
$L80 \times 6H$	14	$80 \times 80 \times 6$						
$L80 \times 7H$	15	$80 \times 80 \times 7$						
$L90 \times 6H$	16	$90 \times 90 \times 6$						
$L90 \times 7H$	17	$90 \times 90 \times 7$						
L100×7	18	$100 \times 100 \times 7$						
$L110 \times 8H$	19	$110 \times 110 \times 8$						
$L125 \times 8D$	20	$125 \times 125 \times 8$						
$L140 \times 12B$	21	$140 \times 140 \times 12$						
L140×14D	22	$140 \times 140 \times 14$						
L180×16D	23	$180 \times 180 \times 24$						
$L200 \times 16D$	24	$200 \times 200 \times 16$						

Table 3: List of cross-sections

4. Numerical Examples

As numerical examples, cross-sections of the members are selected from the 24 types as listed in Table 3. The maximum number of iterations for termination of the SA algorithm is set as 200, while 25 candidate solutions are to be searched in the neighborhood of the current solution in each iteration. The scaling parameter *s* is set to ensure p = 0.5 when the objective function increases 5%.

If we use random values as the initial solution, the stress constraints can hardly be satisfied, resulting in large penalty. Hence, we use the maximum sections for all group as the initial solution.

Moreover, randomness will increase and convergence of SA will become much worse if all variables are changed in the local search. For such problem, only twelve of the variables in each iteration are randomly selected to generate candidate solutions in the neighborhood of the current solution.

Table 4 shows ten cases of solving the optimization problem by using the SA algorithm. Case 0 is for reference, where the stress constraints are not satisfied with G^{\max} larger than 1.0. Within the ten final solutions, Case 8 has the minimum objective function, while the stress constraints are also satisfied at the same time. Hence, Case 8 is considered to be the (nearly) optimal solution for this problem.

From Table 4, the members in groups 21~24 have much larger cross-sections than the members in other groups. This is because the vertical members of the cage (groups 21 and 22) as well as the members of the main legs (groups 23 and 24) are much more effectively in sustaining external forces than other

members. However, the cross-sections types 23 and 24 with the maximum areas are absent in the final solutions.

Moreover, the chords of the crossarms (groups 15 and 19), the diagonal members of the cage as well as those in the tower body (group 17 and 18) also have large cross-sections.

The final solutions in the ten cases are respectively derived in 144, 186, 176, 175, 183, 148, 189, 166, 191, and 194 iterations.

4. Conclusions

Transmission towers are of great importance to the transmission lines, and they are constructed by mass manufacture. Hence, they are necessary to be designed as economic as possible. Meanwhile, their structural performance in severe environment has to be guaranteed. For such purposes, we present an optimal design methodology for large-scale transmission towers using simulated annealing.

From the numerical examples, we have the following observations.

- 1. SA is applicable to the (discrete) size optimization problem of three-dimensional large-scale transmission tower consisting of a large number of members.
- 2. Convergence performance of the optimization process by SA can be greatly improved by limiting the number of variables to be changed in each iteration.
- 3. It is mechanically more effective to increase sections of the members of the main legs as well as the chords of the crossarms.

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Case	F	G^{\max}		Group number																						
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	4.306	2.3651	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	4.420	1.0026	11	1	1	1	1	15	5	5	10	1	1	6	8	19	21	14	20	18	18	7	15	22	22	22
2	4.241	0.9932	1	1	1	1	1	16	5	5	11	1	1	9	7	19	19	14	20	19	18	5	14	21	22	22
3	4.447	0.9903	11	1	1	1	1	16	5	5	7	1	1	11	7	19	19	14	20	19	18	8	9	21	22	22
4	4.314	1.0087	1	2	1	1	1	16	3	5	14	1	5	8	10	19	21	14	20	18	18	11	14	22	22	22
5	4.353	1.0069	1	5	1	6	2	15	5	5	14	8	6	8	9	19	21	14	20	18	18	11	14	22	22	22
6	4.380	1.0098	5	2	1	1	3	15	5	5	10	1	1	6	7	19	22	14	20	18	18	5	14	21	22	22
7	4.310	1.0060	5	2	1	1	1	15	3	5	13	1	1	11	7	19	21	14	20	18	18	5	15	21	22	22
8	4.217	0.9938	1	1	1	1	1	16	3	5	8	5	1	9	8	19	19	14	20	19	18	8	12	21	22	22
9	4.459	1.0051	11	2	1	2	2	16	5	6	15	5	5	9	10	19	20	14	20	18	18	8	13	21	22	22
10	4.234	0.9933	1	1	1	1	1	16	3	5	11	1	1	9	7	19	19	14	20	19	18	6	9	21	22	22

Table 4: Ten cases for the design of transmission tower by application of SA.

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