Optimization of elastically deformed gridshell with partially released joints

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Abstract

An optimization method is presented for form finding and stress reduction of gridshells. Each joint basically consists of a hinge that can rotate along the normal axis of the surface. The translational constraints at some joints are released by assigning 'hinge+slot' joints that can move in the direction of a member axis so that the stress due to bending and axial deformation of members is reduced. Locations of slots are optimized using a heuristic approach called simulated annealing, where the total number of hinge+slot joints is fixed. The efficiency of the proposed method is demonstrated using a gridshell generated by forced displacements at the supports.

Keywords: gridshell, optimization, hinge, slot, simulated annealing, large-deformation analysis, strain energy, combinatorial problem, forced displacement

1. Introduction

Latticed shells are widely used for covering large space for arenas and stadiums. Although latticed shells are environmentally compatible in view of the total amount and weight of material in comparison to the covered area, the construction process is not easy if complex surface is to be designed. By contrast, a gridshell can be generated easily from an orthogonal grid on a plane [1].

Practically applicable gridshells were developed at the Institute of Lightweight Structures of the University of Stuttgart by Frei Otto in 1950s and 1960s. One of the most famous realizations of girdshell is the Mannheim Multihalle in 1975. Recently, owing to development of structural material, construction process, and computational design tool, gridshells utilizing elastic deformation, or so-called active bending, are extensively studied and constructed for reduction of construction cost of latticed free-form shells [2]. The gridshells are environmentally efficient, because natural materials such as timbers can be used, and the members and joints can be reused for temporary structure. Although some researcher call ordinary single-layer latticed shell as gridshell, we consider only the post-formed gridshell with bending at the state without external load [2,3].

In the construction process of a gridshell, long straight members in mutually perpendicular directions are located on the ground and connected by hinges at their intersections. Then forced deformation is given at the boundary to obtain a curved surface. To generate a curved grid with uniform member length, various methods such as compass method have been presented [4]. Kuijvenhoven and Hoogenboom [5] presented a particle-spring method along with the dynamic relaxation method to generate an equilibrium shape close to the target shape. However, generating a desired shape is very difficult, and only limited types of shape can be realized. Furthermore, the stress and strength at joints should be carefully investigated to prevent fracture due to stress concentration. Hernández and Gengnagel [6] investigated the effect of connection types on the shape and stress of gridshells. Pone et al. [3] developed a design tool using Grasshopper and Karamba. Optmization methods have also been presented for generation a desired shape without any stress concentration [4].

In this study, an optimization method is presented for form finding and stress reduction of gridshells. The axial deformation is released by assigning 'hinge+slot' joints that can move in a member direction and rotate along the normal axis of the surface. Locations of hinge+slot joints are optimized using a heuristic approach called

simulated annealing (SA), where the total number of hinge+slots are fixed. The efficiency of the proposed method is demonstrated using the gridshell generated by forced displacements at supports.

2. Properties of gridshell

Consider a gridshell consisting of long bars connected by joints. Bars are initially placed on a plane, as shown in Fig. 1(a), and forced displacements are given at boundary nodes to generate curved surface as shown in Fig. 1(b). Let *member* denote the segment of a long bar separated by joints that are also called *nodes*. Each member is divided into two beam elements.



Figure 1: An example of gridshell; (a) grid on a plane, (b) a curved surface generated by forced displacements.

We use two types of joints as shown in Fig. 2(a) and (b). Most of the pairs of intersecting bars are connected using 'hinge' joints that can rotate around the normal axis (Z-axis in Fig. 2(a)) of the surface. Some pairs of bars are connected using 'hinge+slot' joints that can rotate around the Z-axis and also translate along the bar in Y-direction. The local coordinates (x, y, z) are in the direction of (X, Y, Z) at the initial undeformed state, and they rotate with the deformation.

Forced-displacement analysis is carried out for the gridshell as shown in Fig. 1. In order to generate out-of-plane deformation as a result of in-plane displacement, a fictitious upward gravity force is given before assigning horizontal displacements, and the gravity force is reduced to zero at the end of large-deformation analysis.



Figure 2: Two types of joints; (a) hinge, (b) hinge+slot.

3. Optimization problem and optimization algorithm

The locations of hinge-slot joints are defined by the vector **J** of the 0-1 design variables $\mathbf{J} = J_i, \dots, J_n$, where *n* is the number of design variables, which is equal to the number of free nodes. The hinge and hinge-slot joints are indicated by $J_i = 0$ and 1, respectively; hence, the optimization problem is formulated as a 0-1 programming problem or a combinatorial problem.

The strain energy of element *i* is denoted by S_i (*i* = 1,...,*m*), where *m* is the number of elements. We consider two unconstrained optimization problems designated as P1 and P2 as follows:

P1: Minimize
$$F_1(\mathbf{J}) = \max S_i(\mathbf{J})$$
 (1)

P2: Minimize
$$F_2(\mathbf{J}) = \sum_{i=1}^m S_i(\mathbf{J})$$
 (2)

It is obvious that the objective function value is reduced if the number of hinge-slots are increased. Therefore, we assign the number s of the hinge-slots and optimize their locations.

Optimal solutions are found using a heuristic algorithm SA. The algorithm is summarized as follows:

- Step 1: Assign the initial temperature T_0 , scaling parameter c, and temperature reduction ratio r. Assign, or randomly generate, the initial solution \mathbf{J}^{init} , and compute the objective function value $F^{\text{init}} = F_1(\mathbf{J}^{\text{init}})$ or $F_2(\mathbf{J}^{\text{init}})$. Set incumbent optimal solution and objective value as $\mathbf{J}^{\text{opt}} = \mathbf{J}^{\text{init}}$ and $F^{\text{opt}} = F^{\text{init}}$, respectively. Set the iteration counter k = 0 and let $\mathbf{J}^{(k)} = \mathbf{J}^{\text{init}}$.
- Step 2: Randomly find two elements I_0 and I_1 from the sets satisfying $J_i^{(k)} = 0$ and 1, respectively, and swap the variables as $J_{I_0} = 1$ and $J_{I_1} = 0$ to generate a neighborhood solution. Find *h* neighborhood solutions $\mathbf{J}_1^*, \dots, \mathbf{J}_h^*$ using this process.
- Step 3: Compute the objective function values $F(\mathbf{J}_i^*)$ of all the neighborhood solutions, and find the best solution \mathbf{J}^* . If $F(\mathbf{J}^*) < F(\mathbf{J}^{(k)})$, then let $\mathbf{J}^{(k+1)} = \mathbf{J}^*$; otherwise, we have the following two cases:
 - (a) If a uniform random number $p \in [0,1]$ is less than the probability of acceptance $P = \exp(-\Delta F / (cT))$, then accept the solution as $\mathbf{J}^{(k+1)} = \mathbf{J}^*$ where $\Delta F = F(\mathbf{J}^*) F(\mathbf{J}^{(k)})$.
 - (b) If $p \ge P$, keep the solution as $\mathbf{J}^{(k+1)} = \mathbf{J}^{(k)}$.

Step 4: Increase the counter as $k \leftarrow k+1$. Let $\mathbf{J}^{opt} = \mathbf{J}^{(k)}$ and $F^{opt} = F(\mathbf{J}^{(k)})$, if $F(\mathbf{J}^{(k)}) < F^{opt}$. Go to Step 2, if the termination condition is not satisfied.

Step 5: Output the best solution \mathbf{J}^{opt} and its objective value F^{opt} .

4. Optimization results

We use ABAQUS Ver. 6.13 [7] for large-deformation analysis. Optimal locations of hinge-slots are found for the gridshell model as shown in Fig. 1. The lengths of horizontal and vertical members on the plane are 1 m except the four members of 0.5 m connected to the supports. Material of bars is steel with Young's modulus 210.0×10^3 N/mm², Poisson's ratio 0.3, and mass density 7.85×10^3 kg/m³. Width and thickness of the bars are 50 mm and 5 mm, respectively. The number of free joints indicated by filled circle is 81; i.e., n = 81.



Figure 3: Gridshell with all hinge joints; (a) shape generated by forced displacements, (b) locations of hinges, indicated by '×', with 10 largest y-directional forces, (c) locations of hinges, indicated by '×', with 20 largest y-directional forces.

The parameters for SA are $T_0 = 1.0$ and r = 0.94. The scaling parameter is assigned so that P = 0.6 for $\Delta F = 0.1$ at the initial step, which results in c = 0.1958. The number of neighborhood solutions is equal to the number of hinge-slots. When all joints are hinges; i.e, there exists no hinge-slot, the shape of gridshell is as shown in Fig. 3(a). The locations of hinges with 10 largest y-directional forces are indicated with '×' in Fig. 3(b). Hinge+slots are located at these joints to define the initial solution for optimization using SA. As seen in Fig. 3, the joints near the supports with forced displacements have large y-directional forces.

Problem P1:

Consider first the problem P1 for minimizing the maximum value of strain energy among all elements. Optimal solution found by SA from the initial solution in Fig. 3(b) is shown in Fig. 4. Properties of gridshells with various joint conditions are listed in Table 1. It is seen from Table 1 that the maximum strain energy $F_1(J)$ can be reduced 43% from 69.300 Nm to 39.372 Nm by allowing 10 hinge-slots. Note that the objective value of initial solution is 41.567 Nm, which is 60% of the initial solution. Therefore, the maximum strain energy can be effectively reduced by locating 10 hinge-slots at the joints with 10 maximum y-directional forces for the structure with hinges at all joints. The maximum strain energy is reduced only 6.4% through optimization using SA. The axial forces and y-directional shear forces are also drastically reduced by assigning hinge-slots. However, reduction of z-directional shear force is not significant, because it corresponds to the bending deformation (curvature) of members that is related to the forced displacements that does not change in the process of optimization.



Figure 4. Optimal locations of 10 hinge-slots, indicated by '×' for problem P1, and its equilibrium shape.

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	$F_1(\mathbf{J})$	$F_2(\mathbf{J})$	Axial force	y-directional	z-directional			
	(Nm)	(Nm)	(N)	shear force (N)	shear force (N)			
All hinge joints	69.300	663.63	1021.0	528.0	316.0			
10 hinge-slots (Initial)	41.567	551.84	843.4	356.3	308.6			
10 hinge-slots (Optimal)	39.372	544.58	871.8	241.7	250.3			
20 hinge-slots (Initial)	38.920	522.03	752.7	291.3	230.4			
20 hinge-slots (Optimal)	30.940	505.65	717.5	252.1	201.9			
All hinge-slots	41.44	515.58	1015.0	289.0	203.0			

Table 1: Properties of gridshells with various joint conditions.

We next increase the number of hinge-slots to 20, and carry out SA from the initial solution in Fig. 3(c). Optimal solution is shown in Fig. 5. As seen from Table 1 and Fig. 5, the maximum strain energy is reduced to 45% of the structure with hinges at all joints. Furthermore, the maximum strain energy is reduced more than 20% from the initial solution; hence, optimization using SA is effective if the number of allowable hinge-slots are sufficiently large.



Figure 5. Optimal locations of 20 hinge-slots, indicated by '×' for problem P1, and its equilibrium shape.

Problem P2:

We next solve the problem P2 for minimizing the total strain energy of the beam members. The initial solutions are defined in the similar manner as P1. When 10 hinge-slots are allowed, the optimal solution found by SA from the initial solution in Fig. 3(b) is shown in Fig. 6. Properties of gridshells with various joint conditions are listed in Table 2, which shows that the total strain energy $F_2(J)$ is reduced 22 % from 663.63 Nm to 515.58 Nm by replacing 10 hinges to hinge-slots. The objective value is reduced 6.4% from 551.84 Nm to 515.58 Nm through optimization using SA.



Figure 6. Optimal locations of 10 hinge-slots, indicated by '×' for problem P2, and its equilibrium shape.

Iable 2: Properties of gridshells with various joint conditions.								
	$F_1(\mathbf{J})$	$F_2(\mathbf{J})$	Axial force	y-directional	z-directional			
	(Nm)	(Nm)	(N)	shear force (N)	shear force (N)			
All hinge joints	69.30	663.63	1021.0	528.0	316.0			
10 hinge-slots (Initial)	41.57	551.84	843.4	356.3	308.6			
10 hinge-slots (Optimal)	41.44	515.58	962.7	240.9	234.2			
20 hinge-slots (Initial)	38.92	522.03	752.7	291.3	230.4			
20 hinge-slots (Optimal)	41.93	489.52	717.5	252.1	201.9			
All hinge-slots	41.44	515.58	1015.0	289.0	203.0			

When the number of hinge-slots is increased to 20, the optimal solution obtained using SA from the initial solution in Fig. 3(c) is shown in Fig. 7. As seen from Table 2 and Fig. 7, the total strain energy is reduced more than 26% by locating 20 hinge-slots near the supports. The reduction of total strain energy through optimization is 22% from the initial solution, where the hinge-slots are located intuitively.



Figure 7. Optimal locations of 20 hinge-slots, indicated by '×' for problem P2, and its equilibrium shape.

4. Simple gridshell model

simple models with 5×5 grid are generated using epoxy material. Difference in shape for two models with all hinge joints and with hinge-slots at all free joints can be seen in Photos 1 and 2. If no hinge-slot is used, a shape with positive and negative curvatures can be generated as shown in Photo 1. However, if some hinge-slots are used, then each arch becomes close to a sinusoidal shape that can be at equilibrium without interaction between intersection members.



Photo 1: Models of gridshell generated using epoxy material with all hinge joints.



Photo 2: Models of gridshell generated using epoxy material with hinge+slots at all free joints.

5. Conclusions

An optimization method has been presented for design of a gridshell that is generated by forced displacements from the plane grid. The conclusions obtained from this study are as follows:

- 1. Maximum and total strain energy and the total strain energy in the beam members can be reduced by placing some hinge-slots at the joints near the supports where the forced displacements are applied.
- 2. Optimal locations of hinge-slots with fixed total number can be successfully found using a heuristic approach called simulated annealing.
- 3. Maximum and total strain energy and the total strain energy can be intuitively reduced by assigning hinge slots at joints that vane large y-directional force for the structure when all joints consist of hinges.

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