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Local Search for Multiobjective Optimization of Steel Frames

Makoto Ohsaki[1]

[1]Kyoto University, Kyoto, Japan, ohsaki@archi.kyoto-u.ac.jp

Abstract
A single-point local search method is presented as a simplification of the multiobjective tabu search. Some improvements are made to reach the Pareto front within small number of function evaluations. The performance of the proposed method is first verified by a small mathematical problem. It is shown that accurate Pareto optimal solutions with good diversity are obtained by using the proposed method. Pareto optimal solutions are next found for a 5-story 4-span steel building frame. The objective functions are the total structural volume and the compliance under the specified set of loads. It is shown that about 30 Pareto solutions with good accuracy can be found by carrying out structural analysis 200 times. The proposed method has very few problem-dependent parameters. Therefore, the method can be applied to multiobjective optimization of large structures, for which the population-based method is difficult to be applied.

Keywords: multiobjective optimization, heuristic method, structural optimization, local search, compliance

1. Introduction

Optimal designs of moderately large structures can be obtained if the objective function and the constraints are continuous functions of the design variables. In the fields of civil engineering and architectural engineering, however, the cross-sectional properties of the frames are usually selected from the lists or catalogs of the standard sections. Therefore, the optimization problems are formulated as a combinatorial optimization problem with continuous state variables.

It is very easy to solve a combinatorial optimization problem if the number of variables is small. However, the computational cost increases as an exponential function of the problem size, and it is not possible to solve a practical problem within a practically admissible computational time. However, in the practical design process, it may be enough to obtain an approximate optimal design. Heuristic approaches have been developed to obtain approximate optimal solutions within reasonable computation time, although there is no theoretical proof of convergence [1]. The most popular approach is the genetic algorithm, which can be categorized as a multipoint search or population-based method that has many solutions at each iterative step called generation. Since computational cost for evaluating the objective and/or constraint functions at each step may be very large for structural optimization problems, a multipoint strategy may not be appropriate especially for of large structures. Therefore, single-point search heuristics such as simulated annealing [2] and tabu search [3] are preferred.

Another aspect of optimization of the frame structures is that it usually has multiple performance measures as objective functions, and the design problem should be formulated as a multiobjective programming problem [4], for which several single-point search methods have been developed [5-8]. In this study, local search methods are presented for combinatorial multiobjective structural optimization. It is shown in the numerical examples that approximate Pareto optimal solutions with good accuracy and diversity can be easily found by using various criteria for selecting the seed solution.

2. Multiobjective Combinatorial Optimization Problem of a Frame Structure

Suppose a list of available standard sections is given for an optimization problem of frame structures. Let J_i denote an integer variable for the i th member. $J_i = j \ \ (i = 1, 2, \dots, m)$ indicates that j th section is assigned to the i th member, where m is the number of members. The constraints, if exist, are assumed to be incorporated to the objective function using the standard approach of penalty method.

Let $F_i(\mathbf{J})$ denote the i th objective function, which is a function of $\mathbf{J} = (J_1, \dots, J_m)$. Let p denote the number of objective functions. The multiobjective optimization problem to minimize the objective functions $F_1(\mathbf{J}), \dots, F_p(\mathbf{J})$ is formulated as

$$\text{Minimize} \quad F_1(\mathbf{J}), F_2(\mathbf{J}), \dots, F_p(\mathbf{J}) \tag{1}$$

$$\text{subject to} \quad J_i \in \{1, 2, \dots, r_i\}, \quad (i = 1, 2, \dots, m) \tag{2}$$

where r_i is the number of standard cross-sections for the i th member.

3. Local Search for Multiobjective Combinatorial Optimization Problem

The basic approach of local search is presented as follows as a simplification of multiobjective tabu search by Baykasoglu *et al*. [6]:

Step 1: Randomly generate initial solution $\mathbf{J}^{(0)}$, which is selected as the seed solution $\mathbf{J}^*$. The Pareto list $\mathcal{P}$ and candidate list $\mathcal{C}$ are initialized as $\mathcal{P} = \mathcal{C} = \{\mathbf{J}^{(0)}\}$.

Step 2: Select q neighborhood solutions $\mathcal{N} = \{\mathbf{J}_j^{\mathrm{N}} \mid j = 1, \cdots, q\}$ from $\mathbf{J}^*$. The candidate set $\mathcal{S}$ is defined with the solutions in $\mathcal{N}$ that are not dominated by any solution in $\mathcal{N}$, $\mathcal{P}$ and $\mathcal{C}$.

Step 3: Randomly select the solution $\mathbf{J}^*$ from $\mathcal{S}$. If $\mathcal{S}$ is empty, assign the oldest solution in $\mathcal{C}$ to $\mathbf{J}^*$.

Step 4: Remove the solutions in $\mathcal{P}$ and $\mathcal{C}$, which are dominated by a solution in $\mathcal{S}$.

Step 5: Add $\mathbf{J}^*$ to $\mathcal{P}$, and other candidate solutions to $\mathcal{C}$.

Step 6: Stop if $\mathcal{C}$ is empty and there exists no new candidate solution, or if the number of steps reaches the prescribed limit; otherwise, go to step 2.

4. Selection of Seed Solution

The seed solution should be selected in Step 3 of each iteration so that the Pareto set should have enough accuracy and diversity. Therefore, following five techniques are proposed to generate such Pareto optimal set by the single-point local search. Note that the objective function F_i is normalized by the scaling factor D_i as

$$F_i^* = F_i / D_i\,, \quad (i = 1, ..., p) \tag{3}$$

Method 1: Randomly select a seed solution from the set $\mathcal{S}$, which is the basic method presented in the previous section.

Method 2: Define the congestion N_i in the neighborhood of J_i in $\mathcal{S}$ as

$$N_i = \sum_{\mathbf{J}_j \in \mathcal{P}} s(d(\mathbf{J}_i, \mathbf{J}_j)) \tag{4}$$

where $d(\mathbf{J}_i, \mathbf{J}_j)$ is the distance between the solutions $\mathbf{J}_i$ and $\mathbf{J}_j$, and the *sharing function* $s(d)$ is defined as

$$s(d) = \max\,(0,\, 1-d/\sigma) \tag{5}$$

with the prescribed *sharing radius*, or *niche size*, σ. Then select the solution in $\mathcal{S}$ with smallest N_i as the seed solution.

Method 3: Let $\mathbf{F}^*$ and $\mathbf{F}^{*(i)}$ denote the vectors of the objective functions of the current seed solution and the ith solution in $\mathcal{S}$, respectively. Then select the solution with smallest value of $\sum_{j=1}^{p}(F_j^{*(i)} - F_j)$, which is the sum of increase of the objective functions.

Method 4: Select the solution with smallest value of increase $F_j^{*(i)} - F_j$ for the specified objective function *Fj*.

5. Performance Measures of Pareto Optimal Solution

Let $\mathcal{A}$ denote the exact set of Pareto solutions obtained, e.g., by enumeration. The size of the approximate Pareto optimal set $\mathcal{P}$ obtained by one of the methods presented in the previous section is denoted by n. The performance of the Pareto optimal set is evaluated by the following three measures used in Coello et al. [9,10]:

Error ratio: If the ith approximate Pareto solution in $\mathcal{P}$ is not included in $\mathcal{A}$, then $e_i = 0$, otherwise, $e_i = 1$. Define the error ratio ER as

$$\mathrm{ER} = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{6}$$

The approximate Pareto set is subset of $\mathcal{A}$ if ER = 0.

Generational distance: Let d_i be the minimum distance from the ith approximate solution to a solution in $\mathcal{A}$. The generational distance GD is defined as

$$\mathrm{GD} = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(d_i)^2} \tag{7}$$

The approximate Pareto set is a subset of $\mathcal{A}$ if GD = 0.

Spacing: The minimum Manhattan distance g_i from the ith approximate solution to a solution in $\mathcal{A}$ as

$$g_i = \min_j \left(\sum_{k=1}^{p} \left| F_k^{*(i)} - F_k^{*(j)} \right| \right) \tag{8}$$

where $F_k^{*(i)}$ and tilde $F_k^{*(j)}$ are the kth objective functions of the ith solution in $\mathcal{A}$ and the jth solution in $\mathcal{P}$, respectively. Let $\bar{g}$ denote the mean value of g_i. The spacing S is defined as

$$\mathrm{S} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\bar{g} - g_i)^2} \tag{9}$$

If the approximate solutions are uniformly spaced, then S = 0.

In addition to these performance measures, the span of the Pareto set is defined as
Span:

$$\mathrm{W} = \max_{i,j} \left(\sum_{k=1}^{p} \left| F_k^{*(i)} - F_k^{*(j)} \right| \right) \tag{10}$$

The solution with wide diversity is found if W is sufficiently large.

6. Numerical Examples

6.1. Mathematical Example

The performance of local search is first investigated by a small mathematical problem. Consider the following problem used in Coello *et al*. [9,10]:

$$\text{Minimize} \quad F_1(\mathbf{x}) = 200(2x_1 + \sqrt{2x_2} + \sqrt{x_3} + x_4), \quad F_2(\mathbf{x}) = \frac{1}{100}\left(\frac{2+\sqrt{2}}{x_2} - \frac{\sqrt{2}}{x_3} + \frac{2}{x_4} \right) \tag{11}$$

$$\text{subject to} \quad 1 \le x_1 \le 3, \quad \sqrt{2} \le x_2 \le 3, \quad \sqrt{2} \le x_3 \le 3, \quad 1 \le x_4 \le 3 \tag{12}$$

The real variable x_i is discretized to an integer variable J_i as

$$x_i\ (J_i) = x_i^{\mathrm{L}} + \Delta x_i \times J_i, \quad \Delta x_i = (x_i^{\mathrm{U}} - x_i^{\mathrm{L}})/(r_i - 1) \tag{13}$$

where x_i^{U} and x_i^{L} are the upper and lower bounds of x_i, respectively, and r_i is equal to $2^{14} = 16384$ for x_1 and x_4, and $2^{13} = 8192$ for x_2 and x_3.

In the following figures, the solid lines are the exact Pareto front obtained by enumeration. Note that F_1 is an increasing function of x_1, and F_2 does not depend on x_1. Furthermore, F_1 and F_2 are increasing functions of x_3. Therefore, x_1 and x_3 can be fixed at their lower bounds in the process of enumeration. Hence, the total number of combination is $2^{13} \times 2^{13} = 67{,}108{,}864$.

Neighborhood solutions are generated using the random numbers with normal distribution. The mean values are 0, and the standard deviation is 800 for J_1 and J_4, and 400 for J_2 and J_3. If the value of J_i generated by the random number turns out to be less than 1 or greater than r_i, then J_i is replaced by 1 or r_i, respectively. Note that all variables are simultaneously modified in generating the neighborhood solutions, and the scaling factors are $D_1 = 600$, $D_2 = 0.03$.

The 100 neighborhood solutions generated from a feasible solution indicated by ■ are plotted in Fig. 1. As is seen, the neighborhood solutions are symmetrically spaced around the feasible solution in the objective function space. Therefore, it is expected that the Method 1 of randomly selecting the seed solution can be used to rapidly reach the Pareto front.

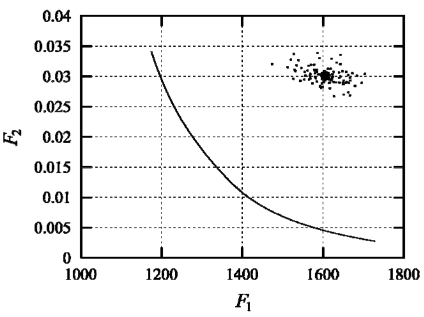


Figure 1. Neighborhood solutions for the mathematical example.

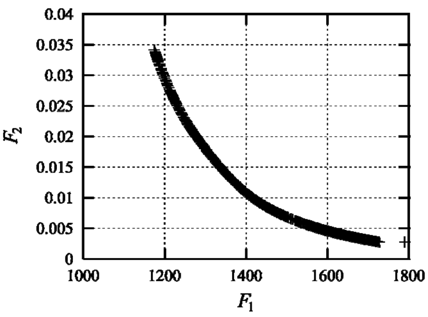


Figure 2. Pareto optimal solutions obtained by 8000 analyses; Method 1, maximum W.

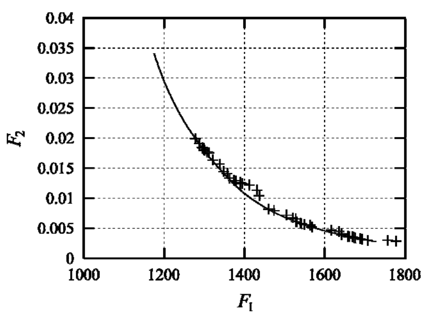


Figure 3. Pareto optimal solutions obtained by 400 analyses; Method 1, maximum W.

The optimization results with function evaluations of 8000, 2000 and 400 times, indicated by Cases 8000, 2000 and 400, are shown in Table 1. The size q of the neighborhood is 10 for Cases 8000 and 2000, and 4 for Case 400; i.e. the numbers of update of seed solution are 800, 200 and 100, respectively. Method 1 is used for selection of seed solution. The Pareto optimal solutions are generated from 30 different initial random numbers for each cases, and their minimum, maximum, mean and standard deviation are computed.

The minimum, maximum, mean and standard deviation of ER in Ref. [10] by 8000 analyses of particle swarm optimization are 0.07, 0.35, 0.23 and 0.063, respectively. Therefore, the results of local search are not better than those by the particle swarm optimization. However, the values of GD and SR by 2000 analyses are less than 1/1000 and 1/100, respectively, which are very small. Hence, very good performance is observed for local search with small number of function evaluations.

The '+' marks in Fig. 2 are the approximate Pareto solutions of Case 8000 by Method 1 with maximum W. Note that the Pareto solutions with good accuracy can be obtained by Case 8000 irrespective of the initial random seed. The results of Case 400 with Method 1 with maximum W are shown in Fig. 3. As is seen from Table 1, the four measures of Case 400 are worse than those of Case 2000. However, it is observed from Fig. 3 that many diverse Pareto approximate solutions with good accuracy have been found within 400 function evaluations.

Table 1. Performances of local dearch with different function evaluation times.

Case 8000				
	ER	GD ($\times 10^{-3}$)	SP ($\times 10^{-2}$)	W
Minimum	0.139	0.011	0.111	1.792
Maximum	0.574	0.360	0.506	2.166
Mean	0.399	0.070	0.275	1.971
Std. Dev.	0.102	0.072	0.104	0.100
Case 2000				
	ER	GD ($\times 10^{-3}$)	SP ($\times 10^{-2}$)	W
Minimum	0.162	0.075	0.321	1.270
Maximum	0.709	7.035	1.838	2.062
Mean	0.440	0.510	0.666	1.739
Std. Dev.	0.153	1.241	0.328	0.265
Case 400				
	ER	GD ($\times 10^{-3}$)	SP ($\times 10^{-2}$)	W
Minimum	0.296	0.528	0.536	0.797
Maximum	1.000	43.61	3.644	1.721
Mean	0.737	5.255	1.764	1.336
Std. Dev.	0.194	9.453	0.665	0.214

6.2. Plane Frame Model

Consider a 5-story 4-span plane frame as shown in Fig. 4 subjected to static loads, where the horizontal loads (kN) are (P_1, P_2, P_3, P_4, P_5) = (7.5, 8.4, 10.1, 13.1, 31.5), and the vertical loads are W_1 = 245 kN, W_2 = 343 kN. The elastic modulus is 205.8 kN/mm^2. The members are divided to 15 groups considering symmetry conditions as indicated in Fig. 4. The variables J_i (i=1, …, 15) are selected from the predefined lists of available sections. See Ref. [8] for the details of the lists. Note that the numbers of candidate sections for beams and columns are 9 and 8, respectively.

The objective functions F_1 and F_2 to be minimized are the total structural volume and compliance, respectively. The scaling factors are D_1 = 4.0, D_2 = 30.0. A uniform random number $\tau \in [0,1)$ are generated, and the section is increased or decreased by 1 for $\tau \geq 0.5$ or $\tau < 0.5$, respectively. If the value of J_i generated by the random number turns out to be less than 1 or greater than r_i, then J_i is replaced by 1 or r_i, respectively. All variables are modified when generating a neighborhood solution. The Pareto set is generated from 30 different initial random seeds for each case. The solid line in the following figures shows the Pareto front obtained by 90 times carrying out local search with 5000 analyses.

The results are presented by using the abbreviation based on number of analyses, method number for selection of seed solution, and the performance measure used for selection of the results from the 30 runs; e.g., Case200-3-W indicates that the number of analyses is 200, and the Pareto set with maximum W is selected from the 30 results by Method 3. For Method 4, the objective function to be minimized is also indicated.

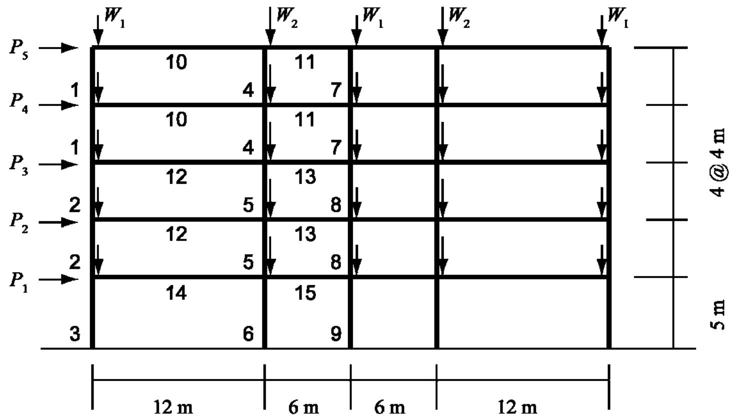


Figure 4. A 5-story 4-span frame.

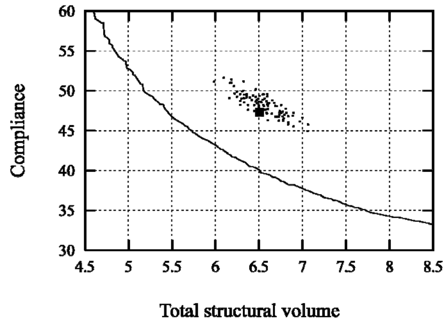


Figure 5. Neighborhood solutions for the 5-story frame.

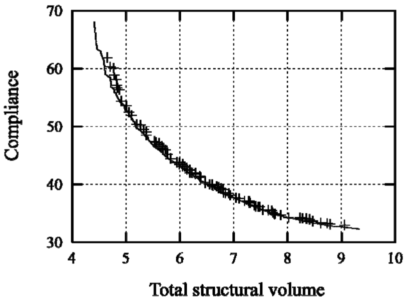


Figure 6. Case-5000-1-W.

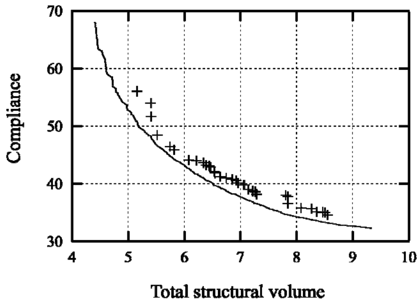


Figure 7. Case-400-1-W.

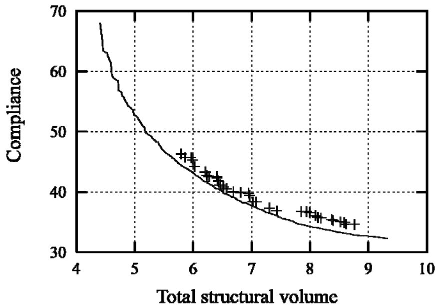


Figure 8. Case-400-1-GD.

Fig. 5 shows the 100 neighborhood solutions generated from a feasible solution indicated by ■. As is seen, there is no solution that improves both of the objective functions; therefore, it is difficult to reach the Pareto front for this problem. The result of Case-5000-1-W is shown in Fig. 6, where q = 10. It is observed from Fig. 6 that the solutions with good diversity have been found.

We next investigate the possibility of generating Pareto solutions with small number of analysis in view of application to structural optimization problems. The result of Case-400-1-W is shown in Fig. 12, where q = 4. A good accuracy is observed except in the region of small F_1. Fig. 13 shows the result of Case-400-1-GD. Although the accuracy around the center region has been improved, the solutions with small F_1 could not been obtained.

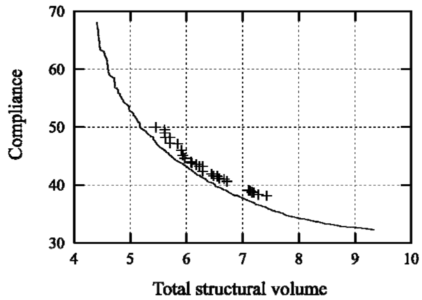


Figure 9. Case-200-1-GD.

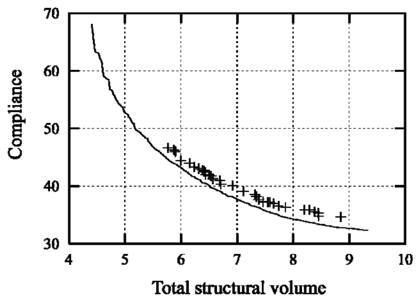


Figure 10. Case-200-3-W.

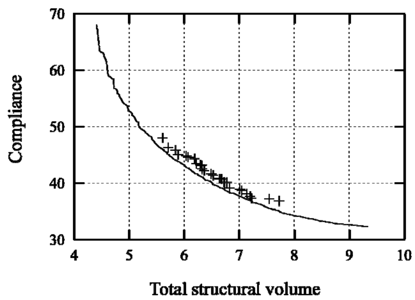


Figure 11. Case-200-3-GD.

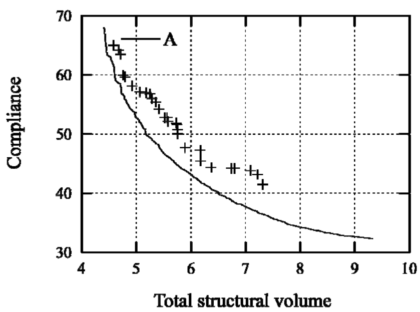


Figure 12. Case-200-4-F1-W.

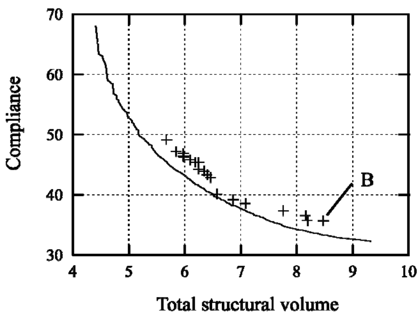


Figure 13. Case-200-4-F2-W.

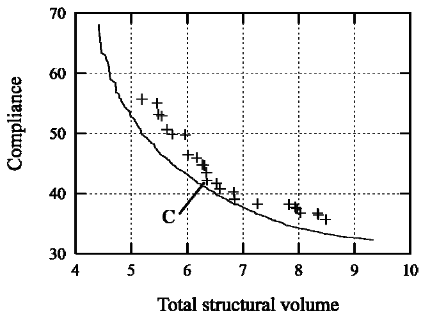
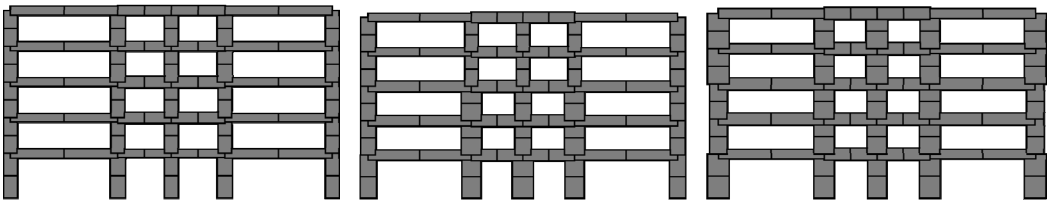


Figure. 14 Case-200-2-W.

The result of Case-200-1-GD is shown in Fig. 9, where $q = 4$. In this case, no exact Pareto solution was found. If q is decreased to 2 to increase the number of update of seed solution from 50 to 100, the process terminated because no candidate solution could be found. The results of Case-200-3-W and Case-200-3-GD are shown in Figs. 10 and 11, respectively. As is seen, it is difficult to improve the accuracy, as observed from Fig. 5, even when the seed solution to most rapidly reach the Pareto front is selected.

Figs. 12 and 13 show the results of Case-200-4-F1-W and Case-200-4-F2-W, which have many solutions with small F_1 and F_2, respectively. Fig. 14 shows the result of Case-200-2-W, where $\sigma = 0.1$. As is seen, the use of shearing function leads to diversity of the solutions, while sacrificing the accuracy. The average number of Pareto solutions is 26.8, which means that the solutions with practically acceptable numbers are generated with 200 analyses. Fig. 15 shows the cross-sectional arras of the solutions corresponding to A, B, and C in Figs. 12-14. As is seen, the cross-sectional areas of the columns are almost uniform for small structural volume. The area of internal column first increases as the structural volume is increased.



(A) (B) (C)

Figure 15. Cross-sectional areas of Pareto solutions.

7. Conclusions

The following conclusions have been drawn from this study:

1. Many Pareto optimal solutions with sufficient accuracy and diversity are obtained by local search with small number of function evaluations. Therefore, the local search is very effective for structural optimization, for which substantial computational cost is needed for function evaluation.
2. Accuracy, diversity and distribution of Pareto solutions in the objective function space can be effectively controlled by using various measures in selection of seed solution from the neighborhood solutions.
3. The effectiveness of the local search can be easily estimated from the distribution of the neighborhood solutions in the objective function space.
4. Since the proposed method has very few problem-dependent parameters, the method can be easily applied to various structural optimization problems.

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