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Robust Design Optimization of Building Frames using Order Statistics and Local Search

Yusei Taniguchi[1], Makoto Ohsaki[2*], Makoto Yamakawa[3]

1Department of Architecture, Hiroshima University, 739-8527, Higashi-Hiroshima, Japan

2Department of Architecture, Hiroshima University, 739-8527, Higashi-Hiroshima, Japan

** Corresponding author: ohsaki@hiroshima-u.ac.jp*

3Department of Architecture, Tokyo Denki University, 120-8551, Tokyo, Japan

Abstract

A method combining pure random search (PRS) and local search (LS) is presented for robust design optimization of building frames. The robust design problem is formulated as a two-stage problem consisting of upper-level optimization problem and lower-level anti-optimization problem. The lower-level problem is solved using PRS, and its stopping rule is defined based on the order statics. The upper-level problem is solved using multistart LS, for which probabilistic stopping rules are investigated. The proposed approach is applied to a building frame subjected to a seismic motion. The objective function is the total structural volume, and a constraint is given for the worst value of maximum interstory drift angle between the roof and base. The results demonstrate the effectiveness of the proposed method.

Keywords: Uncertainty; Two-stage robust design problem; Order statistics; Pure random search; Local search

1. Introduction

In the design process of structures in various fields of engineering, uncertainty in the parameters such as material properties and geometry should be appropriately incorporated. Reliability-based design is the most popular approach, if the probability models of parameters can be appropriately assigned. However, the variations of parameters are usually unknown, and only their bounds may be estimated. Therefore, the design problem turns out to be a two-stage robust design problem, where the worst response is found in the lower problem, and the optimal design variables are found in the upper problem [1,2].

For a practical design problem of complex responses and constraints, it is difficult to find the globally worst response in the lower problem. Furthermore, the probability for taking the extreme value is very small. Therefore, it is desired to develop an approach that does not depend on the model of probability distribution, and approximates the extreme value with specified accuracy.

The authors developed a method for finding the approximate worst value using pure random search (PRS) [3], where the stopping rule is defined based on order statics [4].

In this study, we use our approach for the design of a building frame. The upper problem is a combinatorial problem, which is solved using a multistart local search (LS). The accuracy of the stopping rule for PRS for the lower problem is first verified. The number of local optimal solution in the upper problem is estimated based on the size of attractor that leads to each local optimal solution using a series of deterministic LSs.

2. Two-stage robust design problem

We first formulate a two-level optimization problem. The design variable vector is denoted by $\mathbf{x}$ with its feasible region Ω. The vector consisting of uncertain parameters of building structures is denoted by Θ. The objective function of the upper problem is denoted by $f(\mathbf{x})$. For simple presentation of the method, the uncertainties are incorporated into only one constraint function $g(\mathbf{x},\Theta)$, for which the upper bound $\bar{g}$ is given. Hence, the robust design optimization problem is formulated as

$$\left.\begin{array}{ll}\text{minimize} & f(\mathbf{x}) \\ \text{subject to} & g(\mathbf{x},\Theta) \le \bar{g} \\ & \text{for all } \Theta \in \Omega\end{array}\right\} \tag{1}$$

The worst value $g_{\max}(\mathbf{x})$ of the constraint function $g(\mathbf{x},\Theta)$ is obtained by solving the following lower-level problem:

$$\left.\begin{array}{ll}\text{find} & g_{\max}(\mathbf{x}) = \max\limits_{\Theta} g(\mathbf{x},\Theta) \\ \text{subject to} & \Theta \in \Omega\end{array}\right\} \tag{2}$$

By using (2), we can formulate the upper problem as a robust design problem or a worst-case design problem:

$$\left.\begin{array}{ll}\text{minimize} & f(\mathbf{x}) \\ \text{subject to} & g_{\max}(\mathbf{x}) \le \bar{g}\end{array}\right\} \tag{3}$$

[*]Corresponding author. Tel. and Fax: +81-82-424-7794,
E-mail address: ohsaki@hiroshima-u.ac.jp

3. Pure random search for upper-level problem

In this section, we summarize the stopping rule for PRS proposed in Ref. [3] based on order statistics [4]. For simplicity, we omit the design variable $\mathbf{x}$ as $g_{\max} = \max g(\Theta)$ because we focus on the lower problem.

A PRS is used for solving problem (2). We generate an independent sample $\{\Theta_1, \cdots, \Theta_n\}$ from a uniform probability distribution on Ω, and the corresponding independent sample $\{Y_1 = g(\Theta_1), \cdots, Y_n = g(\Theta_n)\}$ of the objective function values at these points. Suppose Y_j has a probability distribution F_Y.

The sequence $Y_1, \cdots, Y_n$ is arranged in increasing order, and the k th value is denoted by $Y_{k,n}$ such that $Y_{1,n} \le Y_{2,n} \le \cdots \le Y_{n,n}$. The properties of $Y_{1,n}, \cdots, Y_{n,n}$ are obtained by the order statistics.

Since the globally worst value is difficult to obtain, we use the following constraint for the lower problem:

$$Y_{k,n} \le \bar{g} \tag{4}$$

which indicates that the k th value of the total n samples from PRS satisfies the constraint of the upper problem.

Let β and γ $(0 \le \beta, \gamma \le 1)$ denote preassigned constants, and we choose n and k satisfying

$$1 - I_\gamma(k, n-k+1) \ge \beta \tag{5}$$

where I_γ is the incomplete beta function. If Eqs. (4) and (5) are satisfied, then the following equation holds:

$$\Pr\{F_Y(\bar{g}) \ge \gamma\} \ge \beta \tag{6}$$

i.e, we have at least 100 β % confidence that at least a proportion γ of the total n samples is less than $\bar{g}$.

These results indicate that the number of samples n and the k th value in the samples are closely related to the parameters β and γ, and the accuracy of the solution of PRS is ensured by appropriately assigning n and k.

If we assign the lower bund ε for $n-k$, which is given for avoiding obtaining the extreme value, the minimum value for the prescribed confidence is found by solving the following optimization problem:

$$\left.\begin{array}{ll} \text{minimize} & n \\ \text{subject to} & 1 - I_\gamma(k, n-k+1) \ge \beta \\ & 1 \le k \le n - \varepsilon \end{array}\right\} \tag{7}$$

4. Local search for lower-level problem

In this section, we summarize the stopping rules for LSs for solving the upper robust design optimization problem, which is a minimization problem.

The following deterministic algorithm of LS is used:

Algorithm (Local Search)

1. Sample an initial random point $\mathbf{x}_0$ from a uniform probability distribution. Set $k = 0$.
2. Enumerate all N neighborhood solutions of $\mathbf{x}_k$, denoted by $\mathbf{x}_k^i$ $(i = 1, \ldots, N)$, and compute $f(\mathbf{x}_k^i)$.
3. Select the best solution $\mathbf{x}_k^{\min}$, which has the smallest value of $f(\mathbf{x}_k^i)$.
4. If $f(\mathbf{x}_k) > f(\mathbf{x}_k^{\min})$, let $\mathbf{x}_{k+1} \leftarrow \mathbf{x}_k^{\min}$, $k \leftarrow k+1$, and go to 2; otherwise, output $\mathbf{x}_k$ as a local optimal solution and terminate the Algorithm.

Suppose we obtain w local optimal solutions $\mathbf{x}_1^*, \ldots, \mathbf{x}_w^*$ by carrying out LS t times from randomly generated initial solutions. The number of LSs that find $\mathbf{x}_i^*$ is denoted by n_i, i.e., $n_1 + \ldots + n_w = t$. Define X_i as *attractor* or *region of attraction* [5] of $\mathbf{x}_i^*$, which is the set of solutions that leads to $\mathbf{x}_i^*$ by carrying out LS.

The ratio of the size s_i of attractor X_i to number of all the feasible solutions is denoted by c_i. If s_i is defined as the number of initial solutions leading to $\mathbf{x}_i^*$, then $s_i = n_i$ and $s_1 + \ldots + s_w = t$ hold.

Suppose there exist h local optimal solutions, which are not known *a priori*. Then, $c_1 + \ldots + c_h = 1$ is satisfied. Boender and Kan [6] derived the following estimate w_{est} of the number of local optimal solutions based on Baysian approach, where c_i is supposed to be uniformly distributed between 0 and 1 satisfying $c_1 + \ldots + c_h = 1$, and w_{est} is obtained as the mean value of posterior estimate of h

$$w_{\text{est}} = \frac{w(t-1)}{t-w-2} \tag{8}$$

Based on Eq. (8), we can use the following stopping rule of multistart LS:

Rule 1:
Terminate multistart LS if $w_{\text{est}} - w \le 0.5$ is satisfied.

Let a denote the number of local optimal solutions that have not been found after carrying out LSs t times; i.e., $h = w + a$. Since we use a deterministic algorithm for LS, the following two cases are considered for estimating $s_1, \ldots, s_w$ and $c_1, \ldots, c_w$.

C-1: The attractor X_i consists of the initial solutions that reaches $\mathbf{x}_i^*$; i.e., $s_i = n_i$.

C-2: The attractor X_i consists of all feasible solutions along the path between the initial solution and $\mathbf{x}_i^*$.

Furthermore, the sizes $s_{w+1}, \ldots, s_h$ of attractors $X_{w+1}, \ldots, X_h$ of the solutions that have not been found are estimated by one of the following methods:

C-mean: $s_{w+1}, \ldots, s_h$ are equal to the mean value of $s_1, \ldots, s_w$.

C-min: $s_{w+1}, \ldots, s_h$ are equal to the minimum value of $s_1, \ldots, s_w$.

For example, C-1-mean denotes that the methods C-1 and C-mean are used.

Let $c_i^{(h)}$ denote the ratio of s_i of X_i when there exits *h* local optimal solutions, which are estimated by dividing s_i by the sum of $s_1, \ldots, s_h$. Then the likelihood $P_w^{(h)} = P(n_1, \ldots, n_w)$ for n_i times finding $\mathbf{x}_i^*$ $(i = 1, \ldots, w)$ in *t* trials is computed as

$$P_w^{(h)} = \prod_{i=1}^{w} C_{t,w} (c_i^{(h)})^{n_i}, \quad (n_1 + \ldots + n_w = t) \tag{9}$$

where $C_{t,w}$ is a coefficient that vanishes in the following equations. Based on Eq. (9), we can use the following stopping rule of multistart LS:

Rule 2:
Terminate multistart LS if $P_w^{(w+j)} / P_w^{(w)}$ is smaller than a specified small value, where *j* is a specified value.

Finally, suppose we find $c_i^{(h)}$ using the combination of (C-1 or C-2) and (C-mean or Cmin). Then, the probability of missing the (*w*+1)th solution in *t* trials is computed as

$$\overline{P}_w^{(h)} = (1 - c_{w+1}^{(h)})^t \tag{10}$$

Based on Eq. (10), we can use the following stopping rule of multistart LS:

Rule 3:
Terminate multistart LS if $\overline{P}_w^{(h)}$ is less than a prescribed small value.

For the case $h = w + 1$, the following relation holds:

$$c_i^{(w+1)} = \frac{c_i^{(w)}}{1 + c_{w+1}^{(w+1)}} \tag{11}$$

Therefore, it is easily seen that

$$\begin{aligned} \frac{P_w^{(w+1)}}{P_w^{(w)}} &= \prod_{i=1}^{w} \left(\frac{c_i^{(w+1)}}{c_i^{(w)}} \right) \\ &= \left(\frac{1}{1 + c_{w+1}^{(w+1)}} \right)^t \\ &= \left(1 - \frac{c_{w+1}^{(w+1)}}{1 + c_{w+1}^{(w+1)}} \right)^t \\ &= \overline{P}_w^{(w+1)} \end{aligned} \tag{12}$$

is satisfied as confirmed in the following examples. Therefore, we investigate only Rules 1 and 2 in the examples.

4. Example of mathematical problem for LS.

The number of local optimal solutions is estimated for a test function called Shekel-10 [5], which is defined as

$$f(\mathbf{x}) = -\sum_{i=1}^{10} \left(\frac{1}{(\mathbf{x} - \mathbf{A}_i)(\mathbf{x} - \mathbf{A}_i)^T + b_i} \right) \tag{13}$$

where b_i is the *i*th component of $\mathbf{b}$, $\mathbf{A}_i$ is the *i*th row of $\mathbf{A}$, and

$$\mathbf{A} = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 5 & 3 & 3 \\ 8 & 1 & 8 & 1 \\ 6 & 2 & 6 & 2 \\ 7 & 3.6 & 7 & 3.6 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0.1 \\ 2 \\ 2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \\ 0.5 \end{pmatrix}, \quad \mathbf{x} \in [0, 10]^4 \tag{12}$$

This function has 10 optimal solutions. To solve this problem as a combinatorial problem, each component of $\mathbf{x}$ is supposed to have discrete values with uniform interval of 0.1 between the lower bound 0 and the upper bound 10. The number of neighborhood solutions is $3^4 = 81$ including the current solution, because there are four variables and three patterns −1, 0, +1 for the increment of each variable.

Table 1 Ten solutions of problem Shekel-10 for LS-2.

Solution No.	1	2	3	4	5
x_1	4	5	6	3	7
x_2	4	5	6	7	3.6
x_3	4	3	6	3	7
x_4	4	3	6	7	3.6
$f(\mathbf{x}_i^*)$	-10.53	-3.83	-2.86	-2.81	-2.43
s_i (C-1)	51	42	39	38	20

Solution No.	6	7	8	9	10
x_1	6	7.9	2	1	8
x_2	2	7.9	9	1	1
x_3	6	7.9	2	1	8
x_4	2	7.9	9	1	1
$f(\mathbf{x}_i^*)$	-2.42	-0.68	-1.86	-0.63	-1.67
s_i^* (C-1)	16	12	6	5	4

Series of LSs denoted by LS-1, … LS-7 is carried out from randomly generated 7 initial solutions. Ten optimal solutions are found before satisfying Rule 1 in 6 cases. All 10 solutions and corresponding sizes of attractors for LS-2 are listed in Table 1. The number of trials m_{stop} for satisfying Rule 1, and the number of trials m_{10} for obtaining all 10 local optimal solutions are listed in Table 2

Table 2 Number of steps m_{stop} for satisfying Rule 1 and m_{10} when all 10 solutions are found for LS-1, …, LS-7.

LS No.	1	2	3	4	5	6	7
m_{stop}	192	233	233	233	233	233	233
m_{10}	212	81	67	78	182	51	46

Table 3 Ratios $P_{10}^{(11)}/P_{10}^{(10)}$ and $P_{10}^{(12)}/P_{10}^{(10)}$, and the value of $\bar{P}_{10}^{(10)}$ for problem Shekel-10 at m_{stop} trials of LS-2.

	$P_{10}^{(11)}/P_{10}^{(10)}$	$P_{10}^{(12)}/P_{10}^{(10)}$	$\bar{P}_{10}^{(10)}$
C-1-mean	2.267×10^{-10}	3.554×10^{-19}	2.267×10^{-10}
C-1-min	0.01895	0.0003837	0.01895
C-2-mean	2.267×10^{-10}	3.554×10^{-19}	2.267×10^{-10}
C-2-min	0.07262	0.005430	0.07262

Ratios $P_{10}^{(11)}/P_{10}^{(10)}$ and $P_{10}^{(12)}/P_{10}^{(10)}$, and the value of $\bar{P}_{10}^{(10)}$ evaluated at m_{stop} are listed in Table 3. We can see from the table that values of $P_{10}^{(11)}/P_{10}^{(10)}$ $(=\bar{P}_{10}^{(10)})$ are very small if C-mean is used; however, they are not sufficiently small if C-min is used. By contrast, ratios $P_{10}^{(11)}/P_{10}^{(10)}$ $(=\bar{P}_{10}^{(10)})$ evaluated at the step m_{10} using C-mean seems to be appropriate as shown in Table 4.

Table 4 Ratios $P_{10}^{(11)}/P_{10}^{(10)}$ and $P_{10}^{(12)}/P_{10}^{(10)}$, and the value of $\bar{P}_{10}^{(10)}$ for problem Shekel-10 at m_{10} trials of LS-2.

	$P_{10}^{(11)}/P_{10}^{(10)}$	$P_{10}^{(12)}/P_{10}^{(10)}$	$\bar{P}_{10}^{(10)}$
C-1-mean	4.438×10^{-4}	3.857×10^{-7}	4.438×10^{-4}
C-1-min	0.37014	0.13866	0.37014
C-2-mean	4.438×10^{-4}	3.857×10^{-7}	4.438×10^{-4}
C-2-min	0.56077	0.31576	0.56077

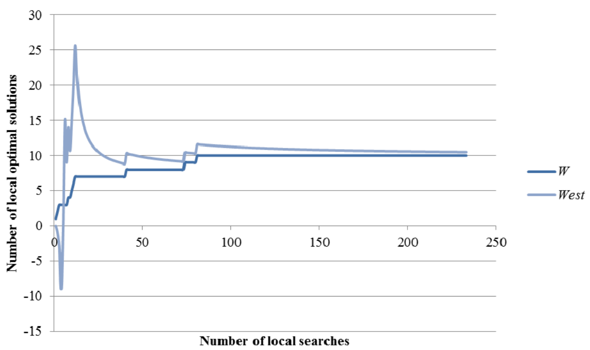

Fig.1 Histories of w_{est} and w for LS-2 of Shekel-10.

Histories of w and w_{est} are plotted for LS-2 in Fig. 1, which shows that w_{est} is a good upper bound for w; however, the convergence of w_{est} to w is very slow also for LS-3, 4, 6, and 7. Therefore, a less strict stopping rule is desired.

5. Example of robust design of a building frame

5.1 Description of model

Effectiveness of the proposed robust design method is investigated for a seismic design problem of a 4-story plane shear frame model as shown in Fig. 2. All columns have the same section C1. Beams are classified into 2 groups, which consist of G1 of 2nd and 3nd floors and G2 of 4th floor and roof. We choose each solution from 10 predefined section in Table 5.

We use frame analysis software OpenSees Ver. 2.4 [7] for time history response analysis. The material is steel with Young's modulus E = 2.05 GPa. Plastic hinges of length 0.2 m can exist at both ends of members, which is modeled as a fiber section with kinematic hardening ratio $0.01E$.

We use an artificial ground motions as shown in Fig. 2, which is compatible to the acceleration response spectrum in Table 5. The duration is 20 sec., and the time step 0.01 sec.

Table 5 Target acceleration response spectrum (damping factor = 0.05).

Period (s)	$T \le 0.16$	$0.16 \le T \le 0.864$	$0.864 \le T$
Acceleration (m/s^2)	$4.80 + 45T$	12.0	$10.37/T$

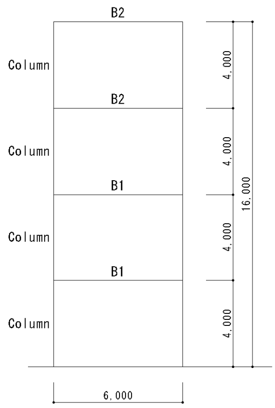

Fig. 2 A 4-story shear frame model.

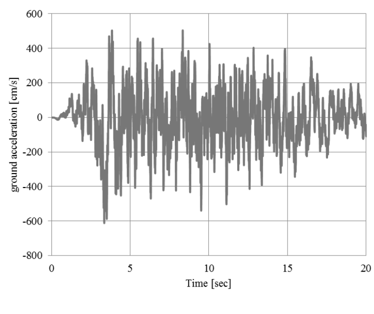

Fig. 3. Seismic motion.

Table 6 List of available sections.

list	Column	B1	B2
1	□-410×20	H-560×200×9×19	H-460×200×7×17
2	□-420×20	H-580×200×10×20	H-480×200×8×18
3	□-430×20	H-600×200×10×20	H-500×200×8×18
4	□-440×20	H-620×200×10×20	H-520×200×8×18
5	□-450×20	H-640×200×11×21	H-540×200×9×19
6	□-460×20	H-660×200×11×21	H-560×200×9×19
7	□-470×20	H-680×200×12×22	H-580×200×10×20
8	□-480×20	H-700×200×12×22	H-600×200×10×20
9	□-490×20	H-720×200×12×22	H-620×200×10×20
10	□-500×20	H-740×200×13×23	H-640×200×11×21

The design variable vector represents the cross-sections of beams and columns. For example, the *j*th section in Table 6 is chosen for the *i*th variable if $x_i = j$, where x_1, x_2, and x_3 correspond to Column, B1, and B2, respectively.

The objective function is the total structural volume $V(\mathbf{x})$ that is to be minimized. Constraints are given so that the worst value of the maximum interstory drift angle $\phi_{\max}(\mathbf{x},\mathbf{\Theta})$ between the roof and base does not exceed the upper bound 0.01.

Uncertainty is given for the yield stresses $\sigma_{\rm C}$ and $\sigma_{\rm G}$, respectively, of columns and beams as

$$\begin{aligned} &\sigma_{\rm C} = 325 + \Theta_1, \quad \sigma_{\rm G} = 235 + \Theta_2 \\ &\mathbf{\Theta} = (\Theta_1, \Theta_2) \\ &0 \le \Theta_i \le 100 \quad (i = 1,2) \end{aligned} \tag{14}$$

The problem of robust design optimization is defined as follows:

$$\left.\begin{aligned} &\text{minimize} && f(\mathbf{x}) = V(\mathbf{x}) \\ &\text{subject to} && g(\mathbf{x},\mathbf{\Theta}) = \phi_{\max}(\mathbf{x},\mathbf{\Theta}) \le 0.01 \\ & && x_i \in \{1, 2, \ldots, 10\}, \quad (i = 1,2,3) \end{aligned}\right\} \tag{15}$$

5.2 Verification of extreme value with specified accuracy

The uncertain parameters representing the yield stresses are assumed to distribute uniformly between the upper and lower bounds, because the formulations in Sec. 3 do not depend on the types of distribution.

By solving problem (7) for $\varepsilon = 3$ and $\gamma = \beta = 0.9$, we obtain $n = 65$ and $k = 63$, which means that the 62nd smallest value among objective values of 65 parameter sets generated by PRS is regarded as the approximate worst value of interstory drift angle.

Table 7 shows the results of four sets PRS-1, …, PRS-4 of PRSs of 65 trials for the design $(x_1, x_2, x_3) = (5,10,5)$.

The number of parameter sets satisfying $g(\mathbf{\Theta}) \le Y_{62,65}$ among randomly generated 1000 sets is also listed. It is confirmed that the numbers are not less than $1000\beta = 900$. The results of 50 sets of 1000 trials are listed in Table 8. Note that the number of parameter sets satisfying $g(\Theta) \le Y_{62,65}$ is less than 900 in 5 trials.

Table 7 Numbers and ratios of parameters sets satisfying $g(\Theta) \le Y_{62,65}$ among randomly generated 1000 sets, and the extreme values for four PRS-1, …, PRS-4.

PRS No.	1	2	3	4
$Y_{62,65}$	0.0103	0.0102	0.0102	0.0102
$Y_{65,65}$	0.0103	0.0103	0.0103	0.0104
Verification by 1000 samples.				
Number of parameter sets satisfying $g(\Theta) \le Y_{62,65}$	960	912	933	923
$Y_{1000,1000}$	0.0104	0.0104	0.0103	0.0104

Table 8 Number of parameter sets satisfying $g(\Theta) \le Y_{62,65}$ among 50 sets of 1000 trials.

960	980	953	957	842
912	973	939	946	915
933	915	881	924	957
923	965	975	973	936
955	950	918	887	888
948	933	952	889	967
929	944	971	903	969
973	919	961	944	946
925	934	936	972	957
956	960	946	981	939

5.3 Optimization results

We carry out two sets of muitistart LSs with *t* = 30 and 50, respectively. Eight solutions found by 30 trials are listed in Table 9. The values of $P_8^{(9)} / P_8^{(8)}$, $P_8^{(10)} / P_8^{(8)}$, and $\overline{P}_8^{(8)}$ are shown in Table 9.The objective values and the sizes of attractors are also listed. The results of *t* = 50 trials from different initial random seed from *t* = 30 are also listed in Tables 11 and 12, where 11 solution are found for this case. Note that the LS used here is not completely deterministic, because the lower problem involves uncertainly.

It is seen from Tables 9 and 11 that the solutions 6 and 7 in Table 9 do not exist in Table 11; therefore, there exist at least 13 local optimal solutions. It can be confirmed from Tables 9 and 11 that $P_w^{(w+1)} / P_w^{(w)}$ and $\overline{P}_w^{(w)}$ have the same value if C-mean is used.

Table 9 Eight solutions found by 30 trials.

	1	2	3	4	5	6	7	8
C1	10	10	6	7	4	10	8	8
G1	1	5	2	2	5	4	2	1
G2	4	2	5	4	1	3	4	5
$V(\mathbf{x})$	0.401	0.416	0.404	0.400	0.396	0.415	0.403	0.403
s_i^*	65	33	24	20	8	4	9	8

Table 10 Ratios $P_8^{(9)} / P_8^{(8)}$ and $P_8^{(10)} / P_8^{(8)}$, and the value of $\overline{P}_8^{(8)}$ for $t = 30$.

$P_8^{(9)} / P_8^{(8)}$	$P_8^{(10)} / P_8^{(8)}$	$\overline{P}_8^{(8)}$

C-1-mean	0.02920	0.001238	0.02920
C-1-min	0.37393	0.144257	0.37393
C-2-mean	0.02920	0.001238	0.02920
C-2-min	0.09997	0.253683	0.53240

Table 11 Eleven solutions found by 50 trials.

	1	2	3	4	5	6
C1	10	7	8	6	9	6
G1	1	2	1	2	6	3
G2	4	4	5	5	2	4
$V(\mathrm{x})$	0.401	0.400	0.403	0.404	0.421	0.405
s_i^*	106	41	25	13	15	10

	7	8	9	10	11
C1	10	9	7	4	4
G1	5	5	3	5	4
G2	2	3	3	1	2
$V(\mathbf{x})$	0.416	0.420	0.401	0.396	0.395
s_i^*	9	5	11	7	3

Table 12 Ratios of $P_{11}^{(12)}/P_{11}^{(11)}$ and $P_{11}^{(14)}/P_{11}^{(11)}$, and the value of $\bar{P}_{11}^{(11)}$ for $t=50$.

	$P_{11}^{(12)}/P_{11}^{(11)}$	$P_{11}^{(14)}/P_{11}^{(11)}$	$\bar{P}_{11}^{(11)}$
C-1-mean	0.01290	5.7974×10^{-6}	0.01290
C-1-min	0.39427	0.06466	0.37153
C-2-mean	0.01290	5.7974×10^{-6}	0.01290
C-2-min	0.54415	0.16467	0.54415

Table 13 Estimated and obtained numbers of local optimal solutions for $t=30$ and 50.

t	30	50
w_{est}	11.60	14.57
w	8	11

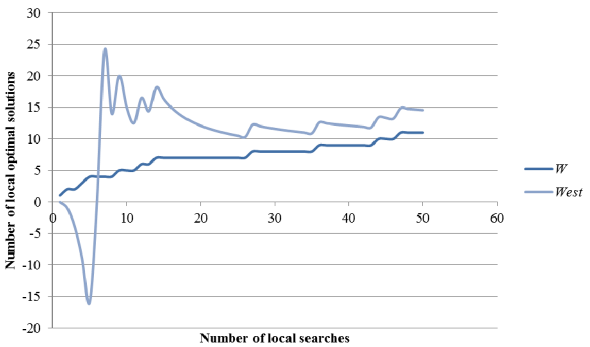

Fig.4 Histories of w_{est} and w for $t=50$.

We can see from these results that $\bar{P}_{11}^{(11)}$ for t = 50 is smaller than $\bar{P}_8^{(8)}$ for t = 30, if C-mean is used. The estimated and obtained numbers of local optimal solutions for t = 30 and 50 are listed in Table 13, which shows that w_{est} is larger than w by about 3.6 for both t = 30 and 50. Therefore, w_{est} exhibits slow convergence also in this example. For Rule 2, the value of $P_{11}^{(14)}/P_{11}^{(11)}$ for C-1-mean has sufficiently small value. Therefore, through further investigation for other problems, we may use Rule 2 for the stopping rule of LSs.

6. Conclusions

A robust design optimization problem has been formulated as a two-stage optimization problem. The worst response is found in the lower problem, and the locally optimal design variables is found in the upper problem.

In the lower problem, the approximate worst value is found using the pure random search and order statistics. The accuracy of the results have been confirmed in the example of a four-story shear frame subjected to seismic motions, where the maximum average drift angle is considered as the representative response.

The upper problem is solved using a multistart local search, where the variables are supposed to take discrete values. Three stopping rules and formula for estimating number of local solutions are compared in a mathematical problem. It has been confirmed that the first rule proposed in Ref. [6] is a little conservative. The second rule proposed in this study may also be used after further investigation for other problems.

It has been shown in the example of robust design of shear frame that there are multiple local optimal solutions; however, application of stopping rules to robust design is difficult, because computational cost for each local search is very large, and we cannot find many local optimal solutions.

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