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Optimization of Energy Dissipation Property of Eccentrically Braced Steel Frames

Makoto Ohsaki[1], Taku Nakajima[2]

[1]Hiroshima University, Higashi-Hiroshima, Japan, ohsaki@hiroshima-u.ac.jp
[2]Kyoto University, Kyoto, Japan, nakajima@archi.kyoto-u.ac.jp

1. Abstract
Eccentrically braced frames are used for passive control of building frames under seismic excitations. The link member between the beams and braces is reinforced with stiffeners in order to improve stiffness and plastic deformation capacity. In this study, we present a method for optimizing the locations and thicknesses of the stiffeners as well as the length of a link member. The optimal solutions are found using a heuristic approach called tabu search. The objective function is the dissipated energy before failure of the link member. The deformation under static cyclic forced displacements is simulated using the commercial finite-element software package called ABAQUS. It is demonstrated in the numerical examples that the dissipated energy can be increased through optimization with small number of analyses.
2. Keywords: Eccentrically braced frame, Energy dissipation, Tabu search, Finite-element analysis, Cyclic deformation

3. Introduction
Owing to recent advancement of computer technologies as well as the algorithms for analysis and optimization, we can optimize complex structures using sophisticated finite-element (FE) analysis for evaluation of elastoplastic responses. For example, the body of automobile can be optimized considering crash properties utilizing approximation methods such as response surface model [1].

In the conventional formulations of optimization problems in the field of building engineering, the stiffnesses of beams and columns of frames are optimized to minimize the total structural volume under constraints on elastic stresses and displacements under static loads [2]. However, one of the criticisms in optimization in building engineering is that the structures in this field are not mass-products; therefore, it is not effective to optimize a structure that is built only once.

Recently, some attempts have been made for optimizing the structural parts such as beams, columns, and joints of building frames. Since the parts are mass-products, it is worthwhile to spend much computational effort for optimization. The authors optimized the cross-sectional shape of the clamping device of frame-supported membrane structures [3]. The first author optimized the flange shape of the beam with reduced section for maximization of plastic energy dissipation under static cyclic loads [4].

In this study, we present a method for optimizing the link member of an eccentrically braced steel frame, which is used as a passive control device for seismic design of building frame. The member is located in the middle or at the end of a beam and is supposed to dissipate seismic energy through plastification under cyclic deformation. The deformation of the link member under static cyclic loads is simulated using the commercial FE-analysis software package called ABAQUS, where a thick shell elements with reduced integration is used. The failure of the link member is predicted by the 'failure index' defined by the triaxiality of the stress.

The objective function is the dissipated energy before failure of the link member. The design variables are locations of stiffeners and the length of the link member, which are discretized to integer values. A heuristic approach called tabu search (TS) is used for optimization. Optimal solutions are first found for the link member subjected to forced static cyclic deformations with increasing amplitude at the ends. We next optimize the link member supported by beams and columns that are modeled by beam elements. It is demonstrated in the numerical examples that the dissipated energy before failure can be increased through optimization with small number of analyses, although global optimality of the solution is not guaranteed.

4. Shape Optimization Problem of Eccentrically Braced Frames
4.1. Failure Index
A braced frame with eccentricity at the joint of beam and brace is called *eccentrically braced frame*

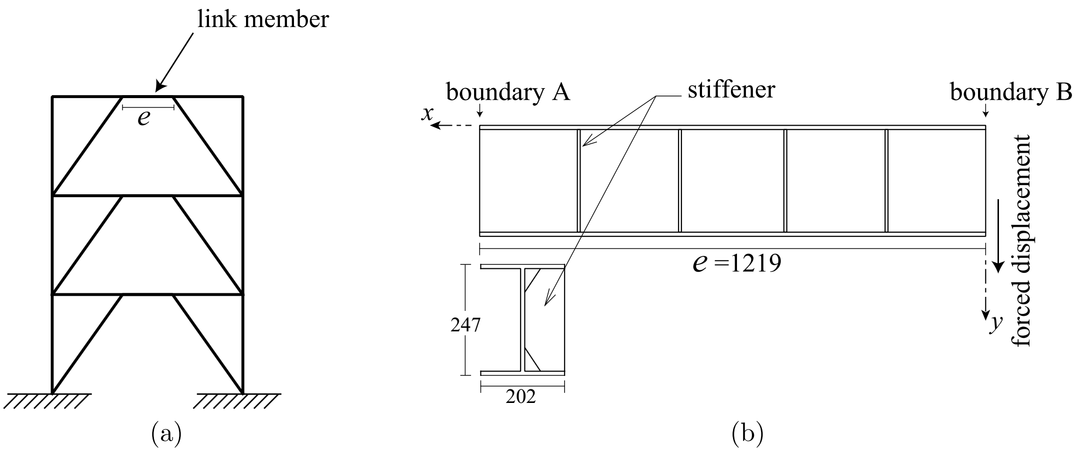

Figure 1: A eccentrically braced frame; (a) frame model, (b) link member

(EBF) [5]. An example of EBF is shown in Fig. 1. In this study, we carry out FE-analysis and optimize the shape of the link member between the two connections of beams and braces. The thicknesses and locations of the stiffeners attached to the link member are optimized to maximize energy dissipation under cyclic static forced deformation. The maximum acceptable deformation of member is defined, as follows, using the failure index.

Let ε_{p} denote the equivalent plastic strain defined as

$$\varepsilon_{\mathrm{p}} = \int_0^t \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{\mathrm{p}}\dot{\varepsilon}_{ij}^{\mathrm{p}}}\mathrm{d}t \tag{1}$$

where $\dot{\varepsilon}_{ij}^{\mathrm{p}}$ is the plastic strain rate tensor, $\dot{(\ )}$ is the derivative with respect to pseudo time t, and the summation convention is used. The critical plastic strain $\varepsilon_{\mathrm{p}}^{\mathrm{critical}}$ is defined as

$$\varepsilon_{\mathrm{p}}^{\mathrm{critical}} = \alpha \exp\left(-1.5\frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{e}}}\right) \tag{2}$$

where σ_{m} is the mean stress and σ_{e} is the von Mises equivalent stress. The ratio $\sigma_{\mathrm{m}}/\sigma_{\mathrm{e}}$ is called stress triaxiality. The parameter α is dependent on material.

The failure index FI is defined as [8]

$$\mathrm{FI} = \frac{\varepsilon_{\mathrm{p}}}{\varepsilon_{\mathrm{p}}^{\mathrm{critical}}} \tag{3}$$

The material is assumed to fracture when FI reaches 1.0.

4.2. Optimization Problem

All variables are discretized into integer values and a heuristic approach called tabu search (TS) is used for optimization. Let $\mathbf{J} = (J_1, \dots, J_m)$ denote the vector of m design variables that can take integer values. Real values $x_1, \dots, x_m$ representing, e.g., the locations of stiffeners, are defined by J_i with the specified values x_i^0 and Δx_i as

$$x_i = x_i^0 + (J_i - 1)\Delta x_i, \quad (i = 1, \dots, m) \tag{4}$$

Therefore, all properties of the link member are functions of $\mathbf{J}$.

We consider two optimization problems. In the first problem, the final time for the specified cyclic displacement as shown in Fig. 2 is specified. The plastic dissipated energy $E_{\mathrm{p}}(\mathbf{J})$ throughout the specified cyclic deformation is maximized, and the upper bound 1.0 is given for the maximum failure index $I_{\mathrm{f}}(\mathbf{J})$ throughout the deformation. Then, the optimization problem is formulated as

$$\mathrm{P1}: \ \text{Maximize } F(\mathbf{J}) = E_{\mathrm{p}}(\mathbf{J}) \tag{5a}$$

$$\text{subject to } I_{\mathrm{f}}(\mathbf{J}) \leq 1.0 \tag{5b}$$

$$J_i \in \{1, \dots, s_i\}, \quad (i = 1, \dots, m) \tag{5c}$$

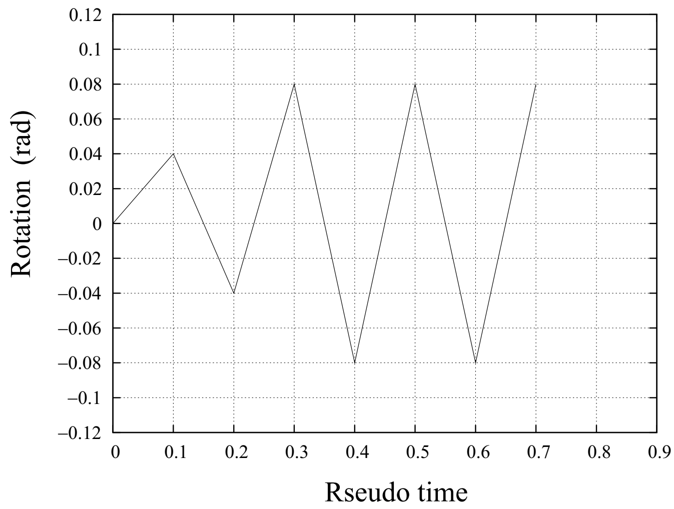

Figure 2: Forced cyclic displacement

where s_i is the number of pre-assigned values for J_i.

We use a penalty function approach, because TS cannot handle the constraints directly. The objective function is replaced by $F^*(\mathbf{J})$, defined as follows, when the constraint is violated:

$$F^*(\mathbf{J}) = F(\mathbf{J}) - p(I_\mathrm{f}(\mathbf{J}) - 1.0)^2 \tag{6}$$

where p is the penalty parameter. Note that a negative penalty is given for the maximization problem.

In the second optimization problem, we maximize the plastic energy $E_\mathrm{p}^\mathrm{f}(\mathbf{J})$ that is dissipated before FI reaches 1.0. The forced deformation terminates at the time when FI reaches 1.0. The optimization problem is formulated as follows as a unconstrained problem:

$$\mathrm{P2}: \ \text{Maximize} \ F(\mathbf{J}) = E_\mathrm{p}^\mathrm{f}(\mathbf{J}) \tag{7a}$$

$$\text{subject to} \ J_i \in \{1, 2, ..., s\}, \ (i = 1, 2, ..., m) \tag{7b}$$

4.3. Tabu Search (TS)

The tabu search (TS) is classified as s single-point search heuristic approach, and is a slight extension of the random local search [2]. TS basically moves to the best neighborhood solution even if it does not improve the solution. A tabu list is used to prevent an unfavorable phenomenon called cycling, in which small number of solutions are selected alternatively. The basic algorithm of TS is summarized as follows:

step 1 Randomly generate a seed solution $\hat{\mathbf{J}}$, and initialize the tabu list T as $T = \{\hat{\mathbf{J}}\}$. Evaluate the objective function and initialize the incumbent optimal objective value as $F^\mathrm{opt} = F(\hat{\mathbf{J}})$.

step 2 Generate a set of q neighborhood solutions $N = \{\mathbf{J}_j^N \mid j = 1, ..., q\}$ from $\hat{\mathbf{J}}$, and evaluate the objective value of each solution.

step 3 Select the best solution that has the maximum value of $F(\mathbf{J}_j^N)$ (or $F^*(\mathbf{J}_j^N)$ if the constraint is violated) in the set N, which is not included in the list T. Assign the best solution to the new seed solution $\hat{\mathbf{J}}$.

step 4 Update the incumbent optimal objective value as $F^\mathrm{opt} = F(\hat{\mathbf{J}})$, if $\hat{\mathbf{J}}$ satisfies the constraint and $F(\hat{\mathbf{J}}) > F^\mathrm{opt}$.

step 5 Add $\hat{\mathbf{J}}$ to the list T. Remove the oldest solution from T, if the number of solutions in T exceeds the upper limit.

step 6 Output F^opt and the corresponding optimal solution if the number of iterations reaches the specified limit; otherwise, go to step 2.

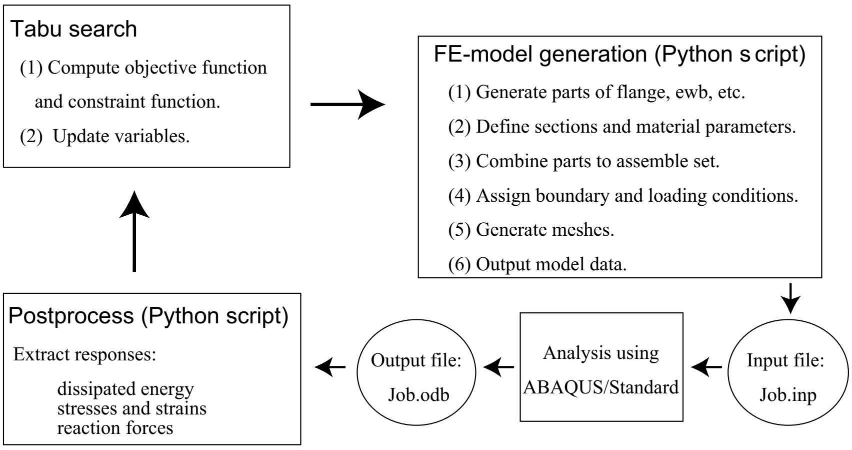

Figure 3: Optimization algorithm using TS and ABAQUS

It is desirable to generate all the neighborhood solutions when selecting the next seed solution. However, one of the purpose of this study is to show that the complex performance of a structure can be improved through small number of analyses; therefore, we limit the size of neighborhood solutions to very small value. Furthermore, the update to a non-improving solution is prohibited, and solutions are searched locally by carrying out TS from several different initial solutions.

Fig. 3 shows the data flow between TS and FE-analysis using ABAQUS. The pre- and post-process are carried out using the Python script. The computations of functions and the process of TS are coded as a Fortran program. In the following, analysis and optimization is carried out using a PC with Intel core i7 CPU, 2.93GHz, 3GB RAM.

5. Optimization of Link Member

The link member has the wide-flange section W10×33 (H-247×202×7×11). The standard solution of the link member has four stiffeners of thickness 10.0 mm in one side, and the length $e = 1219$ mm.

The steel material is ASTM A992, where Young's modulus is 2.0×10^5 N/mm^2, yield stress is 359.0 N/mm^2, tensile strength is 592.0 N/mm^2, and kinematic hardening with the hardening ratio 0.006 is used. The parameter α for the FI in Eq.(2) is 2.6 [8].

According to American Institute of Steel Construction (AISC) [6], the strength and the failure mode of a link member depend on the length e and the ratio of shear strength $V_{\rm p}$ to the fully plastic moment $M_{\rm p}$. The link member to be optimized in this study has the ratio $M_{\rm p}/V_{\rm p}$ =639 mm, and is classified as shear/bending failure mode.

The general purpose FE-analysis software called ABAQUS Ver. 6.9.3 [7] is used for analysis. The shell elements S3R and S4R with reduced integration is used for modeling the link member. The nominal size for automatic mesh generation by Python is between 15 mm and 25 mm. All translational and rotational displacements are fixed at boundary 'A' in Fig. 1(b) except the displacement in x-direction (axial direction). A forced cyclic displacement in y-direction (vertical direction) is given at boundary 'B'.

The locations and thicknesses of the four stiffeners are optimized. In order to preserve the symmetry of link member, the independent variables for the locations X_1 and X_2 (mm) are defined by the two integer variables J_1 and J_2 as

$$X_i = X_i^0 + (J_i - 6)\Delta X, \quad J_i \in \{1, \dots, 11\}, \quad (i = 1, 2) \tag{8}$$

where X_i^0 is the location of the stiffener of the standard model, and $\Delta X = 20$ (mm). Therefore, the lower and upper bounds for $(X_i - X_i^0)$ are -100 mm and 100 mm, respectively.

The thicknesses D_1 and D_2 (mm) of the stiffeners are defined by the variables J_3 and J_4 as

$$D_{i-2} = D_{i-2}^0 + (J_i - 3)\Delta D, \quad J_i \in \{1, \dots, 7\}, \quad (i = 3, 4) \tag{9}$$

where D_i^0 is 10 mm, and $\Delta D = 3$ (mm); hence, the lower and upper bounds of D_i are -4 mm and 22 mm, respectively.

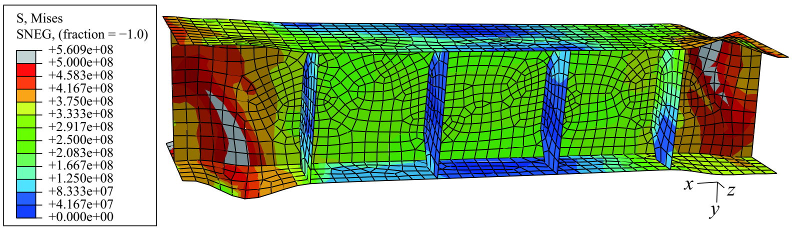

Figure 4: Stress distribution of standard solution

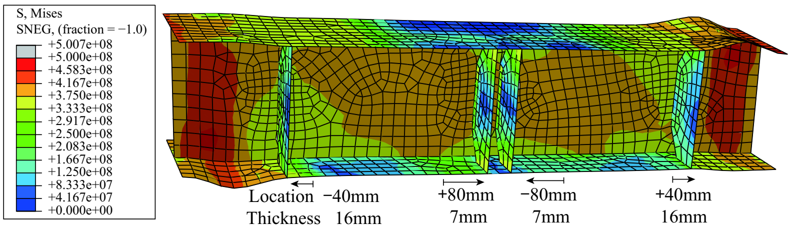

Figure 5: Stress distribution of optimal solution

Table 1: Optimization results of link member

Model	E_{p} (kN· m)	I_{f}	$R_{\max}$ (kN)	$E_{\mathrm{p}}^{\mathrm{f}}$ (kN· m)
Standard	336.0	1.057	416.4	323.0
Optimal	347.0	0.670	417.9	459.9

We solve P1 with fixed number of cycles of forced displacement, which is defined so that the FI of the standard model exceeds 1.0 at the end of loading. Thus, the first cycle has 0.04 rad, and the following 2.25 cycles have 0.08 rad.

For the TS, the number of neighborhood solutions is 3, number of steps is 5, the size of tabu list is 5, and the penalty parameter is $p = 1.0 \times 10^5$, which is large enough so that a solution violating the constraint is not selected.

Note that the best neighborhood solution that does not improve the objective value is not selected as the seed solution of the next step. Although the solution obtained by TS is not the global optimum, the best solution after five trials from different random seed is hereafter called optimal solution. The CPU time for a single analysis is about 520 sec., although it depends on the degree of plastification.

Figs. 4 and 5 show the standard and optimal solutions, respectively, with the contour lines of von Mises stress. The responses of the solutions are listed in Table 1, where $R_{\max}$ is the maximum reaction force. The optimal locations of four stiffeners are $(-40, +80, -80, +40)$ (mm), and the optimal thicknesses are (16,7,7,16) (mm). We can see from these results that the two stiffeners became thicker and moved to the ends, where stresses of the standard solution have large values. Five solutions were rejected by the tabu list.

It is also seen from Table 1 that the maximum value I_{f} of FI is reduced, and the maximum value $R_{\max}$ of the reaction force became slightly larger as the result of optimization. The performances of the standard and optimal solutions are also compared using the dissipated energy $E_{\mathrm{p}}^{\mathrm{f}}$ before FI reaches 1.0. As is seen, $E_{\mathrm{p}}^{\mathrm{f}}$ of the optimal solution is 42% larger than that of the standard solution; hence, the energy dissipation property can be drastically improved through optimization.

6. Shape Optimization of Link Member of Eccentrically Braced Portal Frame

6.1. Frame Model

We next optimize the link member attached to a portal frame as shown in Fig. 6 subjected to lateral cyclic deformation. The length of the link member is included in the design variables to maximize the energy dissipation. The frame has span L =4.0 m, height h =2.0 m, and is rigidly supported at the column bases. Lateral cyclic displacement of magnitude $\Delta = 0.02h = 60.0$ mm is applied at nodes N_1

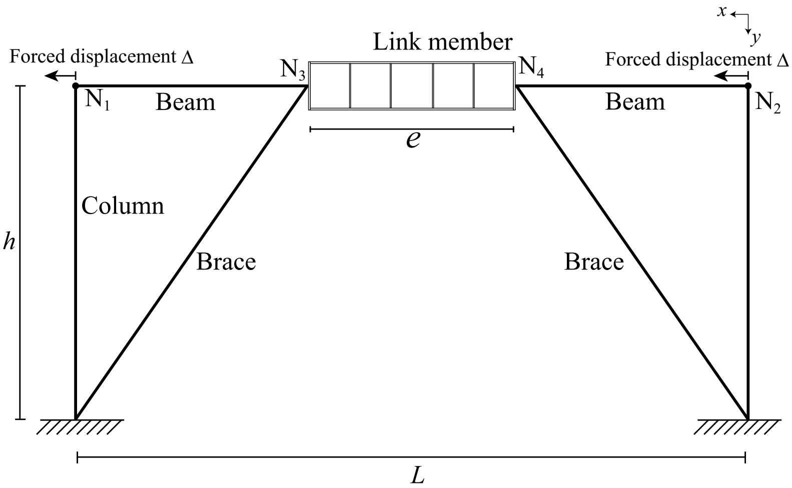

Figure 6: A frame model

Table 2: Sectional properties of beams, columns, and braces

	Section	A (mm^2)	Z_x (mm^3)	I_x (mm^4)
Beam	H-247×202	62.6×10^2	574×10^3	7075×10^4
Column	□-250×250	110×10^2	820×10^3	10300×10^4
Brace	—	60.0×10^2	—	—

and N_2 as indicated in arrows in Fig. 6.

The link member has the length e (mm) and the same sectional properties as the previous section. The flange, web, and stiffener of the link member are discretized into shell elements. The beams have the same section W10×33 as the link member, and columns have the box section □-250×250×12. The beams and columns are modeled by beam elements, and the braces are modeled by truss elements. Young's modulus and Poisson's ratio of beam and truss elements are $2.0\ \times 10^5 \mathrm{N/mm}^2$ and 0.3, respectively. The sectional properties of beams, columns, and braces are summarized in Table 2. Rigid plates are attached at the two ends of the link member.

6.2. Optimization Results

Optimal solution of P2 is found, where the variables are the length e of the link member, and the locations and thicknesses of the stiffeners. In addition to the four variables in the previous section, the length $e =$ (mm) is defined by the integer variable J_5 as

$$e = e_0 + J_5 \Delta e, \qquad J_5 \in \{1, \ldots, 12\} \tag{10}$$

where $e_0 = \Delta e = 100$. Hence, the number of variables for this case is 5. The locations (x-coordinates) and thicknesses are defined by (8) and (9), respectively, where $\Delta X = 0.015e$ and $\Delta D = 3$ (mm). The CPU time for a single analysis is about 1000 sec., although it depends on the degree of plastification.

The objective function is $E_{\mathrm{p}}^{\mathrm{f}}$, which is the dissipated energy before I_{f} reaches 1.0. The number of neighborhood solutions is 4, the number of steps is 5, and the length of tabu list is 5. Optimal solutions are found from six initial solutions with $e =$ 600, 700, 800, 900, 1000, and 1100 (mm). Let N_{cycle}, $\gamma^{\max}$, and $V_{\mathrm{L}}^{\max}$ denote the number of cycles before reaching $I_{\mathrm{f}} = 1.0$, the maximum rotation angle of link member, and the maximum shear force of the link member, respectively.

The optimal thicknesses and locations of the stiffeners are (22,7,7,2) (mm) and $(-18, +36, -36, +18)$ (mm), respectively, which are similar to those of the previous section. There was no solution that was rejected by the tabu list. Fig. 7 shows the locations of stiffeners and the contour of von Mises stress of the optimal solution at $I_f = 1.0$. The optimization results are summarized in Table 3. The dissipated energy $E_{\mathrm{p}}^{\mathrm{f}}$ of the optimal solution at $I_f = 1.0$ is 593.0 kN·m, which is 26.1% larger han that of the standard solution.

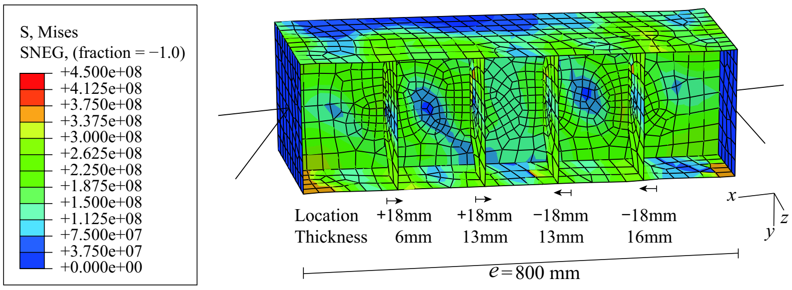

Figure 7: Stress distribution of the optimal frame model at I_{f} =1.0

Table 3: Optimization results of frame model

Model	e (mm)	$E_{\mathrm{p}}^{\mathrm{f}}$ (kN·m)	N_{cycle}	$\gamma^{\max}$ (rad)	$V_{\mathrm{L}}^{\max}$ (kN)
Standard	1219	470.2	7	0.051	409.4
Optimal	800	593.0	7	0.083	505.5

7. Conclusions

It has been shown in this study that TS is very effective for structural optimization problem, for which substantial computational cost is needed for response evaluation and, accordingly, function evaluation cannot be carried out many times. The performance of the solution can be drastically improved within small number of steps, although the global optimality is not guaranteed.

Optimal shapes have been found for the link member of a eccentrically braced frame. The failure of the member is defined using the failure index. It has been shown that non-uniform locations and thicknesses of the stiffeners can drastically improve the energy dissipation property before fracture under cyclic static deformation.

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