6th China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems
June 22–25, 2010, Kyoto, Japan

Multiobjective Hybrid Optimization-Antioptimization for Force Design of Tensegrity Structures

Jingyao Zhang[1], Makoto Ohsaki[2], Isaac Elishakoff[3]

[1]Dept. of Architecture and Urban Design, Ritsumeikan University, Kusatsu, Japan. zhang@fc.ritsumei.ac.jp
[2]Dept. of Architecture, Hiroshima University, Higashi-Hiroshima, Japan. ohsaki@hiroshima-u.ac.jp
[3]Dept. of Mechanical Engineering, Florida Atlantic University, Florida, USA. elishako@fau.edu

Abstract
A hybrid approach of multiobjective optimization and antioptimization is presented for force design of tensegrity structures. The objective functions are to maximize the lowest eigenvalue of the tangent stiffness matrix, and to minimize the deviation of forces from the specified distribution, which are defined as the worst values due to the possible uncertainties. The worst values are found by solving the antioptimization problems using the enumeration of the vertices of the uncertain region of the prestresses. The Pareto solutions for the two-objective optimization problem are found using a nonlinear programming algorithm for minimizing linear-weighted-sum of the objective functions. It is shown in a numerical example that Pareto optimal solutions can be successfully found for a tensegrity grid by solving the two-level optimization-antioptimization problem using vertex enumeration technique combined with a nonlinear programming approach..

Keywords: *Optimization; Antioptimization; Tensegrity; Force design; Vertex enumeration.*

1. Introduction

A tensegrity structure consists of tensile members, cables, and compressive members, struts [1]. Since the tensegrity structure is unstable in absence of prestresses, the shape and stability at the self-equilibrium state strongly depend on the distribution of member forces [2–5]. In this study, we study the tensegrity structures consisting of several independent modes of prestress, and present a hybrid optimization approach for their *robust* design of member forces, while uncertainties are also taken into consideration.

The process of determination of member forces for the tensegrity structure with given shape is called *force design* [6]. There have been several optimization approaches developed for force design of tensegrity structures [4,7]. Mechanical properties such as the lowest eigenvalue of the tangent stiffness matrix and compliance against specified static loads are considered as objective and/or constraint functions.

It is important in the practical design process that the deviation of forces from their designed values should be taken into account for the evaluation of structural performance, due to the unavoidable errors in fabrication and construction processes. Moreover, the antioptimal solutions [8] or worst-case designs should then be considered for the objective and/or constraint functions. Hence, the problem of designing a structure with the highest structural performance turns out to be a hybrid optimization–antioptimization problem [9], while the uncertainties are considered. The antioptimal solution that minimizes the concave function can be found by enumerating the vertices of the convex region [10,11], which is the case for our problem as will be discussed later.

When there are more than one performance measures (objective functions), then the problem turns out to be a multiobjective programming (MOP) problem [12]. Although numerous works have been presented for antioptimization for worst-case design, multiobjective optimization with nonprobabilistic uncertainty has been mainly investigated in the field of fuzzy set based theory [13,14], and no detailed investigation has been made in the framework of standard nonlinear programming (NLP) problem. Rao [15] defined M-Pareto optimal solution in the space of membership functions, and proposed an approach that is similar to a goal programming approach. Loetamonphong et al. [16] used a genetic algorithm for generating Pareto optimal set. Some applications of the fuzzy set based approach are found for tunneling reinforcement design [17] and planning of water resource system [18].

In this paper, we present a multiobjective hybrid optimization–antioptimization method for force design of tensegrity structures. The member forces are defined as a linear combination of the self-equilibrium modes. The coefficients of the forces are optimized for maximization of the lowest eigenvalue of the tangent stiffness matrix and minimization of the deviation of forces from the target values. In the numerical example, a set of Pareto solutions are found for a tensegrity grid that has four self-equilibrium force modes. Since the lowest eigenvalue is concave and the force deviation is convex with respect to the coefficients for the force modes, the worst-case solutions are found by the enumeration of vertices of the convex region of uncertainty. A hybrid approach is presented as a combination of NLP and vertex enumeration, respectively, for optimization and antioptimization, where the linear-weighted-sum approach is used for finding a set of

Pareto optimal solutions.

2. Basic Formulations of Tensegrity Structures

In this section, we present the basic formulations for tensegrity structures. The following properties are assumed for a tensegrity structure:

(1) Members are connected by pin joints.
(2) Topology (connectivity of nodes and members) is specified.
(3) Self-weight is neglected, and no external load exists at the initial self-equilibrium state.
(4) Members are in elastic range, and neither buckling nor yielding is considered.

Consider a structure with n nodes and m members. In the state of self-equilibrium, the equilibrium equation is written as

$$\mathbf{Ds} = \mathbf{0} \tag{1}$$

where $\mathbf{D} \in \Re^{3n \times m}$ is the equilibrium matrix and $\mathbf{s} \in \Re^{m}$ is the force vector.

Let r denote the rank of $\mathbf{D}$. Then the self-equilibrium equation (1) has $q = m - r$ self-equilibrium force modes, and the force vector $\mathbf{s}$ can be written in terms of the force modes as follows

$$\mathbf{s} = \sum_{i=1}^{q} \alpha_i \mathbf{g}_i = \mathbf{G}\boldsymbol{\alpha} \tag{2}$$

where $\boldsymbol{\alpha}$ is the coefficient vector, and $\mathbf{G}$ is the matrix of the self-equilibrium force modes. Let $\mathbf{b}_i^{\mathrm{T}}$ denote the ith row of $\mathbf{G}$. The force s_i of the ith member can obtained as

$$s_i = \mathbf{b}_i^{T}\, \boldsymbol{\alpha}, \qquad (i = 1, \ldots, m) \tag{3}$$

The tangent stiffness matrix is the sum of the linear stiffness matrix $\mathbf{K}_{\mathrm{E}}$ and the geometrical stiffness matrix $\mathbf{K}_{\mathrm{G}}$

$$\mathbf{K} = \mathbf{K}_{\mathrm{E}} + \mathbf{K}_{\mathrm{G}} \tag{4}$$

where $\mathbf{K}_{\mathrm{E}}$ is determined by the member stiffness as well as configuration, and $\mathbf{K}_{\mathrm{G}}$ is determined by the distribution of prestresses. In the following discussions, we assume that the rigid-body motions are constrained.

Let $\lambda_r (\lambda_1 \le \lambda_2 \le \ldots \le \lambda_{3n})$ and $\boldsymbol{\Phi}_r$ denote the rth eigenvalue and its eigenvector of $\mathbf{K}$, respectively, which have following relation

$$\mathbf{K}\boldsymbol{\Phi}_r = \lambda_r \boldsymbol{\Phi}_r \quad \text{and} \quad \boldsymbol{\Phi}_r^{\mathrm{T}} \boldsymbol{\Phi}_s = \delta_{rs}, \qquad (r, s = 1, \ldots, 3n) \tag{5}$$

where δ_{rs} is the Kronecker delta. Since the magnitude of λ_r represents the stiffness of the structure in the direction of $\boldsymbol{\Phi}_r$, when the external loads applied to the structure are unknown a priori, the best way to strengthen the structure might be to increase its stiffness in the weakest direction. Hence, the lowest eigenvlaue $\lambda_{\min}$ after constraining the rigid-body motions is to be maximized, as the performance measure in the optimization problem defined in the next section.

3. Multiobjective Hybrid Optimization-antioptimization Problem

This section presents the formulation of a multiobjective hybrid optimization-antioptimization problem, to find the optimal structure taking consideration of uncertainties in force distribution.

The upper and lower bounds for the forces of the ith member are denoted by s_i^{U} and s_i^{L}, respectively. Then the conditions for the member forces are written as

$$s_i^{\mathrm{L}} \le s_i \le s_i^{\mathrm{U}}, \qquad (i = 1, \ldots, m) \tag{6}$$

For a cable, s_i^{U} is given as the yield force divided by the associated safety factor, while a small positive value s_i^{L} is given for preventing slackening. By contrast, for a strut, s_i^{L} is given as the Euler buckling force divided by the safety factor, while s_i^{U} may be zero or a negative value with sufficiently small absolute value.

In the process of force design, the geometry (nodal locations) and the topology of the structure are assumed to be specified a priori. Therefore, the design variables are the coefficients $\boldsymbol{\alpha}$ for the self-equilibrium modes. By using the relation (3), the constraints for the optimization problems are given with respect to $\boldsymbol{\alpha}$ as

$$s_i^{\mathrm{L}} \le \mathbf{b}_i^{\mathrm{T}}\, \boldsymbol{\alpha} \le s_i^{\mathrm{U}}, \qquad (i = 1, \ldots, m) \tag{7}$$

Let $\mathbf{s}^*$ denote the most desired values of the member forces based on the material properties and the intuition by the designer. In the following anti-optimization problem, the deviation e of the forces from $\mathbf{s}^*$ is used as one of the performance measures, which is defined as follows as a convex function of $\boldsymbol{\alpha}$:

$$e = (\mathbf{s}^* - \mathbf{G}\ \boldsymbol{\alpha}\ )^{\mathrm{T}} (\mathbf{s}^* - \mathbf{G}\ \boldsymbol{\alpha}\ ) = \mathbf{s}^{*\mathrm{T}}\mathbf{s}^* - 2\mathbf{s}^{*\mathrm{T}}\mathbf{G}\ \boldsymbol{\alpha} + \boldsymbol{\alpha}^{\mathrm{T}}\ \mathbf{G}^{\mathrm{T}}\mathbf{G}\ \boldsymbol{\alpha} \tag{8}$$

Furthermore, the lowest eigenvalue $\lambda_{\min}$ of $\mathbf{K}$ after constraining the rigid-body motions is maximized, i.e., $-\lambda_{\min}$ is minimized, as the global measure of stiffness and stability of the structure. Note that $\mathbf{K}_{\mathrm{G}}$ depends on $\boldsymbol{\alpha}$, while $\mathbf{K}_{\mathrm{E}}$ is independent of $\boldsymbol{\alpha}$, and $\lambda_{\min}$ is a concave function of $\boldsymbol{\alpha}$ [21]. Since it is not generally possible to find an optimal solution that minimizes the two objective functions simultaneously, a compromise solution is selected as a solution to an MOP problem. A feasible solution satisfying all the constraints is called Pareto optimal solution, if there exists no feasible solution in its neighborhood that simultaneously improves the two objective functions [12].

The optimization problem is formulated as

$$\begin{aligned} &\text{Minimize} \quad -\lambda_{\min}(\boldsymbol{\alpha}) \quad \text{and} \quad e(\boldsymbol{\alpha}) \\ &\text{subject to} \quad s_i^{\mathrm{L}} \le \mathbf{b}_i^{\mathrm{T}}\ \boldsymbol{\alpha} \le s_i^{\mathrm{U}}, \quad (i = 1, \dots, m) \end{aligned} \tag{9}$$

Consider uncertainty in member forces at the self-equilibrium state due to the errors in unstressed (initial) lengths of members or change in member lengths after construction resulting from relaxation of high-tensioned cables. Since the member force vector $\mathbf{s}$ has to satisfy the self-equilibrium equation (1), the errors in $\mathbf{s}$ cannot be distributed independently. Note that the variation in nodal locations due to the error in member forces is assumed to be very small in the following; i.e., the equilibrium matrix $\mathbf{D}$ is fixed. Therefore, the vectors $\mathbf{g}_i$ of the self-equilibrium force modes are fixed, and the variation of the member forces is investigated in the space of the coefficient vector $\boldsymbol{\alpha}$.

To describe the realistic situation, we assign the range of uncertainty of s as an interval

$$s_i^0 - \Delta s_i \le s_i \le s_i^0 + \Delta s_i, \quad (i = 1, \dots, m) \tag{10}$$

where s_i^0 is the nominal value, and Δs_i is the maximum possible increase or decrease of s_i. Note that the range is given for the forces s_i, although the independent parameters for the forces are $\boldsymbol{\alpha}$. Let $\boldsymbol{\alpha}^0$ denote the value of $\boldsymbol{\alpha}$ corresponding to the nominal force vectors. Then the inequalities (10) can be rewritten using the increment $\Delta\boldsymbol{\alpha}$ from $\boldsymbol{\alpha}$ as

$$-\Delta s_i \le \mathbf{b}_i^{\mathrm{T}}\ \Delta\boldsymbol{\alpha} \le \Delta s_i, \quad (i = 1, \dots, m) \tag{11}$$

Then the worst values are obtained by solving the following antioptimization problems:

$$\begin{aligned} &\text{Find} \quad \bar{\lambda}_{\min}\ (\boldsymbol{\alpha}) = \min_{\Delta\boldsymbol{\alpha}}\ \lambda_{\min}\ (\boldsymbol{\alpha},\ \Delta\boldsymbol{\alpha}) \\ &\text{subject to} \quad -\Delta s_i \le \mathbf{b}_i^{\mathrm{T}}\ \Delta\boldsymbol{\alpha} \le \Delta s_i, \quad (i = 1, \dots, m) \end{aligned} \tag{12}$$

$$\begin{aligned} &\text{Find} \quad \bar{e}\ (\boldsymbol{\alpha}) = \min_{\Delta\boldsymbol{\alpha}}\ e\ (\boldsymbol{\alpha},\ \Delta\boldsymbol{\alpha}) \\ &\text{subject to} \quad -\Delta s_i \le \mathbf{b}_i^{\mathrm{T}}\ \Delta\boldsymbol{\alpha} \le \Delta s_i, \quad (i = 1, \dots, m) \end{aligned} \tag{13}$$

Finally, the multiobjective optimization problem considering the worst values of the performance measures is formulated as

$$\begin{aligned} &\text{Minimize} \quad -\bar{\lambda}_{\min}(\boldsymbol{\alpha}) \quad \text{and} \quad \bar{e}(\boldsymbol{\alpha}) \\ &\text{subject to} \quad -\Delta s_i \le \mathbf{b}_i^{\mathrm{T}}\ \Delta\boldsymbol{\alpha} \le \Delta s_i, \quad (i = 1, \dots, m) \end{aligned} \tag{14}$$

It is known that the constraint approach is superior to the linear-weighted-sum approach, because the constraint approach can find Pareto optimal solutions even for a nonconvex feasible region in the objective function space. However, the location of vertex that has the worst value of an objective function in the antioptimization problem may vary discontinuously with variation of the design variables $\boldsymbol{\alpha}$, and it is very difficult to satisfy the constraints strictly for a problem with discontinuous sensitivity coefficients. Therefore, the linear-weighted-sum approach is used in the following examples. Optimal solutions are first found for single-objective problems for minimizing $-\bar{\lambda}_{\min}(\boldsymbol{\alpha})$ and $\bar{e}(\boldsymbol{\alpha})$, respectively. Then, the objective function $F(\boldsymbol{\alpha})$ for the linear-weighted-sum approach is given as

$$F(\boldsymbol{\alpha}) = -\frac{C}{\delta_\lambda}\bar{\lambda}_{\min}\ (\boldsymbol{\alpha}) + \frac{1-C}{\delta_e}\bar{e}\ (\boldsymbol{\alpha}) \tag{15}$$

where δ_λ and δ_e are the range of $\bar{\lambda}_{\min}$ and $\bar{e}$,respectively, obtained from the optimal values of the single-objective problems, and $0 \le C \le 1$ is the weight coefficient.

Since $\lambda_{\min}$ and e are concave and convex functions, respectively, the worst values of $\Delta\boldsymbol{\alpha}$ that minimizes $\lambda_{\min}$ and maximizes e exist at the vertices of the region of uncertainty defined by (11). In the numerical example in the next section, the vertices of the feasible region of the optimization and antioptimization problems are enumerated using the software cdd+ [22, 23] based on the efficient procedure called *reverse search* [24]. cdd+ can enumerate the vertices and the associated active constraints of the region defined by linear inequality and equality constraints. Then the antioptimal solution is found by vertex enumeration of the feasible region as follows:

Step 1: Specify the geometry, topology, material property of the tensegrity structure, the upper bound s_i^{U} and lower bound s_i^{L} of the forces, and the radius Δs_i of the uncertainty.

Step 2: Construct the equilibrium matrix **D** and compute the self-equilibrium force modes $\mathbf{g}_i$.

Step 3: Assign bound constraints (6) for the self-equilibrium forces and solve the optimization problem of forces using a nonlinear programming as follows:

3.1 Set initial values for $\boldsymbol{\alpha}$, and assign parameters for NLP.

3.2 Compute the sensitivity coefficients of the objective functions using a finite difference approach, and update the variable $\boldsymbol{\alpha}$ based on the algorithm of NLP, where the worst values of $\lambda_{\min}$ and e are obtained using vertex enumeration at every trial that results in modification of variables.

3.3 Go to 3.2 if convergence criteria of nonlinear programming are not satisfied.

We use SNOPT Ver. 7.2 [25] that utilizes the sequential quadratic programming for optimization of the upper-level problem. In the following examples, optimization is carried out from ten different random initial solutions, and the best solution is chosen as the optimal solution.

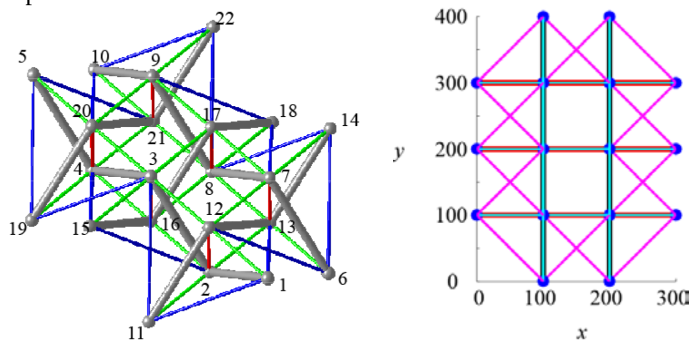
4. Numerical Examples

The tensegrity grid [26], as shown in Figure 1, is used as the example structure for demonstrating the effectiveness of the proposed method. The structure is constructed by consecutively assembling the unit cell as shown in Figure 2 in *x*- and *y*-directions. The thick and thin lines in the figures are struts and cables (or bars), respectively. Note that the members in thin lines that are connected to the boundary nodes do not carry any prestress at the self-equilibrium state; these members are called bars and assumed to have stiffness in both of compression and tension in the structural analysis.

Let r and c denote the numbers of rows (parallel to *x*-axis) and columns (parallel to *y*-axis) of the struts, respectively; i.e., there exist $r + 1$ struts in each column and $c + 1$ struts in each row. Therefore, the structure has $2rc + r + c$ struts and $n = 2(rc + r + c)$ nodes, and the total number of members is $m = 7rc + 5r + 5c - 4$. The rank deficiency of the linear stiffness matrix $\mathbf{K}_{\mathrm{E}}$ after constraining the six rigid-body motions is equal to 1; i.e., this structures has only one infinitesimal mechanism.

The structure in Figure 1 has three and four struts in *x*- and *y*-directions, respectively; i.e., $r = 3$ and $c = 2$. Hence, there are $m = 63$ members and $n = 22$ nodes in total. The *x*- and *y*-coordinates (mm) of the nodes are shown in the plan view of the structure in Figure 1(b), and the height of the grid is 100 mm.

The rank of the equilibrium matrix **D** is $r = 59$. Therefore, the structure has four ($q = 63 - 59 = 4$) force modes at the self-equilibrium state, which are denoted by $\mathbf{g}_1,\ldots,\mathbf{g}_4$ with the coefficients $(s_i^{\mathrm{L}}, s_i^{\mathrm{U}})$. The bounds $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_4)^{\mathrm{T}}$ for the axial forces of the cables and struts are (1, 100) and (-100, -1), respectively. We find the Pareto solutions with uncertainty of radius $\Delta s_i = 0.5$ in the axial forces of all members. Note that no bound or uncertainty is given for horizontal bars that have vanishing axial force irrespective of the value of $\boldsymbol{\alpha}$. The units of length and force are mm and N, respectively, also for this example.



(a) Diagonal view (b) Plan view

Figure 1. Tensegrity grid constructed by assembling the unit cell shown in Figure 2 in *x*- and *y*-directions.

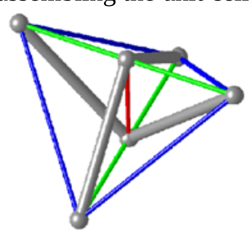


Figure 2. Unit cell of the tensegrity grid in Figure 1.

The elastic modulus of all members is 2000.0 N/mm^2. The cross-sectional areas are 50 mm^2 for struts including the bars without prestress, and 5 mm^2 for cables. The lowest eigenvalue $\lambda_{\min}$ of **K** is positive at all the vertices after constraining the rigid body motions; i.e., the structure is stable at any set of self-equilibrium forces in the feasible region. The center of the feasible region is computed as $\boldsymbol{\alpha}^* = (0.4198, 0.5832, 3.2567, 0.4294)^{\mathrm{T}}$, which is supposed to be the coefficient vector corresponding to the target axial forces $\mathbf{s}^*$.

The values of $(\lambda_{\min}, e)$ for the single objective problem for maximizing $\lambda_{\min}$ and minimizing e, respectively, are $(0.07701, 1.5160\times10^4)$ and $(0.05461, 1.2723\times10^4)$. Therefore the ranges δ_λ and δ_e are defined as $\delta_\lambda = 0.07701 - 0.05461 = 0.02240$ and $\delta_e = 1.5160\times10^4 - 1.2723\times10^4 = 0.2437\times10^4$.

The Pareto optimal solutions are found for the weight coefficient C = 0.1, 0.2, …, 0.9 in (15), which are plotted in Figure 3 in the objective function space. In order to see the distribution more clearly, the relation between the minimum eigenvalue and square-root of force deviation of Pareto optimal solutions is plotted in Figure 4. As is seen, the Pareto solutions that form a convex curve in the objective function space have been successfully found using the proposed method.

5. Conclusions

A hybrid approach of multiobjective optimization and antioptimization has been presented for force design of tensegrity structures. The design variables are the coefficients of the self-equilibrium force vectors. The objective functions are the lowest eigenvalue of the tangent stiffness matrix and the deviation of forces from the specified target distribution, which are defined as the worst values due to the possible uncertainties in the variables.

The upper-level problem of optimization is solved using a nonlinear programming approach, where the sensitivity coefficients are computed by a finite difference approach. The lower-level problems for finding the worst values of the objective functions are found using the enumeration of the vertices of the uncertain region of the prestresses, which is defined with linear inequalities of the variables. It has been shown in the numerical examples that Pareto optimal solutions can be successfully found for tensegrity structures by solving the two-level optimization-antioptimization problem using vertex enumeration combined with a nonlinear programming approach.

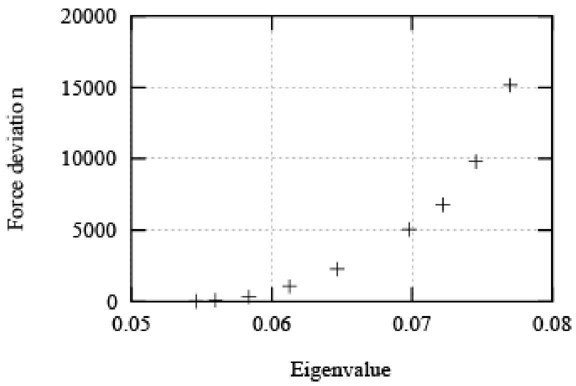


Figure 3. Pareto optimal solutions.

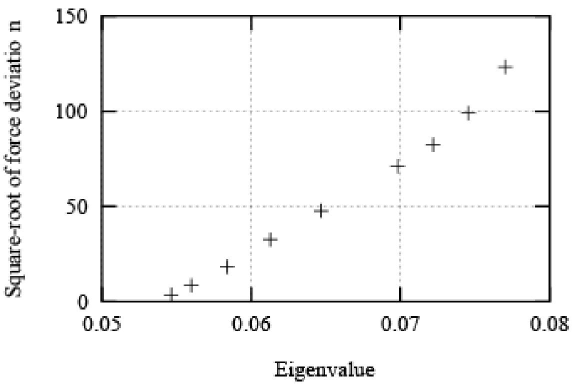


Figure 4. Minimum eigenvalues and square-root of force deviations.

References

1. R. Motro, Tensegrity systems: the state of the art, Int. J. Space Structures. Vol. 7, No. 2, pp. 75–83, 1992.
2. R. Connelly, Tensegrity Structures: Why are they Stable?, Rigidity Theory and Applications, edited by Thorpe and Duxbury, Kluwer/Plenum Publishers, pp. 47–54, 1999.
3. A. G. Tibert and S. Pellegrino, Review of form-finding methods for tensegrity structures, Int. J. Space Structures, Vol.18, No.4, pp. 209–223, 2003.
4. J. Y. Zhang and M. Ohsaki, Optimization methods for force and shape design of tensegrity structures, Proc. 7th World Congress on Structural and Multidisciplinary Optimization, Seoul, Korea, 2007.
5. M. Ohsaki and J. Y. Zhang, Stability conditions of prestressed pin-jointed structures, Int. J. Non-Linear Mechanics, Vol. 41, pp. 1109–1117, 2006.
6. J. Y. Zhang and M. Ohsaki, Adaptive force density method for form-finding problem of tensegrity structures. Int. J. Solids Struct., Vol. 43, pp. 5658–5673, 2006.
7. J. Y. Zhang and M. Ohsaki, Topology and shape of tensegrity structures, in: Proc. 6th Int. Conf. on Computation of Shell and Spatial Structures, Ithaca, IASS-IACM, 2008.

8. Y. Ben-Haim and I. Elishakoff, Convex Models of Uncertainty in Applied Mechanics, Elsevier, Amsterdam, Netherlands, 1990.
9. I. Elishakoff and M. Ohsaki, Optimization and Anti-Optimization of Structures under Uncertainty, Imperial College Press, in press.
10. M. Ohsaki, J. Y. Zhang and Y. Ohishi, Force design of tensegrity structures by enumeration of vertices of feasible region, Int. J. Space Struct., Vol. 23(2), pp. 117–126, 2008.
11. M. Ohsaki, J. Y. Zhang and I. Elishakoff, Optimization and anti-optimization of forces in tensegrity structures, in: Proc. IASS Symposium 2008, Int. Assoc. for Shell and Spatial Struct., Acapulco, Mexico, 2008.
12. J. L. Cohon, Multiobjective Programming and Planning, Vol. 140, Mathematics in Science and Engineering, Academic Press, 1978.
13. F. Massa, B. Lallemand and T. Tison, Fuzzy multiobjective optimization of mechanical structures, Comp. Meth. Appl. Mech. Engng., Vol. 198, pp. 631–643, 2009.
14. F. Tonon, Multiobjective optimization of uncertain structures through fuzzy set and random theory, Computer-Aided Civil and Infrastructure Eng., Vol. 14, pp. 119–140, 1999.
15. S. S. Rao, Multiobjective optimization of fuzzy structural systems, Int. J. Numer. Meth. Eng., Vol. 24, pp. 1157–1172, 1987.
16. J. Loetamonphong, S.-C. Fang and R. E. Young, Multi-objective optimization problems with fuzzy relation equation constraints, Fuzzy Sets and Systems, Vol. 127, pp. 141–164, 2002.
17. F. Tonon, A. Mammino and A. Bernardinic, Multiobjective optimization under uncertainty in tunneling: application to the design of tunnel support reinforcement with case histories, Tunnelling and Underground Space Technology, Vol. 17, pp. 33–54, 2002.
18. M. J. Bender and S. P. Simonovic, A fuzzy compromise approach to water resource systems planning under uncertainty, Fuzzy Sets and Systems, Vol. 115, pp. 35–44, 2000.
19. H. J. Schek , The force density method for form finding and computation of general networks, Comp. Meth. Appl. Mech. Engng., Vol. 3, pp. 115–134, 1974.
20. R. A. Horn and C.R. Johnson, Matrix Analysis. Cambridge University Press (Reprint edition), 1990.
21. S. Boyd and I. Vandenberghe, Convex Optimization, Cambridge University Press, Cambridge, UK, 2004.
22. K. Fukuda, cdd+ Ver. 0.76 User’s Manual, Inst. Operation Res., ETH-Zentrum, Zurich, Switzerland, 1999.
23. D. Avis and K. Fukuda, A pivoting algorithm for convex hulls and vertex enumeration of arrangements and polyhedra, Discrete Comput. Geom., Vol. 8, pp.295–313, 1992.
24. D. Avis and K. Fukuda, Reverse search for enumeration. Discrete Applied Math., Vol. 65(1–3), pp. 21–46, 1996.
25. P. E. Gill, W. Murray and M. A. Saunders, SNOPT: An SQP algorithm for large-scale constrained optimization. SIAM J. Optim., Vol. 12, pp. 979–1006, 2002.
26. R. Motro, Tensegrity Structural Systems for the Future, Butterworth-Heinemann, 2003.