NONLINEAR MULTIMODAL PUSHOVER ANALYSIS METHOD FOR SPATIAL STRUCTURES

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ABSTRACT

An improved nonlinear multimodal pushover analysis method is presented to incorporate contributions of multiple modes and the effects of their interactions to the responses. The key step for the method is to obtain equivalent static seismic force for the important vibration modes. An existing method for building structures based on SRSS combination of the nonlinear modal responses is first extended to spatial structures. A new method is next presented to combine the modal loads before conducting pushover analysis. The proposed method is general and independent of the types of the structures, and does not rely on the properties for building structures. The proposed method is applied to a long span arch structure to show that the nonlinear multimodal pushover method is capable of estimating the seismic response of spatial structures with moderately good accuracy.

INTRODUCTION

Although a spatial structure is commonly designed so that it remains in elastic range even against severe earthquakes, the inelastic responses such as forces and displacements should be computed to simulate its collapse behavior, particularly under the framework of performance-based design. Nonlinear time history analysis is the most rigorous procedure to compute the response. However, current structural engineering practice prefers to use the nonlinear static procedure, e.g., the pushover analysis in EEMA-356 [1]. It is primarily because a nonlinear static analysis is independent of selection of ground motions and requires less effort for modeling than a nonlinear time history analysis.

Conventional nonlinear pushover analysis, which is commonly applied to regular building frames, is carried out using monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target roof displacement is reached. This procedure is based on the assumption that the response is controlled by the elastic fundamental mode and that the mode shape remains unchanged after the structure yields [2]. However, it is not necessarily straightforward to extend the conventional pushover analysis method to spatial structures primarily due to the following reasons: (1) most of the existing methods are strongly dependent on the properties of the regular frame and use base shear and roof displacement; (2) spatial structures commonly have multiple dominant vibration modes, which are all indispensable for determining the structural response; (3) the dominant vibration modes may significantly interact with each other particularly when the responses increase to plastic range [3-4].

Improvements on the conventional pushover analysis method have been conducted recently. The notable two approaches are multimodal pushover analysis [3, 5-8] and adaptive modal pushover analysis [9-10]. The former considers the contribution of the higher modes to the response in addition to that of the fundamental vibration mode, while the latter accounts for the a redistribution of the inertia forces due to structural yielding and the associated changes in the vibration properties of the structure.

In this study, an improved nonlinear multimodal pushover analysis method, which considers contributions of multiple modes and the effects of their interactions to the responses, is presented. The method is applied to estimate the seismic response of a long span arch structure, and its effectiveness is calibrated. Estimation of seismic loads for specific types of structures without eigenvalue analysis may be very effective for practical application [4]. However, we present in this study a more general approach based on elastic eigenvalue analysis in order to develop a unified approach independent of the types of the structures.

LINEAR MULTIMODAL PUSHOVER ANALYSIS

Consider a finite-dimensional spatial structure, subjected to the ground acceleration history $\ddot{u}_{g}(t)$. Let **m**, **c**, and k denote the mass, classical damping, and stiffness matrices, respectively, of the MDOF system. Denote by \mathbf{u} , $\dot{\mathbf{u}}$, and $\ddot{\mathbf{u}}$ the vectors of the displacement, velocity and acceleration, respectively, relative to the ground. Let *i* denote the influence vector, in which the component in the direction of the input ground motion is 1 and the remaining components are 0. The equations of motion of an MDOF system are written as

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{u}\ddot{u}_{g}(t) \tag{1}$$

The right-hand side of Eq. (1) is the effective earthquake forces, which is expressed as a summation of modal inertia force distribution \mathbf{s}_n as [11]:

$$\mathbf{m} = \sum_{n=1}^{N} \mathbf{s}_{n} = \sum_{n=1}^{N} \Gamma_{n} \mathbf{m} \Phi_{n}$$
(2)

where N is the number of degrees of freedom, Φ_n is the nth undamped natural vibration mode, and the participation factor Γ_n is defined by

$$\Gamma_n = \frac{L_n}{M_n}, \quad L_n = \mathbf{\Phi}_n^T \mathbf{m} \mathbf{i}, \quad M_n = \mathbf{\Phi}_n^T \mathbf{m} \mathbf{\Phi}_n \tag{3}$$

Let ω_n and ζ_n denote the *n*th natural circular frequency and damping ratio, respectively. Eq. (1) is ortho-normalized as

$$D_n + 2\zeta_n \omega_n D_n + \omega_n^2 D_n = -\ddot{u}_g(t) \tag{4}$$

Any response quantity of the nth mode $r_{r}(t)$, e.g., nodal displacements, element forces, etc., can be expressed as

$$r_n(t) = r_n^{st} A_n(t) \tag{5}$$

where r_n^{st} is the static response due to external force s_n , called modal static response, and

$$A_n(t) = \omega_n^2 D_n(t) \tag{6}$$

is the pseudo-acceleration response of the SDOF system. The total response r(t) is computed from

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{st} A_n(t)$$
(7)

The peak value r_{n0} of the *n*th mode contribution $r_n(t)$ to response r(t) is determined from

$$r_{n0} = r_n^{st} A_n \tag{8}$$

Where A_n is the maximum absolute value of $A_n(t)$, which can also be obtained from the pseudo-acceleration response spectrum. The peak total responses may be estimated by the SRSS combination rule as

$$r_0 = \sqrt{\sum_{n=1}^{N} r_{n0}^2}$$
 (9)

(10)

Note that other approaches such as CQC rule can be used for more rigorous estimation.

NONLINEAR MULTIMODAL PUSHOVER ANALYSIS METHOD

Based on the derivations in the previous section, the static analysis of the system subjected to seismic loads

 $\mathbf{f}_{n0} = \Gamma_n \mathbf{m} \mathbf{\Phi}_n A_n$ will provide the same value of r_{n0} , the peak value of *n*th mode responses for a linear system; thus \mathbf{f}_{n0} is regarded as the equivalent modal static seismic forces for the nth mode contribution. Therefore, the modal pushover analysis conforming with SRSS rule, called Procedure A, is summarized as [7]

Procedure A:

- Step 1. Conduct eigenvalue analysis, determine the dominant modes, calculate Γ_n using Eq. (3), and obtain A_n from the response spectrum.
- Step 2. For each dominant mode, compute \mathbf{f}_{n0} using Eq. (10), carry out static analysis against \mathbf{f}_{n0} to obtain the *n*th mode contribution r_{n0} of the response.
- Step 3. Estimate the peak total responses r_0 using Eq. (9).

Another approach will be to combine the forces as follows using the weight coefficient α_n before conducting

pushover analysis to obtain the response r^{B} :

$$\mathbf{f}_0 = \sum_{n=1}^N \alpha_n \mathbf{f}_{n0} \tag{11}$$

Let $\chi_k = (\alpha_1, \dots, \alpha_n)$ denote the *k*th set of weight coefficients. If enough number of χ_k are generated and the corresponding responses r_k^B are combined by an appropriate statistical approach, it leads to a good estimation of the peak response. For instance, using the set defined as

$$\alpha_i = 1 \ (k = m) \ , \ \ \alpha_i = 0 \ (k \neq m)$$
 (12)

for the *m*th mode and SRSS rule for the combination results in exactly the same response as the Procedure A. Therefore, Procedure B is presented as

Procedure B:

Step 1. Conduct eigenvalue analysis, determine the dominant modes, calculate Γ_{μ} using Eq. (3), and obtain A_n from the response spectrum.

- For each dominant mode, obtain the equivalent modal static seismic force \mathbf{f}_{n0} using Eq. (10). Step 2.
- Determine several sets of weight coefficients [Eq. (11)]. Step 3.
- For each set χ_k , combine the equivalent modal static seismic force \mathbf{f}_{n0} to obtain the total equivalent Step 4. static seismic force ${}_{k}\mathbf{f}_{0}$ using Eq. (11). Apply ${}_{k}\mathbf{f}_{0}$ to compute the total response r_{k}^{B} . 5. Conduct statistic evaluation on r_{k}^{B} to determine the peak response r_{0} .
- Step 5.

The key ideas for nonlinear multimodal pushover analysis are summarized as

- Although classical modal analysis is not valid for a nonlinear system, the response can be still approximately represented by the combination of the modal responses. The underlying assumptions and the accuracy of such a treatment are presented in [7]. The elastic mode shape is used for \mathbf{f}_{n0} , and the amplitude is determined by the inelastic response spectra of the equivalent SDOF system.
- Procedure A obtains the response for each mode r_n through the pushover analysis, and then combine them to find the total response r_0 .
- The peak response can be obtained by taking a snapshot of every time step in the dynamic response and finding the maximum value among the snapshots for each response quantity. Therefore, the problem for finding the peak response can be transformed to the problem for finding the static load at the time when the peak occurs. Hence, Procedure B combines \mathbf{f}_{n0} to obtain \mathbf{f}_0 before carrying out pushover analysis.

The equivalent normalized SDOF system for the *n*th mode is defined as follows:

- 1. Conduct pushover analysis of the structural system using a force pattern of *n*th mode shape.
- 2. Select the *n*th mode component of the normalized reference displacement as abscissa and the modal force V_{bn} normalized by modal effective mass M_n^* as ordinate to form a pushover force-displacement relationship, which is approximated by a bilinear relation as shown in Fig. 1(a).



Figure 1. Nonlinear pushover method.

The peak value of the displacement response D_n of the equivalent normalized SDOF system [shown in Fig. 1(a)] for the *n*th mode can be estimated from the inelastic response spectra [11], or directly from the time history analysis of SDOF model for a specific earthquake. Based on the linearization method [12-13], the associated peak pseudo acceleration is obtained as [Fig. 1(b)]

$$A_n = \omega_n^2 D_n \frac{1 + \gamma \mu - \gamma}{\mu}$$
(13)

Where μ is the ductility factor and γ is the hardening coefficient.

SPATIAL STRUCTURE MODEL

A long span arch as shown in Fig. 2 is adopted as the example model, which represents one bay of a structure that is commonly adopted for school gymnasiums [14]. The arch consists of a cylindrical roof and support columns. The span is 80 m, and the column height is 5 m. The lower nodes of the arch are located on a circle with radius 80 m and the open angle is 60 degree. The roof and support columns consist of steel pipes. The roof height and column width are both 1/40 of the span. The distance between the arches in the longitudinal direction of the cylinder is taken to be 8 m. The weight of the roof and the external wall, including both structural and nonstructural components, are 0.98 kN/m² and 1.47 kN/m², respectively. The masses are lumped at the associated nodes, whereas the effect of gravity is neglected in this study.



Figure 2. Structural model.

FEDEASLab, a MATLAB tool particularly suitable for research studies and concept development, is used for structural analyses [15]. Modal analysis is first conducted, and the lowest six modes are plotted in Fig. 3. It can be observed that the 1st, 3rd, and 5th modes are antisymmetric, and are excited by horizontal seismic motions.



Figure 3.	Free	vibration	modes
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Table 1. Results of modal analysis						
Mode I	$\mathbf{Period}(\mathbf{s})$	Participation	Effective	Damping		
	r chioù (s)	factor	Mass (kg)	Ratio		
1	1.054	0.766	46940	0.020		
2	0.695	0.000	0	0.017		
3	0.342	0.547	23937	0.020		
4	0.276	0.000	0	0.023		
5	0.180	0.200	3200	0.031		
6	0.142	0.000	0	0.038		

The natural period, participation factor and effective mass are listed in Table 1. The fundamental period is 1.054 sec. The participation factors of the 1st and 3rd modes, which are 0.766 and 0.547, respectively, are much larger than those of the remaining modes. The effective masses of the 1st and 3rd modes are about 60% and 30%, respectively, of the total mass, which is 80000 kg. Therefore, the 1st and 3rd modes are selected as the dominant modes, which are used for the pushover analysis. Rayleigh damping is adopted, and the damping ratios for the 1st and 3rd modes are equal to 0.02.

GROUND MOTIONS

Five records with 10% exceedance in 50 years, LA21, LA28, LA31, LA33, LA35, taken from LA set of SAC FEMA project [16] are adopted as representative ground motions. The displacement response spectra and pseudo acceleration response spectra are given in Figs. 4 (a) and (b), respectively.



(a) displacement response spectra (b) pseudo acceleration response spectra

Figure 4. Response spectra of ground motions.

VERIFICATION OF LINEAR MULTIMODAL PUSHOVER ANALYSIS

Procedures A and B are applied to estimate the seismic response of the arch model. As shown in Fig. 5, the focused responses are two horizontal displacements d_1 and d_2 , two vertical displacements d_3 and d_4 , and the horizontal vertical reaction forces F_1 and F_2 , respectively, at the column base. Time history analyses are also conducted for comparison purpose, where Newark- β method is adopted as integration scheme and the integration time interval is 0.01 sec. In the results presented below, the responses of pushover analyses are all normalized by the total response of the corresponding time history analysis. Therefore, unity indicates that the result obtained from the pushover analysis is accurate.



Figure 5. Focused responses.

First, consider a linear case. The results of Procedure A are presented in Fig. 6. The dashed lines are the results of individual ground motions, and solid lines are the mean values. The response of the 1st and 3rd modes are plotted in Figs. 6(a) and (b), respectively. Note that the 1st mode dominates in horizontal displacements d_1 and d_2 and the horizontal reaction force F_1 , while the 3rd mode contributes about 10% to 20% to vertical displacements d_3 and d_4 , and almost 100% to vertical reaction force F_2 . The results of SRSS combination of the responses of 1st and 3rd modes are shown in Fig. 6(c). It may be noted that Procedure A, which is essentially the same as the conventional SRSS method for linear case, gives very good estimation of the seismic response.



Figure 6. Procedure A for linear cases.

Procedure B, where the equivalent static seismic forces ${}_{k}\mathbf{f}_{0}$ are obtained from the combination of those of the 1st and 3rd modes (\mathbf{f}_{10} and \mathbf{f}_{30}), is also conducted. The combination rule is defined by the following concept:

- For the displacement response, the peak value should be attained around the time when mode 1 takes the maximum, because the response of mode 1 is much larger than that of mode 3, and the natural period of mode 1 is much longer than that of mode 3.
- For the force response, mode 3 has large portion in the total response. Therefore, the peak value should be attained around the time when mode 3 takes the maximum.

Although probabilistic analysis is needed, we adopt the following four types of combination rules:

$$_{1}\mathbf{f}_{0} = \mathbf{f}_{10} + 0.5\mathbf{f}_{30}, \quad _{2}\mathbf{f}_{0} = \mathbf{f}_{10} - 0.5\mathbf{f}_{30}, \quad _{3}\mathbf{f}_{0} = 0.5\mathbf{f}_{10} + \mathbf{f}_{30}, \quad _{4}\mathbf{f}_{0} = 0.5\mathbf{f}_{10} - \mathbf{f}_{30}$$
 (14)

Since the peak values are not simultaneously reached by 1st and 3rd modes, a weight coefficient of 0.5 is adopted. Four pushover analyses using ${}_{1}\mathbf{f}_{0}$, ${}_{2}\mathbf{f}_{0}$, ${}_{3}\mathbf{f}_{0}$ and ${}_{4}\mathbf{f}_{0}$ are conducted, and the results are presented in Figs. 7(a)-(d). The maximum values among the four cases are adopted as the estimated value, which are shown in Fig. 7(e). It may be observed from these results that Procedure B generally results in accurate estimation for linear case, although it slightly overestimates the horizontal force in this example.



VERIFICATION OF NONLINEAR MULTIMODAL PUSHOVER ANALYSIS

The effectiveness the multimodal pushover analyses is investigated for nonlinear cases, where geometrical nonlinearity and the buckling of steel pipe members are neglected. Nonlinear pushover analysis is first conducted as shown in Figs. 8(a) and (b) for 1st and 3rd modes, respectively, to construct the equivalent SDOF systems.



Figure 8. Pushover analysis of SDOF systems.

The horizontal displacement at the top of the arch roof is chosen as the reference displacement. The equivalent static modal forces for the 1st and 3rd modes (\mathbf{f}_{10} and \mathbf{f}_{30}) can be calculated using Eq. (10), in which the magnitude for each modal force A_n is defined by the result of the time-history analysis of the SDOF system. Note that inelastic response spectra can also be used for this purpose.

The results of Procedure A are as shown in Figs. 9, where Fig. 9(a) and (b) are the responses obtained only from the 1st and 3rd modes, respectively, and Fig. 9(c) is the results of SRSS combination of the 1st and 3rd mode responses. It can be observed that Procedure A gives very good estimation of seismic response for nonlinear cases.



Figure 9. Procedure A for nonlinear cases

Procedure B is next applied to nonlinear cases, where the force patterns in Eq. (14) are also used. The results for the four patterns are presented in Figs. 10(a)-(d). The maximum values among the four cases are adopted as the final estimation. It can be observed that Procedure B significantly overestimates the displacements, although it gives good estimation to the reaction forces. The reasons are believed to be as follows, which is also conceived as the future research subjects:

- The combination of equivalent static modal forces may overestimate the equivalent static loads for the snapshot at the peak response.
- Slightly larger forces generate significantly larger displacement responses in nonlinear pushover analyses, which result in larger ductility ratios. Therefore, the magnitude of the forces should be reduced according to the reduction rules of the pseudo-acceleration spectra based on the equivalent damping ratio. However, in the proposed Procedure B, the ductility ratio is calibrated based on the individual equivalent SDOF system, which only considers the contribution from a single vibration mode.



CONCLUSION

A multimodal pushover analysis method has been presented to estimate the seismic response of spatial structures subjected to horizontal ground motions. Major conclusions are summarized as follows:

- 1. In Procedure A, which is an extension of the multimodal pushover analysis for building frames, the response of each mode is first calculated and then combined to obtain the final response using the SRSS rule. This procedure results in good estimation of responses for both linear and nonlinear cases.
- 2. In Procedure B, the modal forces are first combined using multiple rules to define the equivalent static forces. The forces are applied to the structure to obtain the response for each combination, and the maximum value among the multiple combinations is taken as the final response. This procedure results in good estimation of responses for linear case, but overestimates the displacement responses for nonlinear cases.

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