9th World Congress on Structural and Multidisciplinary Optimization
June 13 - 17, 2011, Shizuoka, Japan

PROBABILITY-BASED OPTIMAL DESIGN OF MOMENT-RESISTING FRAMES USING KRIGING APPROXIMATION MODEL

Jingyao Zhang[1], Makoto Ohsaki[2]

[1] Ritsumeikan University, Kusatu, Japan, zhang@fc.ritsumei.ac.jp
[2] Hiroshima University, Higashi-Hiroshima, Japan, ohsaki@hiroshima-u.ac.jp

1. Abstract
In this study, we present an optimization approach for probability-based design of moment-resisting steel frames, where uncertainties in material properties are taken into consideration. The structural reliability is defined by probability of satisfying specific structural performance evaluated by Monte Carlo simulation (MCS). Because MCS needs a large number of structural responses corresponding to possible material properties, we adopt Kriging method to predict these responses using only a limited number of structural analyses. Simulated annealing , which is a heuristic method for combinatorial optimization problems, is adopted to search for the optimal combination of sections from a specified list of available sections, so as to have the minimum weight for the structure under constraint on the structural reliability.
2. Keywords: Reliability-based design; Steel frame; Uncertainty; Kriging method; Simulated annealing .

3. Introduction

A structure that is constructed in the field of building engineering is essentially different from that is designed, because of the existence of uncertainty in geometrical properties and material properties etc. To take account of these uncertainties, an empirical coefficient called *safety factor* is introduced in conventional (deterministic) design procedure: nominal values of the structural parameters are reduced to 'dependable' values by dividing by safety factors. The basic idea behind this process is intended to design a 'conservative' structure through prediction of the worst values of responses, such as maximum displacements and interstory drift angles, by the combination of the worst cases of inputs—the dependable values of the structural stiffness/strength parameters less than those of the real structures, and the external loads greater than that might possibly occur.

However, conservative design by using safety factor can only be guaranteed in static cases, and may mislead designers in dynamic cases. For example, the seismic responses of a structure significantly depend on characteristics of response spectra of the input motions rather than stiffness of the structure; moreover, less strength in some structural members may lead to less responses of the structure owing to more plastic energy dissipation. Thus, conventional design procedure may not lead to conservative design as expected, and could end up with overestimating capacity of the structures, which indicates that more sophisticated approaches to considering uncertainty are necessary.

To evaluate dynamic characteristics of a building structure subjected to severe earthquakes, time history analysis (THA) is regarded as the most reliable approach, though at the same time, much higher computation cost is necessary in comparison to evaluation using static pushover analysis. Moreover, in the framework of performance-based engineering, dynamic performance of a structure subjected to possible uncertainty involved in structural analysis should be evaluated in the framework of probability theory.

For such purpose, Monte Carlo simulation (MCS) is the most straightforward way; however, it is unlikely to be directly applicable to complex systems. This is because that expensive THA for a large number of possible values of the structural parameters has to be conducted, which results in unacceptably expensive computational cost. Instead of a large number of THA by the direct approach, MCS can also be conducted by approximation approaches, which are to estimate the structural responses by using a limited number of structural analyses. This would of course lead to less accuracy; hence, so that a trade-off between accuracy and computational cost has to be made.

Metamodels are such kind of approximate approaches, interpolating the results (dynamic responses) obtained in preliminary experiments (structural analyses) with smooth nonlinear functions. The responses for the parameter values, for which experiments have not been carried out, are predicted from the nonlinear functions.

There have been a number of metamodels developed so far, such as response surface approximation, radial basis function, artificial neural networks, Kriging method and multivariate adaptive regression splines. Among these, Kriging method has gained much attention in engineering literatures because of its high accuracy and low computational cost [1].

As the structural reliability is available in terms of probability, we consider the problem of finding the moment-resisting steel frames with minimum weight, which is assembled by the available sections. For this typical combinatorial optimization problem, the simulated annealing (SA) is adopted in this study.
Following this introduction, Section 4 gives a brief introduction to Kriging model for response prediction, and a brief summary of SA; Section 5 considers the probability-based optimal design of a two-dimensional moment-resisting frame to demonstrate the availability of the proposed approach; and Section 6 concludes the study.

4. Approximate responses and optimal structure

This section gives a brief description of Kriging method for predicting structural responses from a limited number of numerical experiments, and simulated annealing for finding the optimal combination of available sections to have the least-weight structure while satisfying constraints on structural reliability.

4.1. Kriging method

The nonlinear surrogate function in Kriging method is constructed by minimizing the mean error of the weighted sum of responses at the sampling points, at which experiments are conducted. Kriging method was initially developed for statistical evaluation of mining data, and gained further and much wider applications in other engineering fields from the end of 1980s. In this subsection, we briefly summarize the basic equations of Kriging method as described in [2] for the completeness of the study.
Suppose that we consider the uncertainty in n^d structural parameters, and carry out preliminary analyses at the n^s sampling points $\mathbf{s}_i \in \mathbb{R}^{n^d}$ $(i = 1,...,n^s)$, at which the 'true' responses are denoted by $\mathbf{y} \in \mathbb{R}^{n^s}$. The prediction points, at which responses are to be predicted, are denoted by $\mathbf{x} \in \mathbb{R}^{n^d}$.
Let $\mathbf{R} \in \mathbb{R}^{n^s \times n^s}$ denote the correlation matrix, describing correlations of the responses at the sampling points, and $\mathbf{r}(\mathbf{x}) \in \mathbb{R}^{n^s}$ the correlation vector for sampling points and prediction point: the *i*th entry of $\mathbf{r}(\mathbf{x})$ is the correlation between the prediction point $\mathbf{x}$ and the sampling point $\mathbf{s}_i$, and the (*i*, *j*)-entry of $\mathbf{R} \in \mathbb{R}^{n^s \times n^s}$ is the correlation between the two sampling points $\mathbf{s}_i$ and $\mathbf{s}_j$.
The normalized value $\hat{y}_{\text{nor}}$ of the predicted (approximate) response $\hat{y}$ at the prediction point $\mathbf{x}$ is determined as follows by minimizing the mean square error and using the best linear unbiased predictor:

$$\hat{y}_{\text{nor}}(\mathbf{x}) = \hat{\beta} + \mathbf{r}(\mathbf{x})^{\mathrm{T}} \mathbf{R}^{-1} (\tilde{\mathbf{y}} - \hat{\beta}\mathbf{i}) \quad \text{with} \quad \hat{\beta} = \frac{\mathbf{i}^{\mathrm{T}} \mathbf{R}^{-1} \tilde{\mathbf{y}}}{\mathbf{i}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{i}}, \tag{1}$$

where every entry in $\mathbf{i} \in \mathbb{R}^{n^s}$ is equal to one, and $\tilde{\mathbf{y}}$ is the normalized version of $\mathbf{y}$. Thus, the predicted response $\hat{y}$ is computed by

$$\hat{y} = \sigma_y \hat{y}_{\text{nor}} + \bar{y}, \tag{2}$$

where σ_y and $\bar{y}$ are the standard deviation and mean of the responses at the sampling points, respectively.
The correlation is usually defined as a function of correlation parameters and distances between the relevant points: the (*i*, *j*)-entry $R(\boldsymbol{\theta}, \mathbf{s}_i, \mathbf{s}_j)$ of $\mathbf{R}$ and the *i*th entry $r(\boldsymbol{\theta}, \mathbf{x}, \mathbf{s}_i)$ of $\mathbf{r}$ is written as

$$R(\boldsymbol{\theta}, \mathbf{s}_i, \mathbf{s}_j) = \prod_{k=1}^{n^d} R(\theta_k, d_{ij}^k), \quad r(\boldsymbol{\theta}, \mathbf{x}, \mathbf{s}_i) = \prod_{k=1}^{n^d} r(\theta_k, d_i^k), \tag{3}$$

where θ_k, d_{ij}^k and d_i^k are respectively the *k*th entries of correlation parameter vector $\boldsymbol{\theta}$, distances $\mathbf{d}_{ij}$ between sampling points and distances $\mathbf{d}_i$ between sampling points and prediction point. The correlation parameters $\boldsymbol{\theta}$ are unknown and are determined by minimizing prediction errors at specific (verification) points in this study.
More sampling points are needed, if the responses cannot be predicted from the existing set of sampling points with high enough accuracy. There are several approaches for adding new sampling points as summarized in [2], and we adopt the approach that adds the point having the maximum mean square error of prediction to the set of sampling points.

4.2. Optimization method

Using the approximate responses predicted by Kriging method, the structural reliability in terms of probability can be easily computed by carrying out MCS with assumptions on probability densities of uncertain parameters in the structural analysis. The structural reliability in terms of dynamic performance can be defined in various manners. In a numerical example in Section 5, the performance of a moment-resisting frame is defined by the probability of maximum interstory drift angle to exceed the specified value.

Satisfying certain dynamic performances as constraints, our next step is to find the optimal structure with the minimum weight. To consider the design problem in practice, the members are selected from a given list of available sections.

The members are classified into m groups, where members in each group have the same section. The design variable vector is denoted by $\mathbf{J} = (J_1, \ldots, J_m)$, which has integer values. For example, if $J_i = k$, the section of the i th group has the k th section of the list.

Let $F(\mathbf{J})$ denote the total volume of steel materials, equivalently structural weight, of the moment-resisting frame. The maximum value of the maximum interstory drift angle among all stories is denoted by $G(\mathbf{J})$ with the specified upper bound $\overline{G}$. The lower bound $\overline{P}$ is given for the probability $\Pr[G(\mathbf{J}) \geq \overline{G}]$ of $G(\mathbf{J})$ to exceed $\overline{G}$. Then the optimization problem is formulated as

$$\begin{aligned} &\text{Minimize} && F(\mathbf{J}) \\ &\text{subject to} && \Pr[G(\mathbf{J}) \geq \overline{G}] \leq \overline{P} \\ & && J_i \in \{1, \ldots, s_i\}, \quad (i = 1, \ldots, m) \end{aligned} \tag{1}$$

where s_i is the number of admissible values for J_i. This is a typical combinatorial optimization problem, and we adopt simulated annealing as summarized in the next subsection.

4.3. Simulated annealing method

As its name implies, simulated annealing (SA) exploits an analogy between the metal annealing process and the process of searching for the best solution in an optimization problem [3]. Gradients of the objective or constraint functions are not necessary, and the major advantage of SA over other heuristic approaches is the ability to find the global optimum.

There are in total five processes involved in SA: (a) initial solution, (b) local search, (c) transition of solutions, (d) cooling schedule, and (e) termination condition. The typical flowchart for these processes is shown in Figure 1.

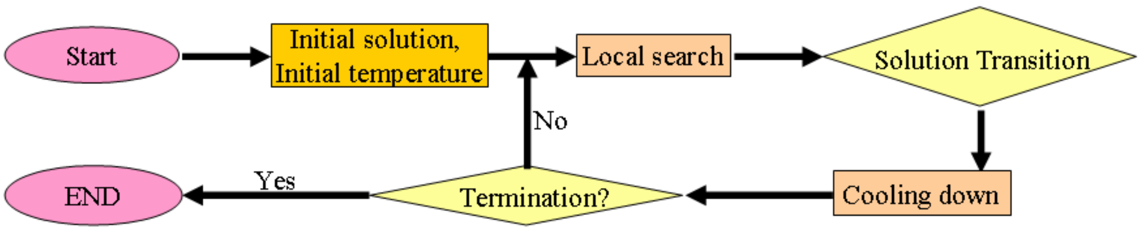


Fig. 1: The flowchart of classical simulated annealing method.

Among these processes, solution transition is the key for jumping out from a local optimum, since it ensures that acceptance of non-improving solution is also possible. To be specific, solution transition will occur if a randomly generated number $\overline{P} \in (0,1)$ is less than the probability P of transition calculated by

$$P = \min\{1, e^{\Delta f_i / t_i}\}, \tag{4}$$

where Δf_i is the increase of the objective value for a minimization problem and t_i is the temperature at the current iteration i. It is obvious from Eq. (4) that, transition to an improving solution is always accepted, and transition to a non-improving solution is possible but becomes more and more difficult along with the continuously decreasing temperature according to the cooling schedule.

5. Numerical examples

A three-span nine-story plane steel frame as shown in Figure 2 is considered as the example structure.

5.1. Model description

The height and width of the frame are 34.7 m and 12.8 m, respectively. The frame has moment-resisting joints, and is rigidly supported at the column-bases. The weights of the roof and the other floors, including structural and nonstructural components, are 4.6 kN/m^2 and 3.3 kN/m^2, respectively, where the depth of 6.4 m is assumed for this plane frame model. The conventional assumption of rigid floor is used. The masses are concentrated at the beam-to-column connections (nodes).

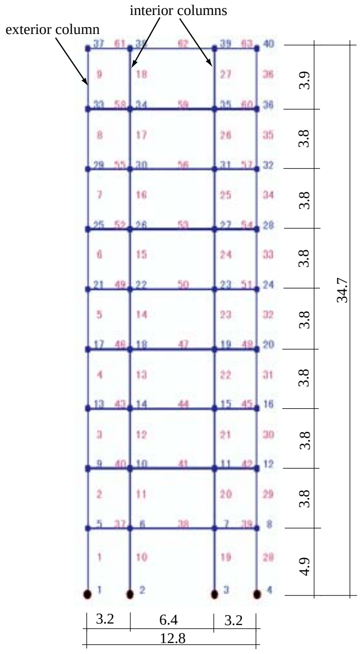


Fig. 2: A three-span nine-story plane steel frame.

The members are separated into twelve groups; i.e., m = 12: the exterior and interior columns (or beams) every three stories. The sections of the columns are selected from 113 available sections, and those of beams from 191 available sections.

The artificial seismic motion as shown in Figure 3 is generated by the standard superposition method of sinusoidal waves, corresponding to the life-safe performance level during the very rare earthquakes specified in Notifications 1461 and 1457 of the Ministry of Land Infrastructure and Transport, Japan. The phase difference spectrum of El-Centro 1940(EW) has been used. To consider extreme loads, the seismic motion is scaled by three and applied at the base in horizontal direction. The deformation due to self weight is not considered for simplicity. The members are classified to groups to preserve the symmetry of structure.

THA is carried out using the open source solver OpenSees [4]. The effect of geometrical nonlinearity istaken into consideration. Rayleigh damping is adopted for THA, with the same damping ratio h=0.02 for both of the 1st and 2nd modes. The time step for integration by the Newmark- β method ($\beta = 0.25$) is 0.01 second.

5.2 Uncertainty and optimal structure

The steel materials are idealized by a bilinear constitutive model defined by Young's modulus, yield stress and hardening coefficient, of which uncertainties are considered. Damping ratio of the structure is also an important structural parameter for dynamic analysis, but will not be considered in this example because it has monotonic relation to the maximum responses of the structure: the larger is the damping ratio, the less is the response.

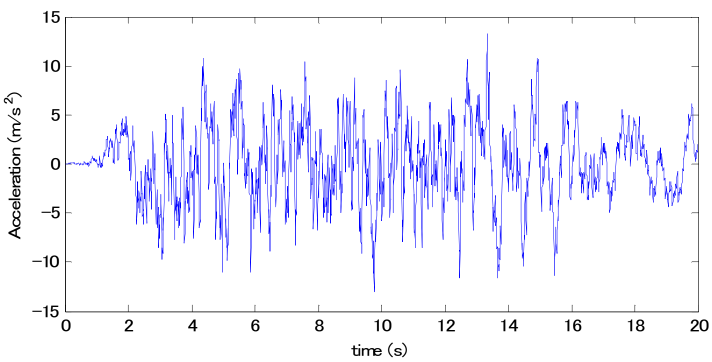


Fig. 3: The artificial seismic motion.

The nominal values of the yield stress, Young's modulus and hardening coefficient are 3.25×10^8 N/m^2, 2.05×10^{11} N/m^2 and 1/100, respectively. The upper and lower bounds for uncertainties are respectively set as 1.2 and 0.8 times of their nominal values. Furthermore, to carry out MCS, these structural parameters are supposed to have uniform probability distribution densities.
To apply Kriging method for response prediction, we start from nine initial sampling points, which are the combination of the upper and lower bounds of any two of the three parameters, in addition to the sampling point with nominal values. New sampling points, which lead to the maximum reduction of MSE are consecutively added as sampling points in order to refine the surrogate function.
Reliability of the structure is evaluated by the probability of exceedance of a specific interstory drift angle, say 5% in this example. The probability should be less than a specific target value to guarantee a safe structure, which is set as 10% as a constraint condition.
Moreover, we use the Gaussian spatial correlation function, which is preferable for a differentiable response function [5]; the lower and upper bounds of the correlation parameters are assigned as 0.1 and 10.0, respectively. The correlation parameters are found by minimizing the prediction errors at specified verification points, using the optimization tool *fmincon( )* provided in MATLAB [6].
The initial temperature for SA is assigned as 3.0, and coefficient for the *linear* cooling procedure is 0.9. The process of finding new solutions will be terminated when the temperature is less than 0.01. Convergence performance of search procedure is illustrated in Figure 4. It shows that SA has ability of jumping out from local optimum, and gradually converges at (nearly) global optimum.
The distributions of cross-sectional areas of the initial and optimal structures are illustrated in Figure 5. To illustrate the distribution clearer in the figure, width of each member is plotted in proportion to its cross-sectional area.

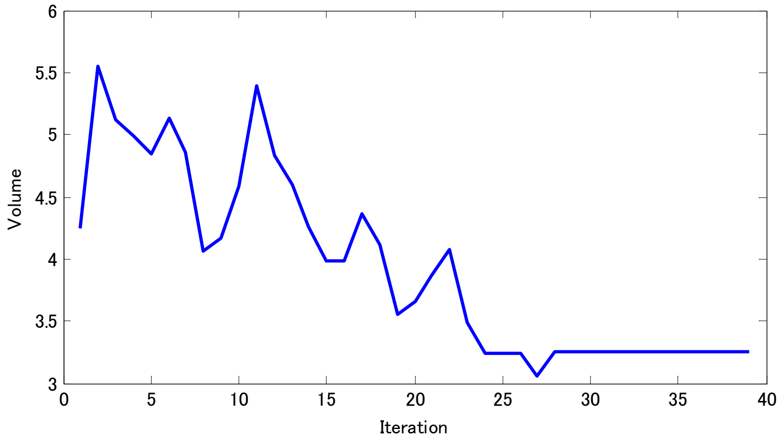
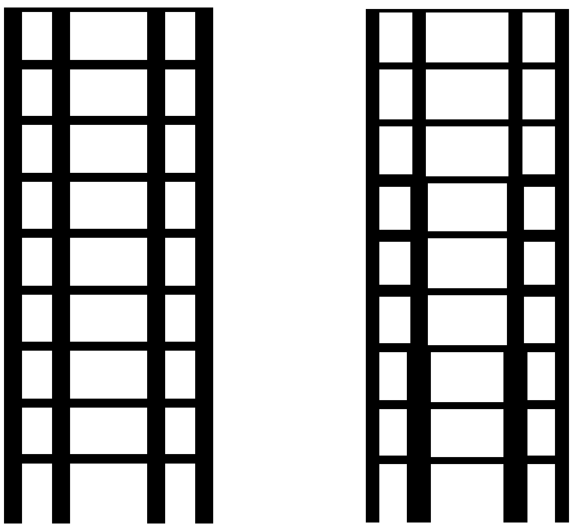


Fig. 4: Convergence performance.



(a) Initial structure (Volume: 4.25m^3) *(b) Optimal structure (Volume: 2.98m^3)*

Fig. 5: The optimal cross-sectional areas.

6. Conclusions

In this study, we have studied the reliability-based design methodology for moment-resisting frames, subjected to possible uncertainty involved in the parameters of structural analysis. The approach can be applied to other structural systems, for example two-dimensional arch model [7] and three-dimensional single-layer lattice shells [8] in our previous studies.

The study has shown that uncertainties in material properties can be easily considered by Kriging method to evaluate reliability of a structure against specific external loads. However, external loads, especially ground motions, are highly uncertain, and hence, their influence on dynamic responses should also be carefully investigated, which is the future topic of the study.

Moreover, we have considered only uniform probability distribution for uncertainties in material properties in the numerical example; but any other type of probability distribution can be incorporated, with minor modification in applying MSC.

Furthermore, other than the single-objective optimization in the example, more structural performance measures and more objective functions could be considered.

6. References

[1] Sakata S, Ashida F and Zako M, Structural optimization using kriging approximation, *Comput. Methods Appl. Mech. Engrg.*, **192**, pp. 923-939, 2003.

[2] Lee TH and Jung JJ, Kringing metamodel based optimization, *Optimization of Structural and Mechanical Systems*, edited by Arora JS, World Scientific, 2007.

[3] Kirkpatrick S, Gelatt CD Jr. and Vecchi MP, Optimization by simulated annealing, *Science*, **220**:**4598**, 671-680, 1983.

[4] Neuenhofer A and Filippou FC, Geometrically nonlinear flexibility-based frame finite element, *Journal of Structural Engineering*, 1998, **124**:**6**, 704-71.

[5] Mitchell TJ and Morris MD, The spatial correlation function approach to response surface estimation, *Winter Simulation Conference archive Proceedings of the 24th conference on Winter simulation table of contents*, Arlington, Virginia, United States. pp. 565-571, 1992.

[6] Borse GJ, *Numerical Method with MATLAB*. International Thomson Publishing Inc. 1997.

[7] Zhang JY and Ohsaki M, Reliability-based optimization of spatial structures using approximation model. *Proc. International Association for Shell and Spatial Structures*, Valencia, Spain, Sep. 2009.

[8] Zhang JY and Ohsaki M, Reliability-based design of single-layer lattice shells using Kriging approximation. *Proc. International Association for Shell and Spatial Structures*, Shanghai, P.R. China, Nov. 2010.