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Design of deployable structures using limit analysis of partially rigid frames with quadratic yield functions

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Abstract

The linear programming approach to generating deployable structures is extend to allow a hinge rotating around an axis in arbitrary direction. Plastic limit analysis problem with quadratic yield function with respect to member-end moments is solved to generate hinges in arbitrary directions of a partially rigid frame. The directions of hinges are obtained from the member-end momemts along the local axes. Using the proposed method, three-dimensional mechanisms and deployable structures with small number of hinges are successfully obtained. The manufacturability of a deployable structure is confirmed by assembling a small-scale model.

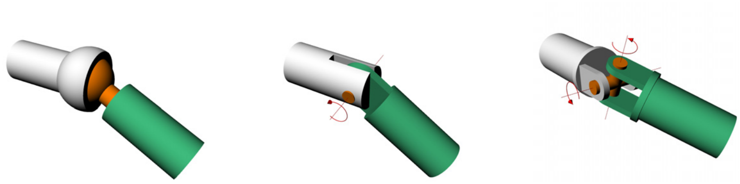
Keywords: link mechanism, deployable structure, quadratic programming, plastic limit analysis

1. Introduction

A deployable structure usually consists of bars and actuators connected by pin joints (Akgün [1], Hoberman [5]). It is kinematically indeterminate and has a mechanism corresponding to the desired deformation without external load. However, three-dimensional mechanisms with ideal pin joints that can rotate in three directions, e.g., spherical joint as shown in Fig. 1(a), are likely to have large degrees of kinematic indeterminacy, and it is difficult to control such a mechanism in the deployment process. Therefore, a deployable structure is desirable to be composed of frames with revolute joints in one direction as shown in Fig. 1(b), or universal joints that can rotate in two directions as shown in Fig. 1(c), to ensure a small degree of kinematic indeterminacy. Fig. 2 shows an example of three-dimensional frame with partially rigid connections; two members are rigidly connected at node A, and the remaining node and supports consist of revolute joints in various directions.

The authors presented a design method of three-dimensional mechanisms of partially rigid frame based on plastic limit analysis that is formulated as a linear programming problem (Ohsaki *et al*. [4], Tsuda *et al*. [6,7]). However, in the previous studies, the hinges are restricted to rotate around specified axes in the orthogonal directions of local coordinates. Therefore, many hinges are needed to generate the desired deformation.

In this study, we extend our method to allow a hinge rotating around an axis in arbitrary direction. Limit analysis with quadratic yield function with respect to member-end moments is used to generate hinges. The directions of hinges are obtained from the ratios of member-end moments. Using the proposed method, three-dimensional mechanisms and deployable structures with small number of hinges are successfully obtained, and their manufacturability is confirmed by assembling small-scale models.



(a) (b) (c)

Figure 1: Hinge connections; (a) spherical joint in three directions, (b) revolute joint in one direction, (c) universal joint in two directions.

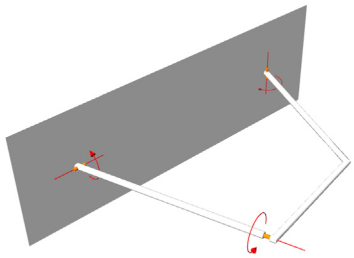
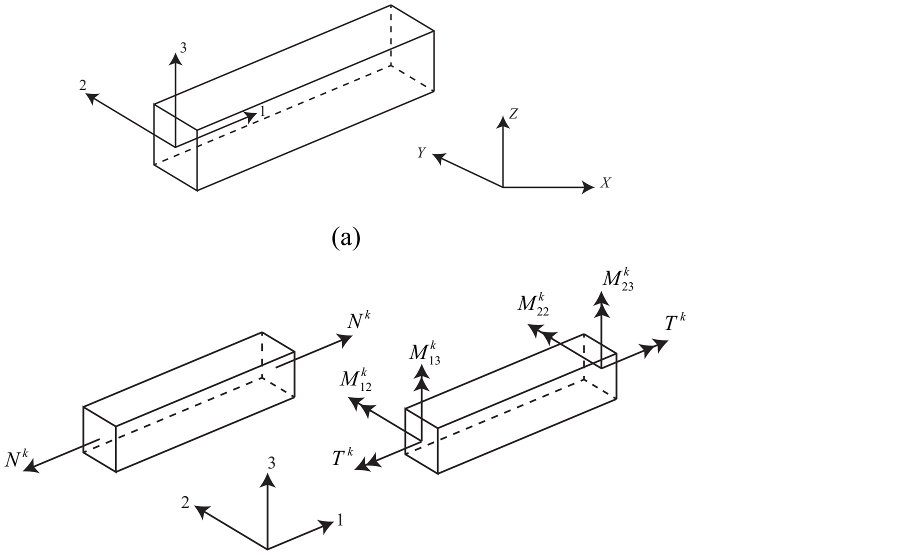


Figure 2: A three-dimensional frame with a partially rigid connection.

2. Definition of variables

We consider deployable structures consisting of frame members. The local member coordinates are defined as shown in Fig. 3(a). The two nodes connected by member k is denoted by 1 and 2. Axis 1 is directed from node 1 to 2, and axes 2 and 3 are the principal axes of the cross-section. Let N^k, M_{12}^k, M_{13}^k, and T^k denote the axial force, bending moment around axis 2, bending moment around axis 3, and torsional moment, respetively, at node 1. Those at node 2 are defined similarly. Note that the axial forces and torsional moments at nodes 1 and 2, respectively, are in equilibrium states; therefore, the forces in the direction of axis 1 and moments around axis 1 at nodes 1 and 2, respectively, have the same magnitude and the oposite direction. Accordingly, each member has independent six components of member-end forces as shown in Fig. 3(b).



(b)
Figure 3: Definition of member coordinates and independent member-end forces; (a) local and global coordinates, (b) independent six member-end forces.

Let $\mathbf{f} = (f_1, \ldots, f_{6m})^T$ denote the vector of member-end forces of m members; i.e., each component of $\mathbf{f}$ corresponds to N^k, M_{12}^k, M_{13}^k, M_{22}^k, M_{23}^k, or T^k. The generalized member-end strain vector is denoted by $\mathbf{c} = (c_1, \ldots, c_{6m})^T$; i.e., c_i corresponds to member extension and rotations around 1, 2, 3 axes, respectively, if f_i is axial force, torsional moment, and bending moments around 2 and 3 axes. The vector $\mathbf{c}$ is related to the nodal displacement vector $\mathbf{u}$ through the compatibility matrix $\mathbf{H}^T = (\mathbf{h}_1, \ldots, \mathbf{h}_{6m})^T$ as

$$\mathbf{c} = \mathbf{H}^T \mathbf{u} \tag{1}$$

3. Quadratic programming problem for generating deployable structure

A plastic limit analysis problem with quadratic yield functions is formulated for generating deployable structures. The member-end forces are related to one of the component of $\mathbf{f}$; therefore, they are regarded as functions of $\mathbf{f}$ as $N^k(\mathbf{f})$, $M_{12}^k(\mathbf{f})$, $M_{13}^k(\mathbf{f})$, $M_{22}^k(\mathbf{f})$, $M_{23}^k(\mathbf{f})$, and $T^k(\mathbf{f})$. To generate a deployable structure that has a desired deformation, input loads are applied at the nodes in the direction of forced deformation, while output loads are applied at the nodes that moves in the desired direction. The load vectors corresponding to the input and output decrees-of-freedom are denoted by $\mathbf{p}_{\text{in}}$ and $\mathbf{p}_{\text{out}}$, respectively.

We assign yield functions for member-end moments and axial force, respectively. An upper bound is given for the sum of squares of three components of moments at each member-end. The optimization problem for maximizing the load coefficient λ_{in} corresponding to the input load is formulated as follows:

$$\begin{aligned} &\text{maximize } \lambda_{\text{in}} \\ &\text{subject to } \sum_{i=1}^{6m} f_i \mathbf{h}_i = \mathbf{p}_{\text{out}} + \lambda_{\text{in}} \mathbf{p}_{\text{in}} \\ &\qquad (T^k(\mathbf{f}))^2 + (M_{j2}^k(\mathbf{f}))^2 + (M_{j3}^k(\mathbf{f}))^2 \le \alpha w^{\text{b}}, \ (k = 1, \dots, m;\ j = 1, 2) \\ &\qquad (N^k(\mathbf{f}))^2 \le \alpha w^{\mathbf{a}}, \ (k = 1, \dots, m) \end{aligned} \tag{2}$$

where w^{b} and w^{a} are the weight coefficients for moment and axial force, respectively, and α is a scaling parameter.

Using $\mathbf{u}$, c_{kj} , and c_{k0} as Lagrange multipliers for the first, second, and third constraints, the following equations are obtained from the optimality conditions (KKT conditions):

Normalization of $\mathbf{u}$:

$$1 - \mathbf{p}_{\text{in}}{}^T \mathbf{u} = 0 \tag{3}$$

For f_i corresponding to bending moment:

$$\mathbf{h}_i{}^T \mathbf{u} + 2 M_{jp}^k(\mathbf{f}) c_{kj} = 0, \ (k = 1, \dots, m;\ j = 1, 2;\ p = 2, 3) \tag{4}$$

For f_i corresponding to torsional moment:

$$\mathbf{h}_i{}^T \mathbf{u} + 2 T^k(\mathbf{f})(c_{k1} + c_{k2}) = 0, \ (k = 1, \dots, m;\ j = 1, 2) \tag{5}$$

For f_i corresponding to axial force:

$$\mathbf{h}_i{}^T \mathbf{u} + 2 N^k(\mathbf{f}) c_{k0} = 0, \ (k = 1, \dots, m) \tag{6}$$

with the complementarity conditions

$$[(T^k(\mathbf{f}))^2 + (M_{j2}^k(\mathbf{f}))^2 + (M_{j3}^k(\mathbf{f}))^2 - \alpha w^{\text{b}}] c_{kj} = 0, \ c_{kj} \ge 0, \ (k = 1, \dots, m;\ j = 1, 2) \tag{7}$$

$$[(N^k(\mathbf{f}))^2 - \alpha w^{\mathbf{a}}] c_{k0} = 0, \ c_{k0} \ge 0, \ (k = 1, \dots, m;\ j = 1, 2) \tag{8}$$

Let θ_{jp}^k denote the rotation angle of node j $(= 1, 2)$ around the local axis p $(= 1, 2, 3)$ of member k . The torsional angle θ_1^k around axis 1 is defined as

$$\theta_1^k = \theta_{j2}^k - \theta_{i1}^k \tag{9}$$

From (1), (4), (5), and (9), we have

$$\text{Bending:}\quad c_k = \mathbf{h}_k{}^T\mathbf{u} = \theta^k_{jp} = 2M^k_{jp}c_{kj},\ \ (k=1,\ldots,m;\ j=1,2;\ p=1,2) \tag{10}$$

$$\text{Torsion:}\quad c_k = \mathbf{h}_k{}^T\mathbf{u} = \theta^k_1 = \theta^k_{j1} - \theta^k_{i1} = 2T^k(c_{k1}+c_{k2}),\ \ (k=1,\ldots,m;\ j=1,2;\ p=1,2) \tag{11}$$

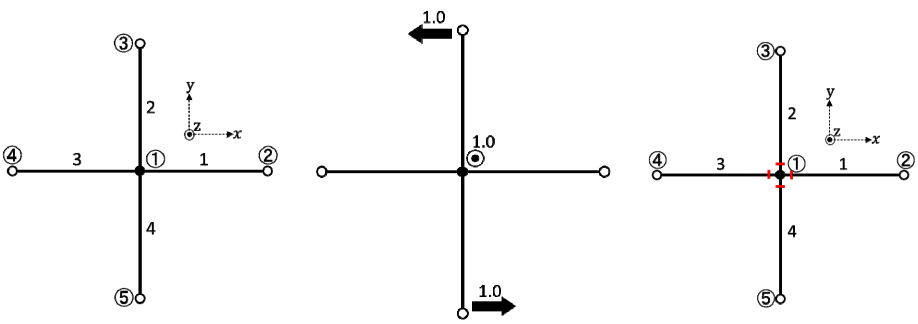
Therefore, the direction of hinges $\mathbf{R}^k_1$ and $\mathbf{R}^k_2$ at nodes 1 and 2, respectively, of member k are obtained as follows:

$$\mathbf{R}^k_1 = \begin{pmatrix} -\theta^k_1 \\ \theta^k_{12} \\ \theta^k_{13} \end{pmatrix} = c_{k1}\begin{pmatrix} -T^k \\ M^k_{12} \\ M^k_{13} \end{pmatrix},\quad \mathbf{R}^k_2 = \begin{pmatrix} \theta^k_1 \\ \theta^k_{22} \\ \theta^k_{23} \end{pmatrix} = c_{k2}\begin{pmatrix} T^k \\ M^k_{22} \\ M^k_{23} \end{pmatrix} \tag{12}$$

i.e., rotation at the member end is proportional to the bending/torsional moment. Therefore, the direction of hinge is obtained by solving the quadratic programming problem, and a deployable structure with inclined hinges is generated. Note from (7) that c_{kj} vanishes and hinge is not generated, if the yield condition for the moments is not satisfied with equality. The optimization library SNOPT Ver. 7.2 (Gill et al. [6]) is used for solving the quadratic programming problem.

4. Numerical examples

Two types of simple mechanisms are generated using the proposed method. For both examples, all members with the length 1 m have pipe cross-section with radius 50 mm and thickness 2 mm. The material is steel with Young's modulus 200 GPa.

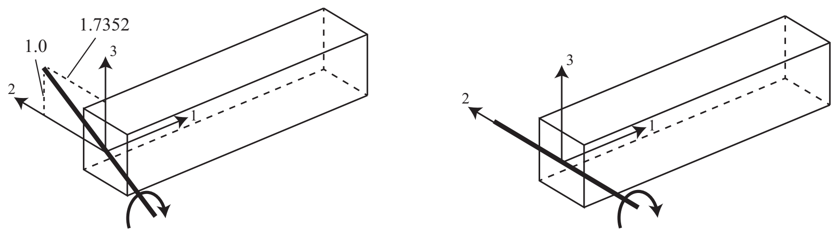


(a) (b) (c)

Figure 4: A simple four-bar mechanism, (a) node and member numbers, (b) input and output loads, (c) location of hinges indicated by red bar.

We first consider a simple four-bar model as shown in Fig. 4(a). All translational and rotational components except z-directional components are constrained at node 1, and z-directional displacement is constrained at nodes 2, 3, 4, and 5. A mechanism is given so that the output nodes 3 and 5 moves to left and right, respectively, as a result of pulling the input node 1 in z-direction. For this purpose, the input load is given at node 1, and the output loads are applied at nodes 3 and 5, as shown in Fig. 4(b).

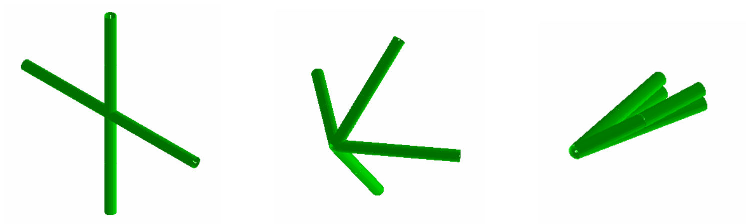
The optimization problem is solved to find the hinge locations as indicated with red line in Fig. 4(c). Note that the hinges of members 2 and 4 are inclined as shown in Fig. 5.



members 2 and 4 members 1 and 3

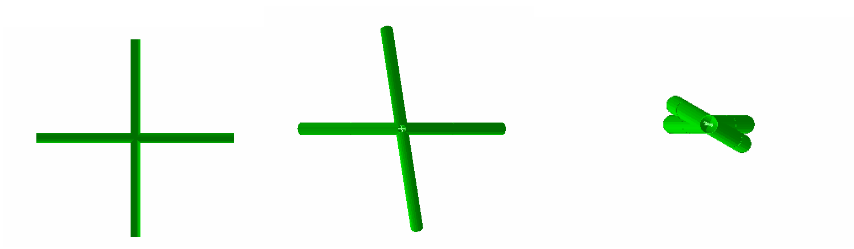
Figure 5: Directions of hinges of member-ends at node 1.

Since only small deformation is considered in the process of generating a mechanism by solving the optimization problem, large-deformation analysis is carried out using ABAQUS Ver. 6.13 (Dassault Systemes [2]) for verification purpose.Let t denote the path parameter that is increased from 0 to 1. The deformation process is shown in Fig. 6. It has been confirmed that no force is needed until t reaches a slightly smaller value than 1, where four members are almost in z-direction and members can be in tensile state.



$t = 0.0$ $t = 0.6$ $t = 1.0$

(a)



t = 0.0 0.6 1.0

(b)

Figure 6: Deformation process of the four-bar mechanism; (a) diagonal view, (b) top view.

We next consider a square grid as shown in Fig. 7. Nodes 2 and 4 are supported in y- and z-directions, and nodes 3 and 5 are supported in x- and z-directions. A mechanism is generated so that the output nodes 6–9 moves in z-direction as a result of pulling the input node 1 in negative z-direction. The input and output loads are applied at the input and output nodes, to find the locations of hinges as indicated with red line in Fig. 7. Note that the hinges at nodes 6–9 are inclied in diagonal directions. The number of hinges is 12, while 28 hinge are needed in Refs. 3 and 5, where only the hinges in the directions of local axes are allowed.

Geometrically nonlienar analysis is carried out using ABAQUS. The deformation process is shown in Fig. 8. Note that the nodes 6–9 first moves in the positive z-direction; however, they later turn to move in the negative z-direction. In this example, large force is needed at node 1; i.e., the mechanism obtained by solving the quadratic programming problem is not a finite mechanism. At t = 0.89, the force at node 1 is 1.706 kN, and the maximum stress is 4.815 GPa. Therefore, additional hinges are needed to generate a finite mechanism.

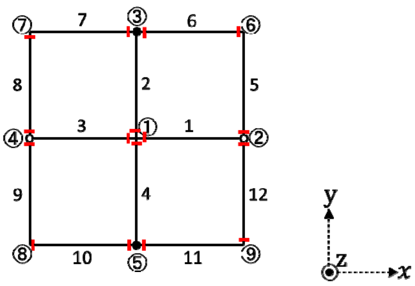
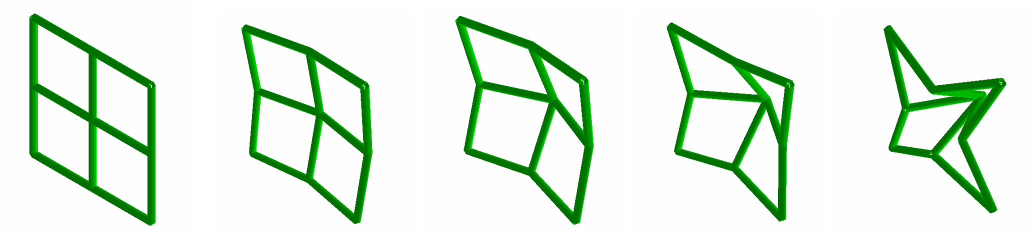


Figure 7: Member number, node number, and locations of hinges of a simple square grid.

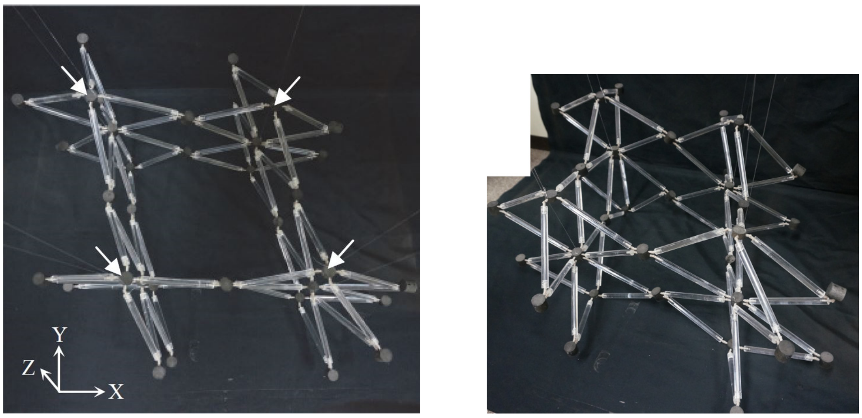


t = 0.0 t = 0.2 t = 0.4 t = 0.6 t = 0.89

Figure 8: Deformation process of the simple square grid model.

Finally we manufactured a small physical model using the acrylic material. The parts are shown in Fig. 9(a). The revolute joint in one direction is used as the hinge connection. The length and the diameter of the acrylic bar are 120 mm and 10 mm, respectively. The four-bar model in Fig. 6 are assembled to generate a deployable structure as shown in Fig. 9(b). We located the circular connectors at the end of the bars to connect them each other. By giving forced displacement in z-direction at the four connecters indicated by white arrows in Fig. 9(b), we obtained expected deformation without large force.

(a)



(b)

Figure 9: Small physical model using epoxy material.

4. Conclusions

A new method has been presented for generating a deployable structure by solving a quadratic programming problem that is regarded as a limit analysis problem with a quadratic yield function of the member-end moments. It has been shown in numerical examples that a mechanism with inclied hinges can be found by the proposed approach. By allowing an inclined hinge, the number of hinges can be reduced compared with the previous study, where only the hinges around the local axes are allowed. However, only small deformation is considered in the problem formulatedn; therefore, additional hinges may be needed to generate a finite mechanism.

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