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Optimum Design of Steel Frames Considering Uncertainty of Parameters

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1. Abstract
A hybrid approach of optimization and anti-optimization is presented for worst-case seismic design of steel building frames. Uncertainties are considered in yield stresses and cross-sectional geometries of members. The concept of unknown-but-bounded is introduced, and the parameters are assumed to exist in a bounded region of uncertainty. An anti-optimization problem is formulated for finding the worst value of the global response defined by the maximum value of the interstory drift among all the stories. Optimal cross-sections of beams and columns are then selected from the list of available sections under constraints on worst responses. The regions of uncertainty of parameters are discretized into several values, and a heuristic approach called tabu search is used for optimization and anti-optimization. It is shown in the numerical examples that a good approximate solution of the global optimal solution, which can be found by enumeration, is successfully found by tabu search within practically acceptable number of analyses. A random selection approach is also carried out to efficiently find approximate solutions.
2. Keywords: Uncertainty, Steel frame, Seismic response, Anti-optimization, Tabu search

3. Introduction
Most of the optimization methods of building frames are developed for application to minimum weight design under constraints on elastic responses against static loads [1]. The limit design of regular frames subjected to static proportional loads is also a traditional problem in 1960's, because analytical solutions can be found, or the solutions can be found by solving a linear programming problem. However, recent building codes in those countries that are prone to seismic hazards demand nonlinear dynamic analysis for relatively tall buildings.

Another important aspect in structural optimization is that uncertainty in structural and problem parameters should be taken into account appropriately for application to practical design. There are various approaches to such purpose; namely, reliability-based approach, probabilistic approach, and worst-case design. It should also be noted that evaluation of elastoplastic dynamic responses using time-history analysis requires much computational time; therefore, we cannot carry out structural analysis many times for optimization.

In this study, we present a new approach to worst-case design of building frames. Uncertainties are considered in yield stresses and cross-sectional geometries of members. The concept of unknown-but-bounded is introduced [2], and the parameters are assumed to exist in a bounded region with prescribed upper and lower bounds. The analysis program called OpenSees [3] is used for evaluation of maximum responses under seismic motions that are compatible to the specified design response spectrum. An anti-optimization problem is formulated for finding the worst value of the global response defined by the maximum value of interstory drifts of the frame. Optimal cross-sections of beams and columns are then selected from the list of available sections under constraints on worst responses.

The regions of uncertainty of parameters are discretized into several values. Therefore, the anti-optimization problem as well as the optimization problem turns out to be an integer programming problem, which is also called a combinatorial problem. Hence, the heuristic approach called tabu search (TS) [4] is used for optimization and anti-optimization. It is shown in the numerical examples that a good approximate optimal solution can be successfully found by TS within practically acceptable number of analyses. A random search approach [5] is also carried out to show that the approximate solutions can be successfully found by a simple algorithm without any problem-dependent parameter.

4. Optimization Problem
Consider a problem for optimizing the cross-sections of steel building frames. The sections are selected from the pre-assigned list of standard sections. The members are classified into m groups, where members in each group have the same section. The design variable vector is denoted by $\mathbf{J} = (J_1, \dots, J_m)$, which has integer values. For example, if $J_i = k$, the the section of the i th group has the k th section of the list.

Let $F(\mathbf{J})$ denote the objective function, which represents, e.g., the total structural volume. The constraint functions representing, e.g., the maximum interstory drifts, are denoted by $G_i(\mathbf{J})$ $(i=1,\ldots,n)$, where n is the number of constraints. Then optimization problem is formulated as

$$\begin{aligned} &\text{Minimize} \quad F(\mathbf{J}) \\ &\text{subject to} \quad G_i(\mathbf{J}) \le G_i^{\mathrm{U}},\ (i=1,\ldots,n) \\ &\qquad\qquad\quad J_i \in \{1,\ldots,s_i\},\ (i=1,\ldots,m) \end{aligned} \tag{1}$$

where G_i^{U} is the upper bound for $G_i(\mathbf{J})$, and s_i is the number of admissible values for J_i.

Since the structural and material parameters have uncertainty, we assign constraints on the worst values $G_i^{\mathrm{worst}}(\mathbf{J})$ of responses, and formulate the optimization problem as.

$$\begin{aligned} &\text{Minimize} \quad F(\mathbf{J}) \\ &\text{subject to} \quad G_i^{\mathrm{worst}}(\mathbf{J}) \le G_i^{\mathrm{U}},\ (i=1,\ldots,n) \\ &\qquad\qquad\quad J_i \in \{1,\ldots,s_i\},\ (i=1,\ldots,m) \end{aligned} \tag{2}$$

Let $\mathbf{X}$ denote the vector of parameters that have uncertainty. Then, the constraint function is written with parameter vector as $\hat{G}_i(\mathbf{J},\mathbf{X})$, and $G_i^{\mathrm{worst}}(\mathbf{J})$ is obtained by solving the following antioptimization problem:

$$\begin{aligned} &\text{Find} \quad G_i^{\mathrm{worst}}(\mathbf{J}) = \max_{\mathbf{X}} \hat{G}_i(\mathbf{J},\mathbf{X}) \\ &\text{subject to} \quad \mathbf{X}^{\mathrm{L}} \le \mathbf{X} \le \mathbf{X}^{\mathrm{U}} \end{aligned} \tag{3}$$

where $\mathbf{X}^{\mathrm{U}}$ and $\mathbf{X}^{\mathrm{L}}$ are the upper and lower bounds for $\mathbf{X}$, which can be obtained from measurements and experiments. Hence, the optimal solution considering the worst values of responses can be found by solving a two-stage hybrid optimization-antioptimization problem.

5. Tabu search

The simplest heuristic approach is a local random search that consecutively selects the best solution in the neighborhood of the current solution. The convergence property to the global optimal solution may be enhanced if many solutions are searched before moving to a neighborhood solution, or preferably, all neighborhood solutions are searched to select the best neighborhood solution. A neighborhood solution that does not reduce (for a minimization problem) the objective value can also be selected to improve the possibility of reaching the global optimal solution. However, in this case, a so-called cycling or loop can occur, where a set of neighboring solutions is chosen iteratively. TS has been developed to prevent cycling utilizing the tabu list containing the prohibited solutions that have already been searched.

The algorithm of TS is summarized follows, where the superscript (k) denotes a value at the k th iteration:

Step 1: Assign an initial solution $\mathbf{J}^{(0)}$, and initialize the tabu list T by $\mathbf{J}^{(0)}$. Set the iteration counter $k=0$.
Step 2: Generate neighborhood solutions $\mathbf{J}_i^{\mathrm{N}}$ $(j=1,\ldots,n^{\mathrm{N}})$ of $\mathbf{J}^{(k)}$ and move to the best solution $\mathbf{J}^*$, satisfying the constraints, among them that is not included in the tabu list T.
Step 3: Add $\mathbf{J}^*$ to T. Remove the oldest solution in T if the length of the list exceeds the specified value.
Step 4: Let $\mathbf{J}^{(k+1)} = \mathbf{J}^*$ and $k \leftarrow k+1$. Go to Step 2 if the termination condition is not satisfied; otherwise, output the best solution satisfying the constraints, and terminate the process.

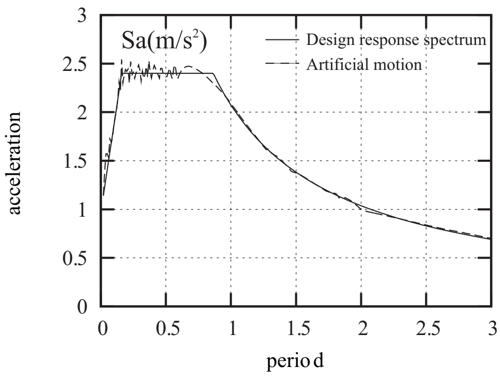

Figure 1: Design acceleration response spectrum

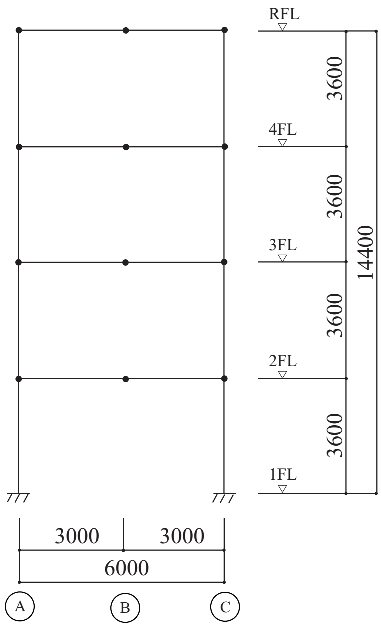

Figure 2: A 4-story plane frame

6. Frame model and seismic motions

Consider a 4-story single-span plane frame model as shown in Fig. 2. The steel sections of beams and columns of the standard design is listed in Table 1. The steel material has a bilinear stress-strain relation, where Young's modulus is 205 kN/mm^2, the nominal value of hardening ratio is 0.01, and the nominal values of yield stresses of beams and columns are 235 N/mm^2.

The artificial seismic motions are generated by the standard superposition method of sinusoidal waves, where the phase difference spectrum of El-Centro 1940 (EW) is used. The target spectrum is the design acceleration response spectrum, as shown in Fig. 1, for 5 % damping specified by Notification 1461 of the Ministry of Land, Infrastructure and Transport (MLIT), Japan. The amplification factor for the ground of 2nd rank is used as defined in Notification 1457 of MLIT. The seismic motion is applied at the base of the frame in horizontal direction. The acceleration response spectrum of an artificial motion is also shown in Fig. 1.The wave is scaled by 7.5 to design the frame with seismic performance of rank 3 for the life-safety level. The mean value of maximum response against five artificial waves is taken as the representative response.

A general purpose frame analysis software called OpenSees [3] is used for seismic response analysis of the frame. The members are divided into fiber sections. Since only plane frame analysis is carried out, the flange and web of the beam are discretized to 4 and 16 fibers in the directions of thickness and depth, respectively. The integration is carried out using the Gauss-Lobatto rules that has integration points at the ends of the element, where the number of integration points is 8; thus, the plastification at the member ends can be accurately detected. The standard Newmark-β method ($\gamma = 0.5$, $\beta = 0.25$) is used for integration in time domain with the increment 0.01 sec.

The stiffness proportional damping is used with the damping ratio 0.02 for the first mode. The duration of seismic motion is 20 sec. The fundamental period of the standard model is 0.71 sec, which means from Fig. 1 that the response acceleration reduces as the period becomes larger as the result of plastification.

Table 1: Sections of standard frame model

beam		column	
floor	section	story	section
4th, roof	H-350×175×7×11	1st–4th	□-300×300×16
2nd, 3rd	H-400×200×9×19		

7. Optimization results

In the following optimization problem, a constraint is given for the maximum value among the maximum interstory drifts, which is simply denoted by *maximum interstory drift*, of stories so that the frame does not collapse under severe earthquakes.

7.1 Antioptimization

We first carry out parametric study to investigate the effect of various parameters on the maximum interstory drift. It has been found through the preliminary parametric study that the maximum interstory drift is a monotonically decreasing function of the yield stress of columns and the hardening ratios of beams and columns. Therefore, the smallest possible values should be chosen for these parameters to obtain the worst response; hence, the yield stress of column is the lower bound value 325 N/mm^2, and the hardening coefficient is the small value 0.01.

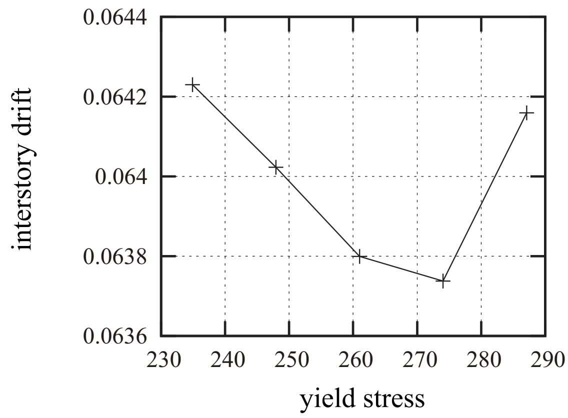

Figure 3: Relation between yield stress (N/mm^2) of beams and maximum interstory drift (m)

In contrast, the maximum interstory drift is not a monotonic function of the yield stress σ_Y^b of the beam as shown in Fig. 3. This is because a stronger beam leads to a column collapse mechanism that has small energy dissipation and larger local interstory drift. In addition to material parameter, the cross-sectional geometry also has uncertainty. Therefore, we consider uncertainty in σ_Y^b and flange thicknesses in two groups of beams; hence, there are four uncertain parameters. The ranges of uncertainty are 20 % of the nominal value for all parameters, which are discretized into five equally spaced values.

Since the nominal value 235 N/mm^2 of yield stress indicates the lower bound, σ_Y^b can take 235, 248, 261, 274, and 287 N/mm^2. The nominal value of flange thickness is assumed to be a mean value; hence, the possible values are detrmined by multiplying 0.95, 0.975, 1.0, 1.025, and 1.05 to the nominal value. Therefore, we have $5^4 = 625$ possible combinations of parameters.

Although the distribution of each parameter can be modeled parametrically using, e.g., normal distribution, we use the uniform distribution, because no information is available for the distribution. Note that the following methods and results can be extended to any distribution by modifying the sampling spaces.

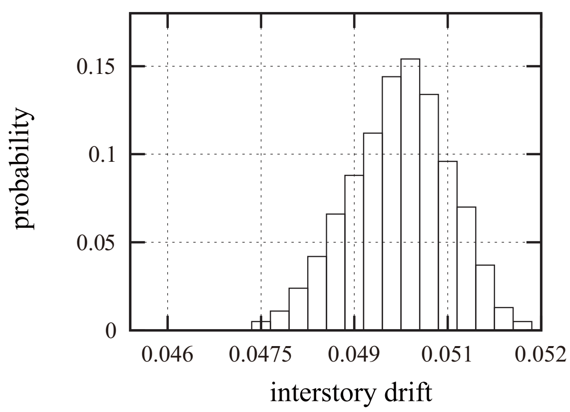

Figure 4: Probability density function by enumeration

Fig. 4 shows the discretized probability density function of the roof displacement for the 625 samples. The maximum and minimum values are 0.0518 m and 0.0475 m, respectively. The purpose here is to find the approximate maximum value within small number of analyses. For this purpose, we assume the parameter set corresponding to the maximum interstory drift up to the 50th maximum value are assumed to be approximate worst parameter sets.

Suppose we select parameter sets 50 times randomly from 625 samples. Then, the probability that no approximate worst parameter set is found through this random selection is $(575/625)^{50} = 0.0154$, which is very small. Therefore, we carry out 50 analyses in the following; i.e., for TS, the number of neighborhood solutions is 5, and the number of steps is 10. We find approximate optimal solutions four times starting with different initial values of random number for verification purpose. Comparison is also done with random sampling of 50 parameter sets.

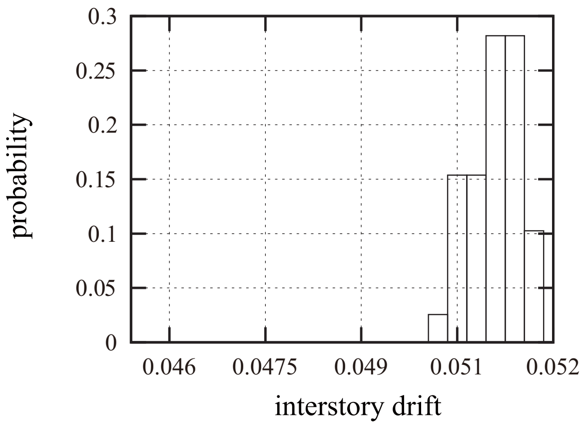

Figure 5: Probability density function by TS

Table 2 Results of TS

Initial value of random number	100	200	300	400
Maximum	0.0518	0.0514	0.0517	0.0516
Minimum	0.0500	0.0495	0.0498	0.0495
Average	0.0511	0.0506	0.0501	0.0508
Standard deviation ($\times 10^{-4}$)	4.2527	4.7602	3.6175	4.4306
Order	1st	9th	3rd	5th

The maximum value, minimum value, mean value, and the standard deviation of the maximum interstory drift obtained through the process of TS is listed in Table 2, where the last row is the order of the worst (maximum) value in the original list of 625 parameter sets. As is seen, a parameter set within the 8th has been found for all cases. Furthermore, the standard deviation is very small, which is less than 1/10 of the difference between the maximum and minimum values in the original list, which means that TS searches the solutions with limited range of function value. This way, a good approximate worst value can be found through analyses of less than 1/10 of the size of the original list. Fig. 5 shows the probability density function of 50 solutions for a single run of TS with ransom seed 500. It is verified from Figs. 4 and 5 that TS searches the solutions with larger responses.

Table 3 Results of random selection

Initial value of random number	100	200	300	400
Maximum	0.0511	0.0517	0.0513	0.0517
Minimum	0.0481	0.0482	0.0481	0.0477
Average	0.0497	0.0499	0.0498	0.0499
Standard deviation ($\times 10^{-4}$)	7.4996	9.2502	7.3484	9.1734
Order	28th	3rd	18th	3rd

Table 3 shows the results of random selection, in which the worst order of the solution is 28th. This result is worse than TS; however, a solution within 50th has been successfully found for all cases within the analyses of 1/10 of the total number of solutions. Furthermore, there is no problem-dependent parameter for ransom selection, which is superior to TS.

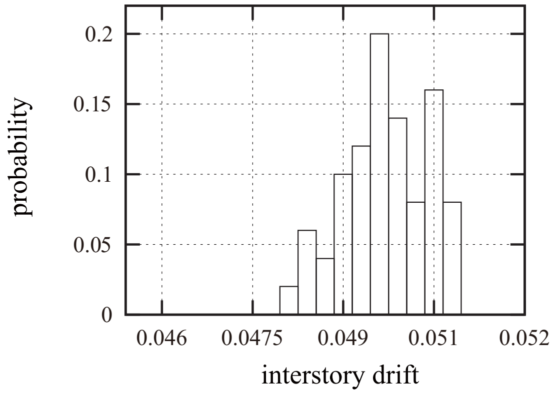

Figure 6: Probability density function by randomization

Table 4: List of standard sections

Beam 1 (2F, 3F)	Beam 2 (4F, RF)
H-400x200x9x12	H-250x125x6x9
H-400x200x9x16	H-300x150x6.5x9
H-400x200x9x19	H-350x175x7x11
H-400x200x9x22	H-400x200x8x13
H-400x200x12x22	H-450x200x9x14

Column
□-300x300x9
□-300x300x12
□-300x300x16
□-300x300x19
□-300x300x22

Table 5 Optimization result

Beam1 (2F, 3F)	H-400x200x9x12
Beam2 (4F, RF)	H-300x150x6.5x9
Column	□-300x300x12

Yield stress:	
Beam 1	278 N/mm^2
Beam 2	248 N/mm^2
Thickness of flange:	
Beam 1	8.55 mm
Beam 2	11.4 mm

7.2 Optimization of frame section

In the upper-level optimization problem, we select member sections from the pre-assigned list of standard sections in Table 4. A constraint is given such that the worst value of maximum interstory drift is not more than 0.072 m, which is equivalent to 2% of the interstory drift angle. The objective function is the total structural volume. In the lower-level antioptimization problem, uncertainty is considered in the yield stress and flange thicknesses of beams. TS is used for both optimization and antioptimization, where the numbers of neighborhood solutions and steps are 5 and 10, respectively.

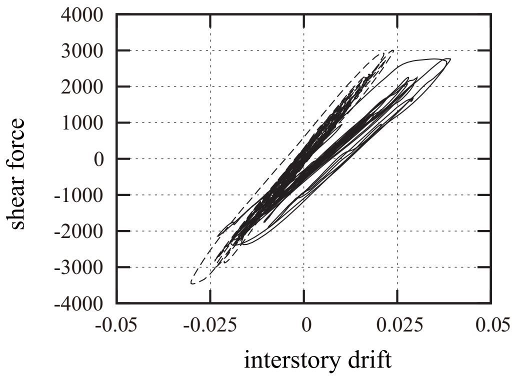

Figure 7: Relation between interstory drift angle and shear force of 1st story

The optimal sections and the worst parameter set at optimum are shown in Table 5. The relation between the interstory drift and shear force of the 1st story is plotted in Fig. 7 for the optimal design with worst parameter set subjected to one of the five seismic motions. The dotted lines are the result for the standard section with nominal parameter set. The envelopes of maximum interstory drift angles are plotted in solid line in Fig. 8. The dotted line is the result of the standard frame model with nominal parameter values. Note that the maximum interstory angles of the optimal frame is almost equal to 2%.

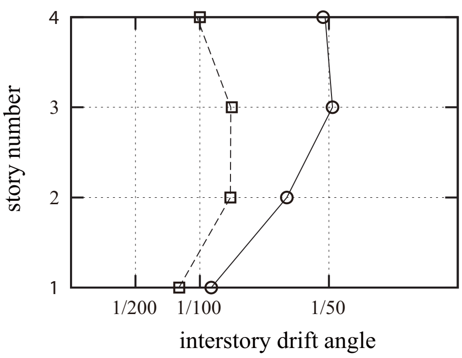

Figure 8: Maximum interstory drift angles

8. Conclusions

A method has been presented for cross-sectional optimization of building frames considering uncertainty of parameters for material and cross-sectional geometry. The objective function is the total structural volume, and a constraint is given for the worst value of the representative response of the frame subjected to seismic motions. The cross-sections are selected from the list of available standard sections. The parameters are also descretized into integer values. Therefore, the optimum design problem and the antioptimization problem for finding the worst response are formulated as combinatorial problems.

A new concept is introduced for the antioptimization problem, where the accuracy of the solution is defined by the order of the response rather than the value of the response among the possible combinations of the parameters. Hence, the approximate values for the optimization and antioptimization problems are found using a heuristic approach called TS.

The performance of TS is compared with that of random selection. It has been shown that TS is better than random selection in view of accuracy; however, the latter has superiority because it does not have any parameter that should be assigned by intuition.

9. References

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