Optimization of an arch-type truss under seismic excitations considering uncertainties of geometrical and material parameters

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ABSTRACT: A random sampling approach is presented to solve the two-stage problem of optimization and anti-optimization of an arch-type truss under seismic excitation. In the lower-level anti-optimization problem, uncertainties are considered in material parameters, which are assumed to exist in bounded intervals. Optimal cross-sections are selected in the upper-level optimization problem under constraints on worst responses. The accuracy of solution is defined based on the order of the objective value. Performances of random sampling and tabu search are compared in the numerical examples. It is shown that a good approximate optimal solution can be successfully found by random sampling.

1 INTRODUCTION

In the conventional optimization methods for structures in civil and architectural engineering, the parameters representing the structural and material properties are given deterministically. However, in the practical design process, uncertainty in those parameters should be appropriately taken into account (Elishakoff & Ohsaki 2010). Especially for building structures subjected to seismic motions, the use of nominal value with moderately large safety factor does not always lead to a conservative estimate of responses, because the dynamic response nonlinearly depends on the stiffness and strength; hence, a smaller strength may lead to a less response owing to a larger plastic energy dissipation.

Reliability-based and probabilistic approaches are widely used for incorporating uncertainties of parameters. However, it is not always possible to find appropriate probability distributions of parameters. Therefore, we utilize the concept of unknown-but-bounded (Elishakoff et al. 1994), and assume that the uncertain parameters exist in the specified bounded intervals. Constraints are assigned on the worst values of the structural responses. In this case, the optimization problem turns out to be a two-stage problem of optimization and anti-optimization.

Heuristic approaches have been developed for obtaining approximate optimal solutions of a highly nonlinear combinatorial problem within reasonable computational cost. They can be classified into population based approaches and those based on local search (Ohsaki 2010). Among various approaches based on local search, tabu search (TS) (Glover 1989) can avoid convergence to a local optimum, even if the number of neighborhood solutions to be searched is limited in a similar manner as the local random search (Ohsaki 2001).

Recently, random sampling (RS) approach, or randomized algorithm (Mitzenmacher & Upfal 2005, Lipton & Naughton 1995), has been studied extensively for knowledge discovery (Domingo et al. 1999), estimation of average and worst computational costs of an algorithm, and finding an approximate optimal solution of a combinatorial problem.

In this study, we apply an RS approach (Ohsaki & Katsura 2012) to find approximate worst-case designs under constraints on maximum strains against seismic motions considering uncertainty in material parameters.

2 OPTIMIZATION PROBLEM

Consider a problem of optimizing the cross-sections of framed structures such as building frames, latticed shells, and arches. The member sections are selected from the pre-assigned list of standard sections. The members are classified into m groups, each of which has the same section. The design variable vector is denoted by $\mathbf{J} = (J_1, \dots, J_m)$, which has integer values. For example, if $J_i = k$, then the members in the ith group have the kth section of the list. Let $F(\mathbf{J})$ denote the objective function representing, e.g., the total structural volume. The constraint functions defined by structural responses are denoted by $G_i(\mathbf{J})$ $(i=1,\ldots,n)$, where n is the number of constraints. Then, the optimization problem is formulated as

$$\begin{aligned} &\text{Minimize} && F(\mathbf{J}) \\ &\text{subject to} && G_i(\mathbf{J}) \le \bar{G}_i, \quad (i=1,\ldots,n) \\ &&& J_i \in \{1,\ldots,s\}, \quad (i=1,\ldots,m) \end{aligned}$$

where $\bar{G}_i$ is the upper bound for $G_i(\mathbf{J})$, and s is the number of sampling values of variables.

We incorporate uncertainty in material parameters such as yield stress, and find the worst parameter set in the lower-level problem. In order to solve the two-stage problem as a combinatorial problem, each parameter is sampled to q values using integer variables $\mathbf{I}=(I_1,\ldots,I_r)$; i.e., $I_i = k$ means that the kth sampling value is assigned to the ith parameter. Then, the structural response is given as a function of $\mathbf{J}$ and $\mathbf{I}$ as $\tilde{G}_i(\mathbf{J},\mathbf{I})$. We assign constraints on the worst values $\hat{G}_i(\mathbf{J})$ of responses, and formulate the optimization problem as

$$\begin{aligned} &\text{Minimize} && F(\mathbf{J}) \\ &\text{subject to} && \hat{G}_i(\mathbf{J}), \quad (i=1,\ldots,n) \\ &&& J_i \in \{1,\ldots,s\}, \quad (i=1,\ldots,m) \end{aligned}$$

The worst value $\hat{G}_i(\mathbf{I})$ is obtained by solving the following anti-optimization problem:

$$\begin{aligned} &\text{Find} && \hat{G}_i(\mathbf{J}) = \max_{\mathbf{I}} \tilde{G}_i(\mathbf{J},\mathbf{I}) \\ &\text{subject to} && I_i \in \{1,\ldots,q\}, \quad (i=1,\ldots,r) \end{aligned}$$

Hence, the optimal solution considering the worst values of responses can be found by solving a two stage problem of optimization and anti-optimization.

3 TABU SEARCH

Many heuristic approaches have been developed for obtaining approximate optimal solutions of a combinatorial problem within reasonable computational cost. Among various approaches based on local search, tabu search (TS) (Glover 1989) utilized the tabu list to avoid convergence to a local optimal solution and cyclic selection of a set of small number of solutions. The algorithm is summarized as follows:

Step 1 Randomly generate an initial seed solution, and initialize the tabu list to be empty.

Step 2 Generate neighborhood solutions from the seed solution. Select their best solution, which is not included in the tabu list, as the next seed solution, and add it to the tabu list.

Step 3 Terminate the process and output the best solution, if all neighborhood solutions are included in the tabu list, or the number of steps reaches the specified limit; otherwise, go to Step 2.

4 SEISMIC MOTION

Artificial seismic motions are generated using the standard superposition method of sinusoidal waves. The target acceleration spectrum is the design acceleration response spectrum for 5% damping specified for the ground of 2nd rank by Notification 1461 of the Ministry of Land, Infrastructure and Transport (MLIT), Japan, as shown in Figure 1, which is to be scaled by the factor 7.5. The seismic motions with duration 20 sec. are applied at the base in horizontal direction. The maximum response is evaluated as the mean-maximum responses among five artificial motions.

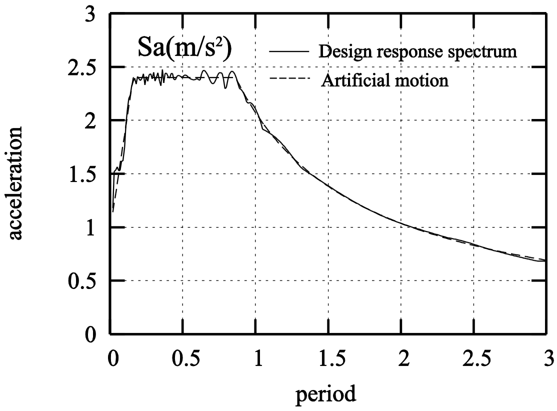

Figure 1. Design acceleration response spectrum and the acceleration response spectrum of an artificial seismic motion.

A general purpose frame analysis software called OpenSees Ver. 2.2.2 (PEERC 2011) is used for seismic response analysis. The standard Newmark-β method (β = 0.25, γ = 0.5) is used for integration in time domain with the increment of 0.01 sec. The stiffness-proportional damping is used with 2% for the lowest mode.

5 OPTIMIZATION RESULTS

5.1 *Arch-type truss model*

We consider a pin-jointed arch-type truss, called simply as arch, supported by two columns as shown in Figure 2, which represents one bay of a spatial structure. The span is 80 m, and the column height is 3.5 m. The lower nodes of the arch are located on a circle with radius of 80 m, and the half-open angle is 20 degrees. Both of the height of the roof truss and width of the column trusses are 1/40 of the span.

The members of the arch are steel pipes modeled as truss elements. The members are classified into nine groups as shown in Figure 3. The cross-sectional areas of the rigid members indicate in Figure 3 are fixed at a sufficiently large value. Therefore, the number of design variables is eight; i.e., m = 8. The cross-sectional areas of members in each group are selected from the list of standard sections as listed in Table 1. Each group has five candidate sections; i.e., $s = 5$.

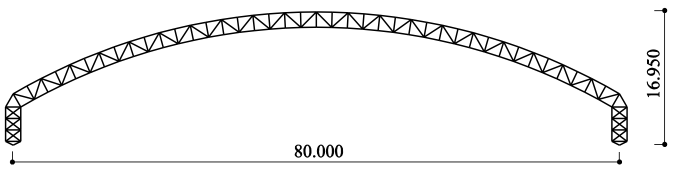

Figure 2. An arch-type truss model.

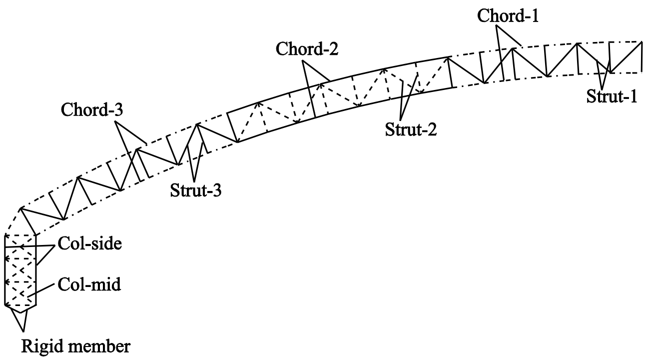

Figure 3. Member groups.

Table 1. List of cross-sectional areas of standard sections (m²).

	Chord-1 (J_1)		Chord-2 (J_2)
1	8.04×10^{-3}	1	7.02×10^{-3}
2	9.89×10^{-3}	2	8.63×10^{-3}
3	10.62×10^{-2}	3	9.37×10^{-3}
4	11.85×10^{-2}	4	10.33×10^{-2}
5	15.71×10^{-2}	5	12.01×10^{-2}
	Chord-3 (J_3)		Col-side (J_4)
1	10.09×10^{-2}	1	14.17×10^{-2}
2	12.41×10^{-2}	2	17.46×10^{-2}
3	13.49×10^{-2}	3	18.32×10^{-2}
4	14.88×10^{-2}	4	19.85×10^{-2}
5	19.76×10^{-2}	5	27.87×10^{-2}
	Strut-1 (J_5)		Strut-2 (J_6)
1	2.99×10^{-3}	1	1.93×10^{-3}
2	3.32×10^{-3}	2	2.03×10^{-3}
3	3.51×10^{-3}	3	2.12×10^{-3}
4	3.84×10^{-3}	4	2.23×10^{-3}
5	5.36×10^{-3}	5	2.52×10^{-3}
	Strut-3 (J_7)		Col-mid (J_8)
1	2.99×10^{-3}	1	2.99×10^{-3}
2	3.32×10^{-3}	2	3.32×10^{-3}
3	3.51×10^{-3}	3	3.51×10^{-3}
4	3.84×10^{-3}	4	3.84×10^{-3}
5	5.36×10^{-3}	5	5.36×10^{-3}

Table 2. Natural period (sec.) of the design $J_i = 3$ for all groups.

Mode	1	2	3	4	5
Natural Period	1.054	0.695	0.342	0.275	0.180

The nodal masses are 1800.0kg for eight nodes at the exterior sides of two columns, and 1600.0 kg for the lower nodes of arch. Note that the nodal mass is assumed to include the mass of steel members. The steel materials are idealized by a bilinear constitutive model with Young's modulus 2.05×10^8 kN/m² and hardening ratio 0.01. The nominal value of yield stress is 235 N/mm². The effect of geometrical nonlinearity is not considered.

The natural periods of the arch are shown in Table 2 for the design $J_i = 3$ for all groups; i.e., each group has the third section in Table 1. It is seen from Figure 1 and Table 2 that the response acceleration will reduce if the lowest natural period increases as a result of plastic deformation.

5.2 *Preliminary investigation for anti-optimization*

We first investigate the diversity of responses by carrying out preliminary analysis for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$. The nominal value is assigned for the yield stress of all members. In this case, members 1-5 indicated in Figure 4 undergo significant plastic deformation under the seismic excitations. Therefore, in the following, uncertainties are considered in the yield stresses of these members.

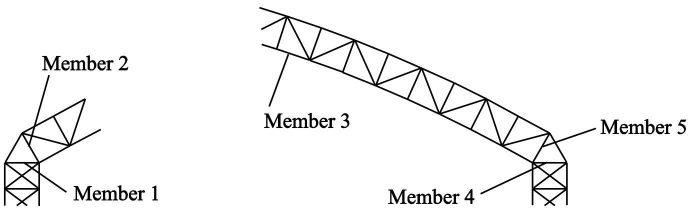

Figure 4. Members with significant plastification.

The maximum absolute value of strains among all members, which is simply called maximum strain, is taken as the representative response quantity. The yield stresses of members 1-5 are sampled to five values 235, 248, 261, 274, and 287 (N/mm²), because the nominal value indicates the lower bound. Therefore, the total number of combinations of the parameters is $5^5 = 3125$.

The maximum strains of all 3125 parameter sets are computed to find the discrete probability density as shown in Figure 5. The maximum and minimum values are 1.414×10^{-2} and 5.0321×10^{-3}, respectively.

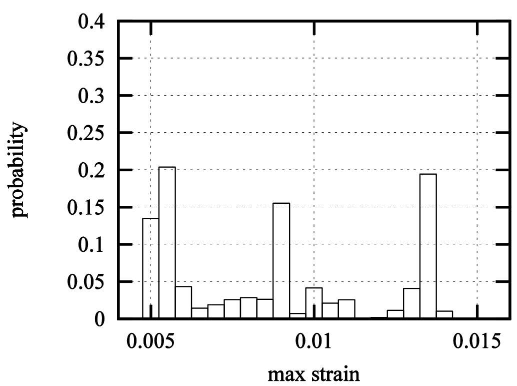

Figure 5. Discrete probability density of maximum strain by enumeration for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$.

In order to find approximate worst (maximum) value with small computational cost, 200 worst solutions among 3125 solutions are assumed to be approximate worst solutions. Suppose we carry out 100 random sampling with replacement. Then, the probability that no approximate solution is found is $(2925/3125)^{100} = 0.00134$, which is sufficiently small. Therefore, in the following, the number of analyses is limited to 100.

5.3 *Anti-optimization for finding worst values*

The performance of RS for anti-optimization is compared with that of TS, where the number of neighborhood solutions is 5 and the number of steps is 20; i.e., the total number of analyses is 100, which is the same as RS. Anti-optimal solutions are found from four different random seeds, which are denoted by Cases 1-4.

The maximum, minimum, and mean values as well as the standard deviation of the maximum strain by TS are listed in Table 3 for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$. The order of the maximum value is also listed in the last row. As is seen, good approximate worst values are found for all four cases. The mean values are close to the maximum values, which means that the solutions near the worst solutions are extensively searched.

The discrete probability density of the maximum strains of the solutions searched by TS is plotted for Case 1 in Figure 6. It is seen from Figures 4 and 6 that TS searches the solutions in the dense regions in the objective function space.

Table 3. Anti-optimization results of TS for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$.

Case	1	2	3	4
Max (10^{-2})	1.3892	1.3892	1.3892	1.3888
Min (10^{-3})	5.0686	5.2790	5.1118	5.2986
Mean (10^{-2})	1.1542	1.1490	1.1783	1.2146
Std. dev.(10^{-3})	2.6428	2.6991	2.6986	2.4496
Order	64	64	64	73

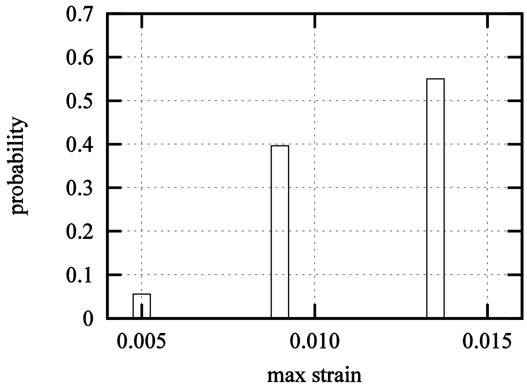

Figure 6. Discrete probability density of maximum strain by TS for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$.

The results of RS are listed in Table 4. The discrete probability density for Case 1 is plotted in Figure 7. As is seen, good approximate worst values are found for all four cases with 100 analyses, which is less than 1/30 of the total number of the solutions. The advantage of RS over TS is that no problem-dependent parameter exists for RS. It is also noted that RS searches the solutions widely in the objective function space.

Table 4. Anti-optimization results of RS for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$.

Case	1	2	3	4
Max (10^{-2})	1.3901	1.4003	1.3892	1.4078
Min (10^{-3})	5.1694	5.2081	5.2132	5.2576
Mean (10^{-2})	8.8714	8.5366	9.4025	8.4621
Std. dev.(10^{-3})	3.1837	3.3662	3.1827	2.9676
Order	48	31	64	9

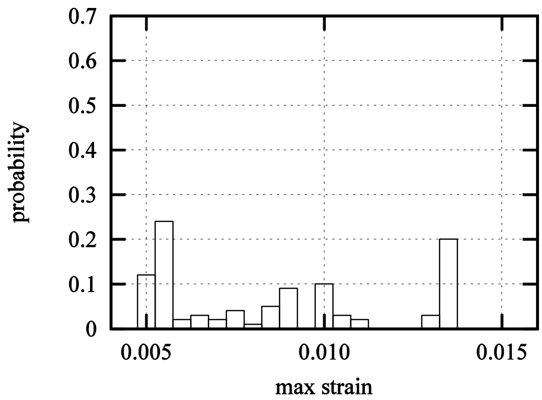

Figure 7. Discrete probability density of maximum strain by RS for the design $\mathbf{J} = (4,5,4,3,5,2,5,5)$.

5.4 *Performance of optimization and anti-optimization by RS*

The cross-sectional areas of eight groups are optimized considering uncertainty in yield stresses of five members. RS is used for both optimization and anti-optimization, where the number of sampling is 100; hence, the total number of analyses is 10000. The upper-bound strain is 1.016×10^{-2}, which is 8 times as large as the yield strain.

Table 5. Optimal cross-sectional areas obtained by RS.

Member group	Cross-sectional area (m^2)
Chord-1	9.89×10^{-3}
Chord-2	9.37×10^{-3}
Chord-3	13.49×10^{-3}
Col-side	14.17×10^{-3}
Strut-1	5.36×10^{-3}
Strut-2	2.03×10^{-3}
Strut-3	3.84×10^{-3}
Col-mid	5.36×10^{-3}

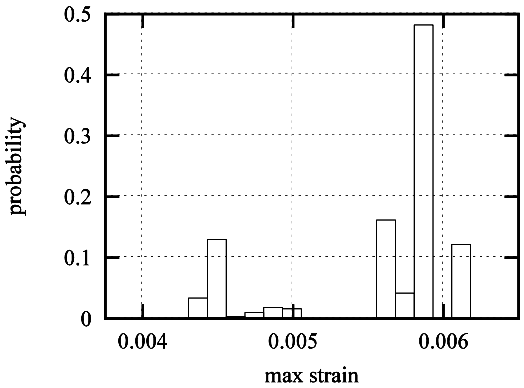

Figure 8. Discrete probability density of maximum strain by enumeration for the optimum design by RS.

The optimal cross-sectional areas are listed in Table 5. Enumeration is first carried out for all the parameter sets of the optimal solution to obtain the discrete probability density in Figure 8. The maximum strain is 6.181×10^{-3}, which is far less than the upper bound. In order to investigate the performance of RS in anti-optimization, RS is carried out for the optimal solution from four different random seeds to obtain the results in Table 6. Enumeration is also carried out to find the order in Table 6. The discrete probability distribution for Case 1 is plotted in Figure 9.

As seen from Table 6, good approximate solutions are found by RS, because the orders of the solutions are sufficiently small. Furthermore, the maximum values are close to 6.181×10^{-3} in the process of optimization, which indicates applicability of RS for anti-optimization as the lower-level problem of optimization.

Table 6. Anti-optimization results of RS for the optimum design obtained by TS.

Case	1	2	3	4
Max (10^{-2})	6.1813	6.1808	6.1813	6.1808
Min (10^{-3})	4.4766	4.4754	4.4754	4.4766
Mean (10^{-2})	5.5337	5.6282	5.5695	5.6546
Std. dev.(10^{-3})	6.5308	5.6508	6.3199	5.6015
Order	45	135	45	135

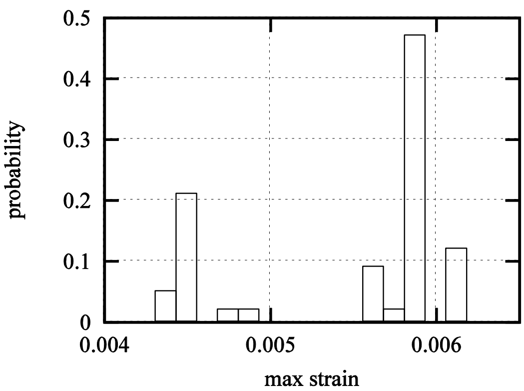

Figure 9. Discrete probability density of maximum strain by RS with different random seed for the optimum design by RS.

5.5 *Comparison of optimization and anti-optimization by RS and TS*

The optimal cross-sectional areas to minimize the total structural volume are found using TS and RS, respectively, for both of optimization and anti-optimization. The numbers of neighborhood solutions and steps in TS are 5 and 20, respectively, and the number of selections in RS is 100 for both of optimization and anti-optimization.

Table 7. Result of optimization and anti-optimization result by TS.

Member group	Cross-sectional area (m^2)
Chord-1	9.89×10^{-3}
Chord-2	9.37×10^{-3}
Chord-3	10.09×10^{-3}
Col-side	19.85×10^{-3}
Strut-1	3.51×10^{-3}
Strut-2	2.12×10^{-3}
Strut-3	3.84×10^{-3}
Col-mid	3.51×10^{-3}
Member number	**Yield stress (N/mm^2)**
1-5	287

The optimal cross-sectional areas and worst parameter set of the optimal solution are listed in Tables 7 and 8 for TS and RS, respectively. The total structural volume by TS is 7.281m^3, and the maximum strain is 7.074×10^{-3}. For RS, the total structural volume is 7.575 m^3, and the maximum strain is 6.181×10^{-3}.

The reduction of total structural volume (8.828 m^3) from the standard model is 7.5% by TS and 14.2% by RS. In this case, a better solution has been found by RS than TS.

Table 8. Result of optimization and anti-optimization result by RS.

Member group	Cross-sectional area (m^2)
Chord-1	9.89×10^{-3}
Chord-2	9.37×10^{-3}
Chord-3	13.49×10^{-3}
Col-side	14.17×10^{-3}
Strut-1	3.32×10^{-3}
Strut-2	2.23×10^{-3}
Strut-3	5.36×10^{-3}
Col-mid	5.36×10^{-3}
Member number	**Yield stress (N/mm^2)**
1, 2	261
3	274
4	235
5	287

6 CONCLUSIONS

Worst-case designs have been found for a pin-jointed arch subjected to seismic motions. The design problem is formulated as a two-level problem of optimization and anti-optimization. The optimal cross-sectional areas are found in the upper problem to minimize the total structural volume from the list of standard sections under constraint on the worst value of the maximum strains. The worst value is found by solving the lower anti-optimization problem considering uncertainty in the yield stresses of steel members.

The performances of TS and RS are have been first investigated for the anti-optimization problem in comparison to the enumeration results. It has been shown that good approximate solutions are consistently found by RS without any problem-dependent parameters. TS and RS has been next applied to the two-level problem of optimization and anti-optimization. The results show that RS can be successfully applied to a two-level problem with integer design variables and parameter sets.

It should also be noted that RS is very effective when many response quantities such as maximum displacements, accelerations, etc., should be considered.

ACKNOWLEDGMENTS

This work was partly supported by Grant-in-Aid of Scientific Research (B), No. 23360248, from MEXT, Japan.

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