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Optimization of locations of slot connections of gridshells modeled using elastica

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Abstract

An optimization method is presented for gridshells, which are generated by bending beams through forced displacements at supports. The target shape of a curved beam is defined as an elastica, which is the shape of a buckled beam-column with large deflection. The length of beam of the initial configuration on the ground is computed from the target shape. It is shown that the interaction forces at hinges between perpendicularly connected beams are reduced by designing the curve as elastica. The interaction forces are further reduced by assigning hinge+slot joints that can move along a member. A heuristic approach called simulated annealing is used for optimizing the locations of hinge+slot joints. It is shown in the numerical examples that maximum forces at joints are effectively reduced by appropriately placing a small number of hinge+slots.

Keywords: gridshell, elastica, optimization, simulated annealing, slot joint

1. Introduction

Among various types of roof structures that cover a large space, a gridshell (Matsuo et al. [1], Douthe et al. [2], Pone et al. [3]) is one of the most efficient structure in view of construction time and cost. Long straight members in mutually perpendicular directions are located on the ground and connected by hinges at their intersections. Then forced deformation is given at the boundary to obtain a curved surface.

To generate a curved surface with uniform grid size, various methods such as compass method (Bouhaya et al. [4]) and particle-spring method (Kuijvenhoven and Hoogenboom [5]) have been presented. However, generating a desired shape is very difficult, and only limited types of shape can be realized. Furthermore, appropriate connection types should be selected (Hernández and C. Gengnagel [6]) to prevent fracture due to stress concentration.

The shape of a buckled beam-column with large deflection is called *elastica* (Watson and Wang [7]). Therefore, it is natural to design the target shape of a gridshell using elastica to reduce interaction forces between beams. In this study, the optimization method in Ref. [1] is extended to reduce internal stresses of gridshells. The lengths of beams placed on the ground are defined from the target shape as assemblage of elastic beams. Gridshells are generated by forced displacements at supports. Some hinge+slots are placed at the joints to reduce the internal forces between beams.

A heuristic approach called simulated annealing (SA) (Aarts and Korst [8]) is used for optimizing the locations of hinge+slot joints. It is shown in the numerical examples that maximum forces at joints are effectively reduced by designing the target shape as elastica, and by appropriately placing the hinge+slots.

2. Definition of elastica

Elastica is defined as the shape of a buckled beam under point loads at both ends. Let s denote the arc-length parameter of a beam. The bending moment, bending stiffness, and curvature of the beam are denoted by $M(s)$, EI, and $\kappa(s)$, respectively. The relation between $M(s)$ and $\kappa(s)$ is given as

$$M(s) = EI\kappa(s) \tag{1}$$

Let $\phi(s)$ denote the deflection angle of the beam. Then the following relation holds:

$$\kappa(s) = \frac{\partial \phi(s)}{\partial s} \tag{2}$$

From Eqs. (1) and (2), we have

$$\frac{\partial \phi(s)}{\partial s} = \frac{M}{EI} \tag{3}$$

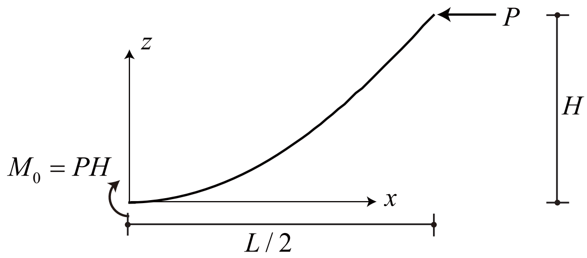


Figure 1: A half part of simply supported elastica.

Since it is difficult to derive the equation of elastica analytically, we obtain the shape using a numerical integration approach. Consider a cantilever-type elastica as shown in Fig. 1, which is assumed to be a half part of a simply supported beam. The deflection at the free end is H, and the bending moment at the fixed end is $M_0 = PH$, where the deflection angle vanishes.

We discretize the elastica with the increment Δs of arc-length as $s_i = s_{i-1} + \Delta s$. The coordinate of curve at $s = s_i$ is denoted as (x_i, z_i). The deflection angle ϕ_i at $s = s_i$ is computed from ϕ_{i-1} at $s = s_{i-1} = s_i - \Delta s$ as

$$\phi_i = \phi_{i-1} + \frac{M_i}{EI}\Delta s, \quad M_i = M_0 - Pz_i \tag{4}$$

and the coordinates are updated as

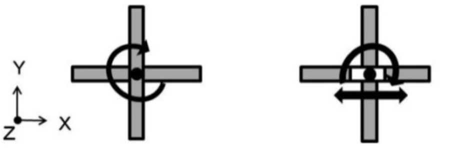
$$(x_i, z_i) = (x_{i-1} + \Delta s \cos\phi_{i-1}, z_{i-1} + \Delta s \sin\phi_{i-1}) \tag{5}$$

We assign M_0, EI, P, and set $i = 0$ and $\phi_0 = 0$ to compute Eqs. (4) and (5) for generating elastica from the fixed end until $M = 0$ is satisfied.

If the shape of a beam of a gridshell is elastica, then such beam can maintain self-equilibrium without any constraint or force from the perpendicularly connected beams. Therefore, elastica is regarded as an ideal shape of a beam of a gridshell.

3. Optimization problem of connection type

A connection of gridshell is assumed to be a hinge (revolute) joint; i.e., a beam can rotate around the normal axis at a connection on the surface of gridshell. In the following optimization method, we add a slot to a hinge so that the joint can move, as shown in Fig. 2, in the direction of one of two beams connected to the joint. This type of joint is called 'hinge+slot' joint.



(a) hinge joint (b) hinge+slot joint
Figure 2: Two types of joints.

We optimize the locations of hinge+slot joints. Let *n* denote the number of joints, where hinge or hinge+slot is to be located. The *i*th component J_i of variable vector $\mathbf{J} = (J_1, \ldots, J_n)$ is 1, if a hinge+slot is located at the *i*th joint, and 0, if a simple hinge is located. A forced displacement analysis is carried out for the gridshell, as described in the next section, to determine the shape at self-equilibrium. The objective function is the deviation of the shapes of selected k beams from the target shapes of elastica, which is formulated as

$$F(\mathbf{J}) = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{n_i} (e_{ij}^z - c_{ij}^z(\mathbf{J}))^2} \quad (6)$$

where e_{ij}^z and $c_{ij}^z(\mathbf{J})$ are the *z*-coordinates of the *j*th joint of the *i*th beam of the target elastica and gridshell, respectively, and n_i is the number of joints of the *i*th beam except the supports. By minimizing $F(\mathbf{J})$, the beam shape becomes close to elastica, and accordingly, the contact forces at joints are reduced. We use SA for optimization.

4. Optimization results of gridshell

Figure 3 shows the initial shape of a gridshell on the ground. Forced displacements are given at the supports as indicated by arrows. The local coordinates (u, v, w) are also defined in Fig. 3.

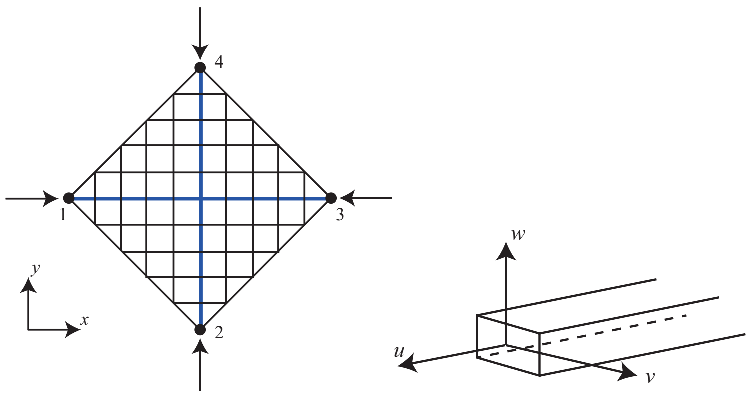


Figure 3: Initial shape and definition of local coordinates of gridshell.

The span lengths in *x*- and *y*-directions after generating a curved surface are 9 m, and the rise at the center is 1.95 m. The beams are steel plates with Young's modulus 210.0 GPa, Poisson's ratio 0.3, and mass density 7.85×10^3 kg/m^3. The following two types of gridshell are considered:

Type 1: All beams have the width 0.060 m and thickness 0.015 m.

Type 2: All beams have the width 0.040 m and thickness 0.015 m except the beams with thick blue lines in Fig. 3 that have width 0.120 m and thickness 0.020 m.

The cross-sectional shapes have been defined so that the two types have almost the same reaction forces at the final equilibrium shape.

The target shapes of beams with thick blue lines in Fig. 3 are generated using Eqs. (4) and (5) with $\Delta s = 0.1$ as shown in Fig. 4, where, $M_0 = 720$ Nm and $P = 369$ N for Type 1, and $M_0 = 3413$ Nm and $P = 1750$ N for Type 2.

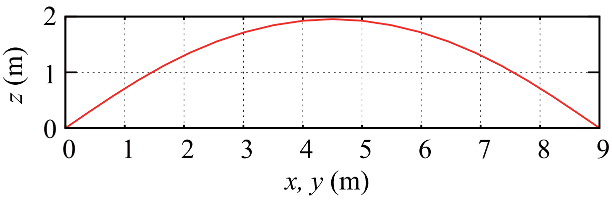
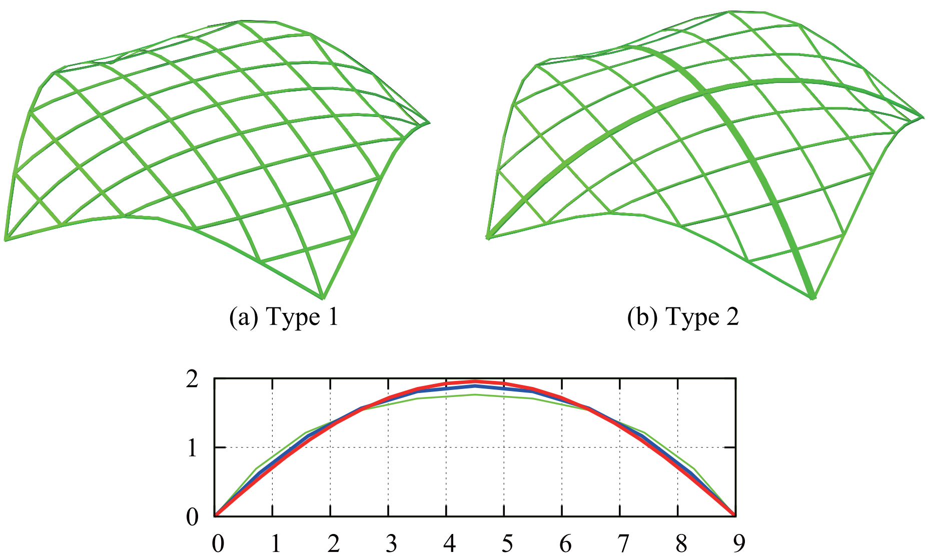


Figure 4: Target shape of elastica.

The arc-length of elastica obtained this way is 10.0 m; i.e., the span lengths in *x*- and *y*-directions of the initial shape on the ground are 10.0 m, and the grid size is 1.0 m. Therefore, the forced horizontal displacements at both ends of elastica are 0.50 m. The number of joints is 61, and the length of a slot is 0.10 m. Each member is divided into two beam elements. We assume all slots are directed to beam axes that are initially in *x*-direction.

ABAQUS Ver. 6.14 [9] is used for large deformation analysis. The loading parameter t is increased from 0.0 to 2.0. Auxiliary upward load equivalent to the self-weight is applied at each joint linearly in the period $0 \le t \le 1.0$ to prevent bifurcation-type buckling while assigning forced horizontal displacements. Next, forced displacements of 0.5 m are applied at four supports indicated with filled triangle in the period $1.0 \le t \le 2.0$. The vertical load is removed also in this period to generate a self-equilibrium shape.



(c) beam shapes in *x*-direction between the supports

Figure 5: Shapes without hinge+slot joints; red: elastica, green: Type 1, blue: Type 2.

The shapes of Types 1 and 2 gridshells with all hinge joints (without hinge+slot joint) are shown in Fig. 5(a) and (b), respectively. The mean absolute values and maximum absolute values of forces at joints are summarized in Tables 1 and 2, respectively, where *Cu*, *Cv*, and *Cw* are the contact forces at hinge joints in *u*, *v*, and *w* directions of the members that are initially in *x*-direction on the ground.

Note that *Cw* is the normal force, and *Cu* and *Cv* are the shear forces in the revolute joint. It is seen from Fig. 5(c) that the shape of the beam undergoing forced displacement of Type 2 is closer to that of elastica 1. It is also seen from Tables 1 and 2 that the mean absolute values and maximum absolute values of forces at joints can be reduced using larger section for members undergoing forced displacements. Therefore, we can assume that the forces at joints can be reduced by reducing the deviation of the beam shape from the target shape of elastica.

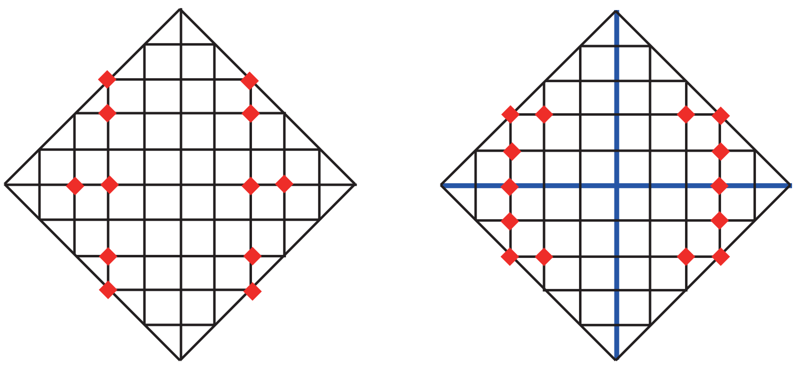
Table 1: Mean absolute values of forces at joints

	Type 1			Type 2		
	All hinge	All hinge+slot	Optimal	All hinge	All hinge+slot	Optimal
Cu (kN)	2.890	1.765	2.116	1.162	0.674	0.894
Cv (kN)	4.288	3.962	4.031	1.948	1.588	1.732
Cw (kN)	0.833	0.963	0.957	0.678	0.537	0.554

Table 2: Maximum absolute values of forces at joints

	Type 1			Type 2		
	All hinge	All hinge+slot	Optimal	All hinge	All hinge+slot	Optimal
Cu (kN)	16.17	13.87	13.85	5.669	4.576	5.676
Cv (kN)	16.40	14.50	14.16	5.885	4.890	5.634
Cw (kN)	4.802	4.232	4.299	2.253	1.900	2.083

If hinge+slots are assigned at all joints, the forces at joints, especially *Cu* in the direction of slot, is reduced, as seen in Tables 1 and 2. Therefore, we optimize the locations of hinge+slot joints to reduce the joint forces using small number of hinge+slot joints. We assume hinge+slot joints are placed symmetrically in *x*- and *y*-directions, and no hinge+slot exists along the beam between support 2 and 4 indicated in Fig. 3, because slot is directed in *x*-direction and a slot along this beam will not have any effect. Therefore, the number of possible locations of hinge+slot joints is 14. We assign four hinge+slot joints; accordingly, the total number of combinations is ${}_{14}C_4 = 1001$. Note that the total number of hinge+slot joints will be less than 16, if some are located along the beam between supports 1 and 3.



(a) Type 1 (b) Type 2

Figure 6: Locations of hinge+slot joints of optimal solutions.

SA is carried out using five different random seeds. The number of steps is 20, and the number of neighborhood solutions is five; i.e., the total number of analysis in each trial is 100. Figures 6(a) and show the optimal locations of slots for Types 1 and 2, respectively. The mean and maximum absolute values of forces at joints are listed in Tables 1 and 2. Note that the same optimal solution has been found by five trials for Type 1, and by four out of fine trials for Type 2. The optimal solutions have been confirmed by enumeration of all 1001 combinations for Type 1 and 2. For comparison, the results of hinge+slot joints for all connections are also listed in Tables 1 and 2. It is seen from these results that optimization of locations of hinge+slot joints leads to reduction of joint forces with small number of hinge+solot joints.

5. Conclusions

1. Joint forces of a gridshell can be reduced by designing the target shape of beam as an elastic, which is a buckled shape of a beam under compression at two ends, because elastica can maintain equilibrium shape without any force from the perpendicularly connected beams.
2. The joint forces can be reduced by assigning hinge+slot joints at all connections. The number of hinge+solot joints can be reduced by minimizing the deviation of the shape of curved beam from the target shape of elastica.

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