Stability of Latticed Shell with Uniform-Length Hexagonal Grid

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Summary

Stability and constructability of a latticed shell with hexagonal grids are investigated. The grid is assembled with pairs of members connected by joints. The latticed shell with uniform lengths of members is first obtained using the optimization approach developed by the first and fourth authors. The lift-up process is also simulated using a large-deformation finite-element analysis. It is shown through the numerical simulation and a prototype physical model that surfaces with various shapes can be constructed using uniform-length hexagonal grids assembled with several types of connections.

Keywords: *latticed shell; hexagonal grid; stability; uniform length; lift-up process*

1. Introduction

There exist many papers on shape optimization of free-form shells modeled by parametric surfaces such as Bézier surfaces and NURBS (Non-Uniform Rational B-Spline) surfaces [1]. However, most of them are concerned with mechanical properties. In architectural design, we have to consider non-mechanical properties including aesthetic properties and constructability that are difficult to define in explicit forms. Especially for a free-form shell, it is very important to formulate the design problem considering mechanical and non-mechanical performances, because there exists a strong interaction between its shape and mechanical performances [2-4].

Shape optimization has also been studied for latticed shells defined by parametric surfaces [5,6]. For a latticed shell, locations of nodes and members are also to be optimized [7], and the constructability plays a key role as the non-mechanical performance. Ogawa *et al*. [8] maximized the linear buckling load under constraint on the variances of lengths of members that are classified into several groups. However, in their work, the surface shape is fixed; therefore, the latticed shell with uniform member lengths was not obtained.

It is very important in practical design and construction of latticed shell that the number of different parts including joints and members should be restricted to reduce the cost and period for construction. It is well known that the grids with uniform mesh can be generated for regular surfaces such as sphere and cylinder [9]. Recently, the latticed shells with hexagonal grids have been extensively studied [10-13]. Hexagonal grids can also be utilized for cable nets [14]. However, effect of joint connectivity on stability, stiffness, and constructability has not been fully explored.

In this study, we investigate mechanical properties of a new type of hexagonal-grid shell. The grid is realized as an assemblage of a unit consisting of a pair of members, which are connected by a joint and have the same length. The units are first assembled on a plane, and the curved surface is formed through a lift-up process. Some degrees-of-freedom (DOFs) are released on the plane to have an unstable mode of mechanism, and the DOFs are consecutively fixed through the lift-up process to ensure the stability when the final surface is formed. A finite-element analysis package called ABAQUS is used for simulation of the lift-up process. The stability of the structure at the final shape and the unstable modes during the lift-up process are investigated through the eigenvalue analysis of free vibration. A prototype model is constructed for a small-scale latticed shell to verify the lift-up process and stability of the final shape simulated using the numerical analysis. The latticed shell proposed in this study may be effectively used for various types of surfaces for temporary structures.

2. Stability of Hexagonal-Grid Latticed Shell

Consider a latticed shell with hexagonal grids as shown in Fig. 1(a). The size of latticed shell is designated by the number D of division in the meridian direction; i.e. $D=3$ for the shell in Fig. 1(a).

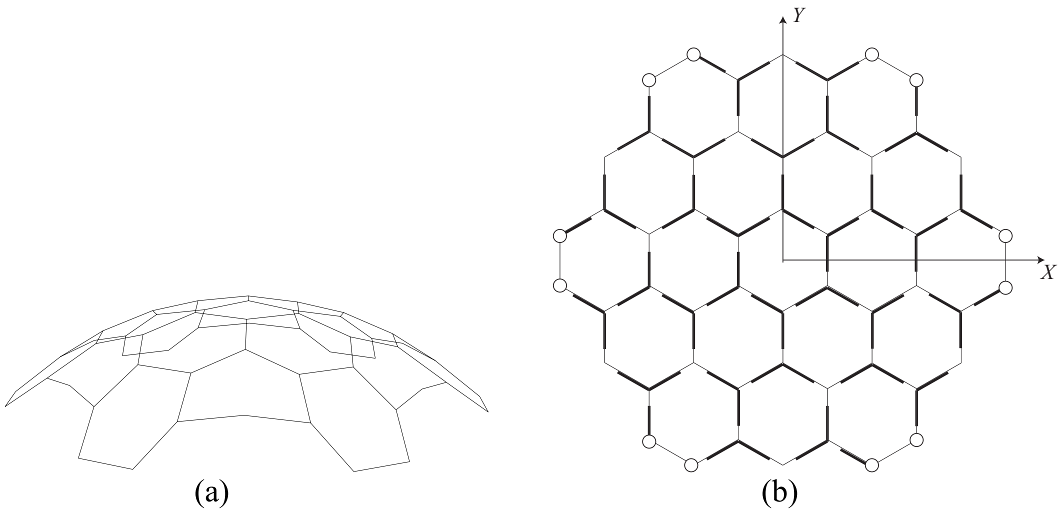


Fig.1: A hexagonal-grid latticed shell (D=3); (a) diagonal view, (b) assemblage of units

The local coordinates of a member are defined as shown in Fig. 2 for a member connecting nodes i and j $(i<j)$. The following notations are used for indicating moments at member ends:

M1: bending around axis 1
M2: bending around axis 2
T: torsion around axis 3

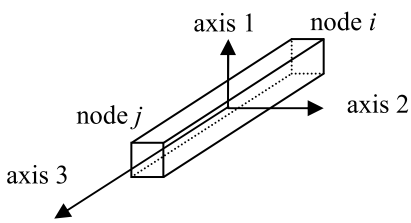


Fig 2: Definition of local coordinates

Latticed shells with hexagonal grids of uniform member lengths are assembled using the pair of members, called units, as shown in Fig. 3 in order to generate various shapes from simple units. The member-end numbers are defined in Fig. 3. Fig. 4 shows an example of release conditions at member ends, which means that M1 and T are released at member ends (1) and (4). This way, various release conditions can be assigned.

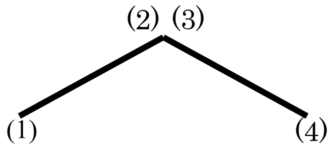


Fig. 3: A unit consisting of a pair of members, and its member-end numbers

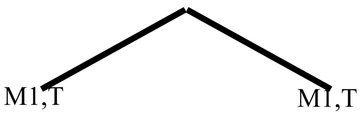


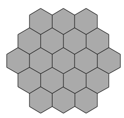
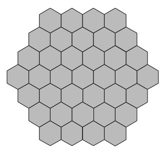
Fig. 4: Example of release conditions at member ends

Since a member that is not supported in space has six generalized force components (three forces and three moments) at each of two member ends and six equilibrium conditions (three translational and three rotational, the number of independent force components of each member is six. Therefore, the statical indeterminacy p of a rigidly-jointed frame with m members, n nodes, and k constraints at the supports is given as [15]

$$p = 6m - 6n + k \tag{1}$$

Let r denote the total number of released moment components at member ends. If the member ends with released moments are distribured appropriately in the structure so that there exist no local instability, the statical indeterminacies of hexagonal-grid latticed shells with $D = 3$ and 4 are given in Table 1.

Table 1: Statical indeterminacies of hexagonal-grid latticed shells

		
Division D in meridian direction	3	4
Hexagonal grids	19	37
Nodes	54	96
Constrained DOFs	k	k
Released member-end moments	r	r
Members	72	132
Statical indeterminacy	$108 + k - r$	$231 + k - r$

3. Stability of Hexagonal-Grid Latticed Shell

The effect of release of moments at member ends on stability of the latticed shell is investigated using a hexagonal grid with $D = 3$ as shown in Fig 1(a). The numbers of some nodes and members are indicated in Fig. 5. *X*- and *Y*-coordinates are located on the plane, and *Z*-coordinate is directed to the upper vertical direction. A general purpose finite-element analysis software called ABAQUS is used in the following. The command “RELEASE” is used for releasing the moments at member ends.

The nodes marked with circle in Fig. 1(b) are fixed in *Z*-direction. Among them, one node is further fixed in *Y*-direction, and another node is further fixed in *X*- and *Y*-directions. Hence, the total number of fixed DOFs is $k = 15$.

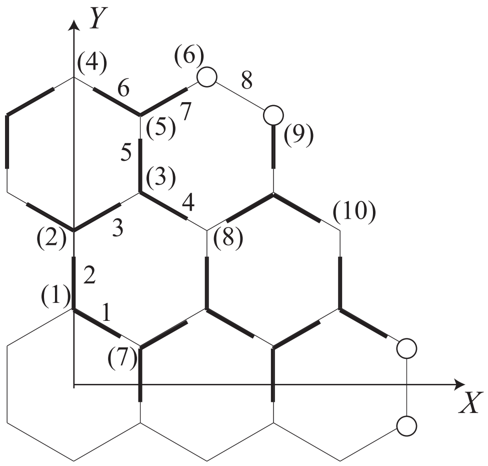


Fig. 5: Node numbers (with parentheses) and member numbers (without parentheses) of one of 12 equal parts of the 3-layer hexagonal latticed shell

The optimization approach presented in Ref. [4] has been used to generate the latticed shell in Fig. 1(a) that has uniform member lengths. The coordinates for one of 12 equal parts is listed in Table 2. The coordinates of other nodes can be found based on the symmetry conditions. Fig. 1(b) shows the developed configuration on a plane, where each unit is indicated by a pair of thick lines connected at a node. As is seen, the hexagonal grids can be constructed as an assembly of pairs of members. The member-end moments are released in various patterns at each node. Note that members are rigidly connected at free node 10 along the boundary.

Table 2: Nodal coordinates (m)

	X	*Y*	*Z*
1	0.000	6.077	10.103
2	0.000	11.917	8.420
3	5.062	14.846	6.766
4	0.000	19.846	4.161
5	6.042	19.883	3.510
6	9.825	23.094	0.000

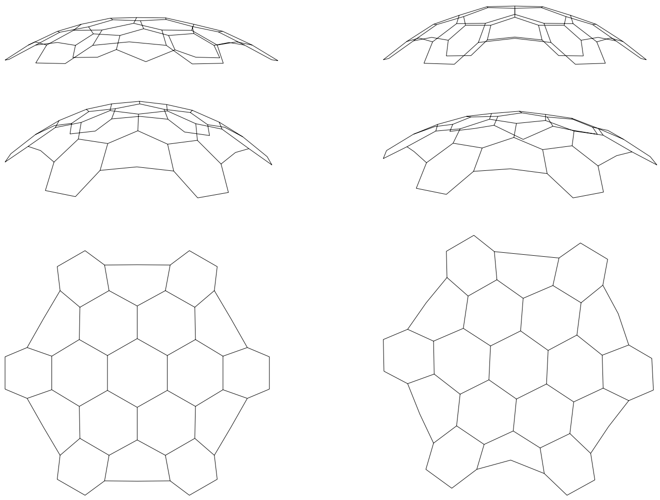
It is seen from Table 1 that the statical indeterminacy of the rigidly jointed latticed shell with $D = 3$ is 133, if no moment is released, i.e. $r = 0$. Because the number of free nodes is $54 - 12 = 42$, stability will not be lost when two moments are released appropriately at each free node.

Stability of a structure can be evaluated from the eigenvalues of the tangent stiffness matrix. However, one of the purpose of this study is to present a method of construction analysis using a general purpose finite-element analysis program, and such program does not have capability of eigenvalue analysis of tangent stiffness matrix. Therefore, we use eigenvalue analysis of vibration, which is readily available in any commercial codes, for evaluation of stability.

Since the latticed shell may have local instability due to rotations of nodes and/or members when too many moments are released at member ends, auxiliary rotational masses are given using the element called “ROTARY INERTIA” of ABAQUS to detect instability due to release of moments. The structure is stable if all eigenvalues of vibration are positive; otherwise, the structure is unstable with zero eigenvalues. Among several approaches of eigenvalue analysis in ABAQUS, only subspace approach with an appropriately small number and bound of eigenvalues was successfully applied to detect all zero eigenvalues of an unstable structure. Static analysis with uniform vertical loads is also carried out for a stable structure for verification purpose.

Table 3: Stability of latticed shells with various release patterns

Case	Release condition		Stability	Statical indeterminacy	Kinematical indeterminacy
1	Release M1 at two ends and center	M1, M1 M1 M1	Stable	0	21
2	Release M1 and T at two ends.	M1,T M1,T	Stable	0	9
3	Release M2 and T at two ends.	M2,T M2,T	Stable	0	9
4	Release T at two ends and M1 at center.	M1, M1 T T	Unstable	1	16
5	Release T at two ends and M2 at center.	M2 M2 T T	Stable	0	15



(a) (b)

Fig. 6: Deformation for Case 2 under uniform vertical loads; (a) undeformed, (b) deformed

Table 3 shows the stability of the latticed shell for various release conditions. We found four release conditions that lead to stable structures. As is seen, Cases 1-3 and 5 are stable. Kinematical and statical indeterminacies are also computed by investigating the size and rank of the equilibrium matrix. As is seen, Case 4 has one unstable mode of deformation.

Among the stable cases, construction process of Case 2 is investigated in the following section. Fig. 6 shows the undeformed and deformed configuration of Case 2 under uniform vertical nodal loads. As is seen, the deformation is slightly asymmetric, and there exists a torsional deformation along the center axis.

4. Lift-up Analysis

We simulate the lift-up process to demonstrate that various shapes can be generated using the hexagonal grids with uniform member lengths. All nodes are first placed on the *XY*-plane, and member-end moments are appropriately released. The final shape is generated after four steps of lift-up, where the nodes are consecutively moved upward from the center.

We first lift-up nodes around the center indicated by circles in Fig. 7(a) to the specified height 1.68274 m, which is the difference between *Z*-coordinates of nodes 1 and 2 in Table 2. This process can be traced using the generalized inverse of equilibrium matrix [15]. However, we show that this can also be done by the forced-displacement analysis of ABAQUS without development of any special purpose program.

It is confirmed that no stress is generated in members during the lift-up process. Some member ends are fixed at the end of this 1st lift-up process illustrated in Fig. 7(a). The final configuration after the 4th step is shown in Fig 7(b), where the nodes with circles are lifted by specified amount. The shape in Fig. 7(b) coincides with the target shape in Fig. 6(a), because the nodes are lifted by the same amount at each step. However, we can generate various shapes by non-uniformly lifting the nodes at each step.

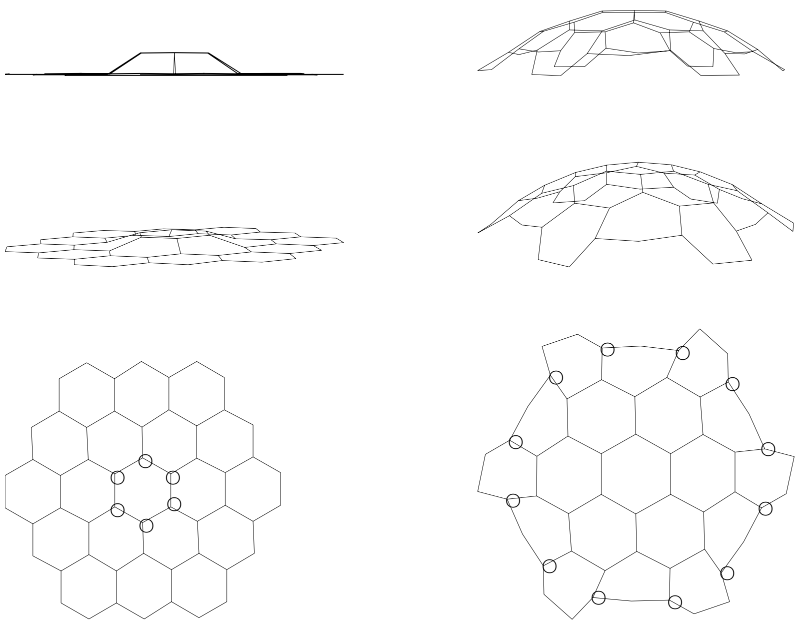
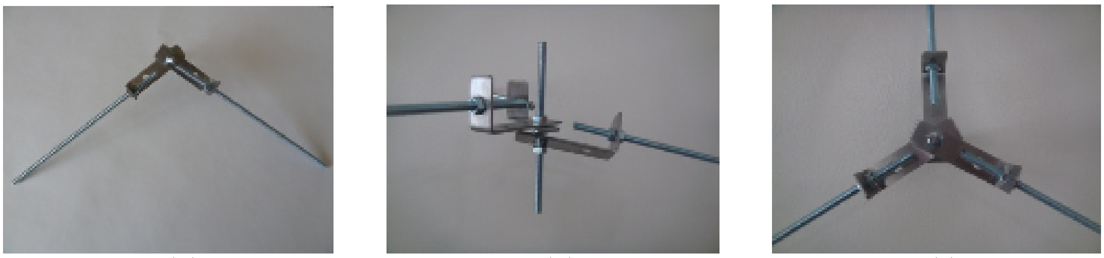


Fig 7: Configuration during lift-up process; (a) after 1st step, (b) after 4th step

In order to verify the stability and constructability of the latticed shell, a prototype-model has been made using steel bars and bolts. The unit is as shown in Fig. 8(a), of which the two bars with solid circular section are connected at the center as shown in Fig. 8(b). Fig. 8(c) shows the connection between two units. Rubber bands are used to enhance stiffness of bars and joints. Fig. 9 shows the configuration on a plane. It can be seen from Fig. 10(a) and (b) that various shapes can be easily generated using this system. Type 1 is almost axisymmetric, and Type 2 has an asymmetric shape.



(a) (b) (c)

Fig. 8: Connections of members; (a) a unit consisting of two bars, (b) center of unit, (c) connection between units

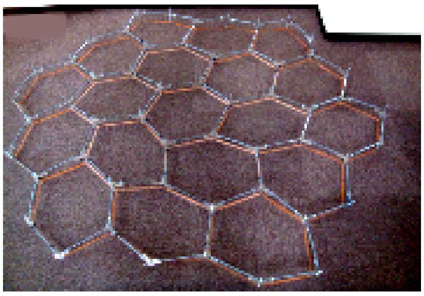
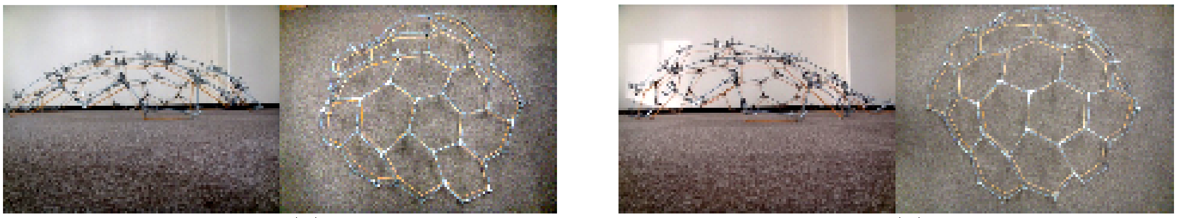


Fig. 9: Configuration on plane



(a) (b)

Fig. 10: Various configurations; (a) Type 1, (b) Type 2

5. Conclusions

Stability and construction process have been investigated for latticed shells with hexagonal grids, which is generated as an assemblage of a unit consisting of a pair of members with the same length. It has been shown that various types of connections are possible to construct stable hexagonal-grid shell with uniform member lengths.

The lift-up process has been simulated using a general purpose finite element analysis software package called ABAQUS. The stability can be verified using eigenvalue analysis of free vibration, and the unstable lift-up process can be simulated using a forced-displacement analysis.

It has been found that various shapes can be generated using hexagonal grids with the same topology and uniform member lengths. Therefore, hexagonal grids can be effectively utilized for generating free-form surfaces with latticed shells.

REFERENCES

[1] E. Ramm, and G. Mehlhorn, On shape finding methods and ultimate load analysis of reinforced concrete shells, Eng. Struct., Vol.13, pp.178.198, 1991.

[2] M. Ohsaki, T. Ogawa and R. Tateishi, Shape optimization of curves and surfaces considering fairness metrics and elastic stiffness, Struct Multidisc Optim, No.27, 250-258, 2004.

[3] S. Fujita and M. Ohsaki, Shape optimization of free-form shells using invariants of parametric surface, Int. J. Space Struct., Vol. 25, pp. 143-157, 2010.

[4] S. Fujita and M. Ohsaki, Shape optimization of shells considering strain energy and algebraic invariants of parametric surface, J. Struct. Constr. Eng., AIJ, Vol. 74, No. 639, pp. 841-847, 2009. (in Japanese)

[5] M. Ohsaki and M. Hayashi, Fairness metrics for shape optimization of ribbed shells, J. Int. Assoc. Shells and Spatial Struct., Vol. 41(1), pp. 31-39, 2000.

[6] H. Ohmori and H. Hamada, Computational morphogenesis of shells with free curved surface considering both designer's preference and structural rationality, Proc. Int. Assoc. Shell and Spatial Struct. (IASS-APCS 2006), Beijing, pp. 512-513, 2006.

[7] M. Ohsaki, T. Nakamura and M. Kohiyama, Shape optimization of a double-layer space truss described by a parametric surface, Int. J. Space Struct., Vol. 12(2), 1997.

[8] T. Ogawa, M. Ohsaki, R. Tateishi, Shape optimization of single-layer latticed shells for maximum linear buckling loads and uniform member lengths, J. Struct. Constr. Eng., AIJ, No. 570, pp. 129-136, 2003. (in Japanese)

[9] M. Ohsaki and S. Fujita, Multiobjective shape optimization of latticed shells for elastic stiffness and uniform member lengths, Proc. Int. Symp. on Algorithmic Design for Architecture and Urban Design (ALGODE2011), Tokyo, 2011.

[10] T. Wester, A geodesic dome-type based on pure plate action, Int. J. Space Struct., Vol. 5(2/3), pp. 155-167, 1990.

[11] J. P. Rizzuto, Rotated mutually supported elements in truncated icosahedric domes, J. Int. Assoc. Shell and Spatial Struct., Vol. 48(1), pp. 3-17, 2007.

[12] M. Turrin, P. von Buelow, A. Kilian and S. Sariyildiz, Performance-based design of SolSt: a roof system integrating structural morphology and solar energy transmittance, Proc. Int. Assoc. Shell and Spatial Struct. (IASS-2010), Shanghai, pp. 2991-3003, 2010.

[13] M. Araya, Y. Hotta and T. Nagai, Design of honeycomb and sandwiched panel structure with the use of aluminum ally, AIJ J. Technol. Des., Nol. 19, pp. 135-140, 2004. (in Japanese)

[14] D. Priour, Analysis of nets with hexagonal mesh using triangular elements, Int. J. Numer. Meth. Eng., Vol. 56, pp. 1721-1733, 2003.

[15] T. H. G. Megson, Structural and Stress Analysis, Elsevier, 1996.

[16] Y. Hangai and K. Kawaguchi, Analysis for shape-finding process of unstable link structures, Bulletin of IASS, Vol. 30, No. 2, pp. 116-128, 1989.