IMPERFECTION SENSITIVITY OF ARCH-TYPE TRUSSES WITH SIMULTANEOUS SNAPTHROUGH AND MEMBER BUCKLING

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ABSTRACT

Imperfection sensitivity properties are investigated for an arch-type truss that has multiple member buckling at a limit point, which is classified as hilltop branching with multiple symmetric bifurcation points. The critical loads of imperfect structures are shown to be governed by a piecewise linear law of the imperfection parameter. Anti-optimization problems are formulated to obtain the worst imperfection modes that most drastically reduce the critical load. A formula for the worst nodal imperfection as a linear combination of the critical modes is also presented. The validity of the formula is ensured by path-following analysis of an arch-type truss with randomly generated imperfections.

1 INTRODUCTION

It is well known that a coincidence of bifurcation loads corresponding to global and local buckling leads to a severe reduction of the critical load due to existence of initial imperfection. The simultaneous buckling was studied in association with optimization. The principle of simultaneous mode design states, "A given form will be optimum if all failure modes which can possibly intersect occur simultaneously [1]." The danger of naive optimization without due regard to imperfection sensitivity and the erosion of optimization by compound branching were suggested [2]. Various kinds of structures were found highly imperfection-sensitive when two or more bifurcation points are nearly or strictly coincident, and are subjected to interaction of buckling modes, such as local and global modes [3–5]. Thompson and Hunt [6] suggested extreme enhancement of imperfection sensitivity due to modal interaction as a result of optimization; imperfection sensitivity of coincident critical points was studied thereafter [7, 8].

Yet such severe enhancement of imperfection sensitivity is absent for another kind of coincident critical points. A nearly coincident pair of a bifurcation point and a limit point of loading parameter was found in (a) numerical simulation of a long tensile steel specimen undergoing plastic instability [9], and (b) mechanical instability of stressed atomic crystal lattices [10]. Such a pair of critical points was approximated by a hilltop branching (bifurcation) point, at which the pair of critical points coincide strictly. This hilltop point was shown to enjoy locally piecewise linear imperfection sensitivity [10, 11], which is less severe than the 2/3-power law for a simple symmetric bifurcation point. A piecewise linear relationship was also observed for other hilltop branching points that occur as the coincidence of (a) an asymmetric bifurcation point and a limit point [12], and (b) a limit point and a double bifurcation point studied by a group-theoretic approach [13].

Ohsaki [14] optimized shallow trusses under constraints on nonlinear buckling and found that the optimum solution usually has a hilltop branching point, which is not sensitive to imperfections. Thus the optimization for nonlinear buckling does not always produce a dangerous structure.

It is noteworthy that, for a pin-jointed truss, member buckling can occur almost independently from global buckling. Therefore, it is possible to create a hilltop branching point at which arbitrary many symmetric bifurcation points can exist at a limit point; i.e., many members buckle simultaneously with global buckling.

In this study, imperfection sensitivity law derived by Ohsaki and Ikeda [15] is verified by carrying out path-tracing analysis for randomly generated imperfect structures. Anti-optimization problems are formulated to find worst imperfection modes [16]. The critical loads of imperfect systems with practically acceptable imperfection norm are investigated. As a consequence to these investigations, it is shown that the 'simultaneous mode design' for this case is not that pessimistic as was cautioned as the 'erosion of optimization by compound bifurcation.'

2 HILLTOP BRANCHING AT THE PERFECT SYSTEM

Consider a general finite-dimensional geometrically nonlinear structure, of which the deformation is described by the nodal displacement vector $\mathbf{U} = (U_1, \dots, U_n)$, where n is the number of degrees of freedom. We assume the existence of the total potential energy $\bar{V}(\mathbf{U}, \Lambda)$ that is a smooth function of $\mathbf{U}$ and loading parameter Λ.

Denote by $\mathbf{H}$ the Hessian of $\bar{V}$ with respect to $\mathbf{U}$, which is called tangent stiffness matrix. The eigenvalue problem of $\mathbf{H}$ is formulated as

$$\mathbf{H}\mathbf{\Phi}_i = e_i \mathbf{\Phi}_i, \;\; (i = 1, \dots, n) \tag{1}$$

where e_i is the ith lowest eigenvalue $(e_i \leq e_{i+1})$, and $\mathbf{\Phi}_i$ is the associated eigenvector normalized by

$$\mathbf{\Phi}_i^\top \mathbf{\Phi}_i = 1, \ \ (i = 1, \dots, n) \tag{2}$$

where $(\ )^\top$ denotes the transpose of a vector.

Consider a case where $m-1$ bifurcation points exist at a limit point; i.e., the critical point is a hilltop branching point with m lowest eigenvalues vanishing simultaneously. The generalized coordinate q_j in the direction of $\mathbf{\Phi}_j$ is defined by the transformation

$$\mathbf{U} = \mathbf{U}^{\mathrm{c}} + \sum_{j=1}^{n} q_j \mathbf{\Phi}_j \tag{3}$$

where $\mathbf{U}^{\mathrm{c}}$ is the displacement vector at the critical point. Then $q_1, \dots, q_m$ serve as active coordinates and $q_{m+1}, \dots . q_n$ as passive coordinates. The increment of the loading parameter from the hilltop point is denoted by λ.

The total potential energy is defined as a function of $\mathbf{q} = (q_1, \dots, q_n)$ and λ and is written as $V(\mathbf{q}, \lambda)$. Differentiation with respect to q_i is indicated by a subscript $(\ \cdot\ )_{,i}$. The equilibrium equations are written as

$$V_{,i} = 0, \ \ (i = 1, \dots, n) \tag{4}$$

Since m lowest eigenvalues e_i $(i = 1, \dots, m)$ vanish at the hilltop point, the following relations hold:

$$V_{,ij} = 0, \ \ (i, j = 1, \dots, m) \tag{5}$$

For the modes $\mathbf{\Phi}_i$ $(i = m+1, \dots, n)$ higher than m, orthogonality conditions

$$\mathbf{\Phi}_i^\top \mathbf{\Phi}_j = 0, \ \ (i, j = m+1, \dots, n; \ i \neq j) \tag{6}$$

should be satisfied so that $V_{,ij}$ is diagonalized such that

$$V_{,ij} = 0, \ \ (i, j = m+1, \dots, n; \ i \neq j) \tag{7}$$

$$V_{,ij} = V_{,ji} = 0, \ \ (i = 1, \dots, m; \ j = m+1, \dots, n) \tag{8}$$

Note that the orthogonality among the eigenvectors $\mathbf{\Phi}_i$ $(i = 1, \dots, m)$ need not be satisfied, because, for multiple eigenvalues, any linear combination of the eigenvectors is also an eigenvector.

From the conditions of bifurcation points and limit point,

$$V_{,i}' = 0, \ (i = 1, \dots, m-1) \tag{9}$$

$$V_{,m}' \neq 0 \tag{10}$$

are to be satisfied, where $(\ )'$ indicates differentiation with respect to λ.

3 IMPERFECTION SENSITIVITY ANALYSIS AT HILLTOP POINT

Let $\mathbf{b}$ denote a vector representing the mechanical properties of the structure such as nodal locations and cross-sectional areas. Indicate by $\mathbf{b} = \mathbf{b}^0$ the perfect structure. The imperfect structures are defined by the imperfection pattern vector $\mathbf{d}$ and the associated imperfection parameter ε as

$$\mathbf{b} = \mathbf{b}^0 + \varepsilon \mathbf{d} \tag{11}$$

The total potential energy of an imperfect system is defined by $n+2$ variables λ, $q_1, \dots, q_n$ and ε as $V(\mathbf{q}, \lambda, \varepsilon)$. Let $\Lambda^{\mathrm{c}0}$ denote the critical load factor of the perfect structure. The increment λ^{c} of the critical load factor Λ^{c} from $\Lambda^{\mathrm{c}0}$ is defined as

$$\lambda^{\mathrm{c}} = \Lambda^{\mathrm{c}} - \Lambda^{\mathrm{c}0} \tag{12}$$

The purpose of imperfection sensitivity analysis is to derive the relation between λ^{c} and ε.

For the bifurcation points corresponding to member buckling, V satisfies the following conditions:

- All the $m-1$ bifurcation points are individually symmetric [7]; i.e.,

$$V_{,ijk} = 0, \ \ (i, j, k = 1, \dots, m-1) \tag{13}$$

- V is symmetric in the direction of bifurcation modes, and is not symmetric in the direction of limit point mode as

$$V_{,ijm} \neq 0, \;\; (i,j = 1,\dots,m-1;\; i \neq j) \tag{14}$$
$$V_{,imm} = 0, \;\; (i = 1,\dots,m-1) \tag{15}$$

- Definition of limit point leads to

$$V_{,mmm} \neq 0 \tag{16}$$

Although the details are not shown, the following imperfection sensitivity law is satisfied:

$$\lambda^{\mathrm{c}} = -\frac{\dot{V}_{,m}}{V'_{,m}}\varepsilon - \frac{1}{V'_{,m}}\sqrt{V_{,mmm}C_m}\;|\varepsilon| \tag{17}$$

where C_m is a constant independent of ε as

$$C_m = \sum_{i=1}^{m-1}\sum_{j=1}^{m-1} V_{,ijm}\tilde{q}_i\tilde{q}_j \tag{18}$$

$\tilde{q}_i$ is obtained by solving the following $m-1$ linear equations:

$$\sum_{j=1}^{m-1} V_{,ijm}\tilde{q}_j + \dot{V}_{,i} = 0, \;\; (i = 1,\dots,m-1) \tag{19}$$

Note that the direction of the limit point mode $\mathbf{\Phi}_m$ is defined such that $V'_{,m} > 0$, and $V_{,mmm}C_m > 0$ are satisfied.

4 WORST IMPERFECTION

The worst imperfection mode is defined so that the critical load of imperfect system is reduced most drastically for a given norm of imperfection mode. Let $\mathbf{B}$ denote the matrix for which (j,i)-element is defined by $\partial^2 V/\partial U_j \partial b_i$. Also let $h_k = \dot{V}_{,k}$ for simplicity. Then the relation

$$h_k = \mathbf{\Phi}_k^\top \mathbf{B}\mathbf{d}, \;\; (k = 1,\dots,s) \tag{20}$$

holds. In the following, the components of the vectors are written as $\mathbf{h} = \{h_i\}$, $\tilde{\mathbf{q}} = \{\tilde{q}_i\}$, etc.

The vectors $\mathbf{h}$ and $\tilde{\mathbf{q}}$ are divided into the components corresponding to the bifurcation mode $\mathbf{h}^B = (h_1,\dots,h_{m-1})^\top$, $\tilde{\mathbf{q}}^B = (\tilde{q}_1,\dots,\tilde{q}_{m-1})^\top$ and the limit point mode $h^L = h_m$, $\tilde{q}^L = \tilde{q}_m$. The matrix for which the ith row is equal to $\mathbf{\Phi}_i^\top$ $(i = 1,\dots,m-1)$ is denoted by $\mathbf{F}^B$, i.e.,

$$\mathbf{h}^B = \mathbf{F}^B\mathbf{B}\mathbf{d} \tag{21}$$

The $(m-1)\times(m-1)$ matrix that has $V_{,ijm}$ at (i,j)-element $(i,j = 1,\dots,m-1)$ is denoted by $\mathbf{G}$. By using (21), (18) and (19) are reduced to

$$C_m = \mathbf{d}^\top\mathbf{B}^\top\mathbf{F}^{B\top}(\mathbf{G}^{-1})^\top\mathbf{F}^B\mathbf{B}\mathbf{d} \tag{22}$$
$$\tilde{\mathbf{q}}^B = -\mathbf{G}^{-1}\mathbf{F}^B\mathbf{B}\mathbf{d} \tag{23}$$

For simplicity, define $\tilde{\mathbf{G}}$ as

$$\tilde{\mathbf{G}} = V_{,mmm}\mathbf{B}^\top\mathbf{F}^{B\top}(\mathbf{G}^{-1})^\top\mathbf{F}^B\mathbf{B} \tag{24}$$

Then, (17) is rewritten as

$$\lambda^{\mathrm{c}}(\mathbf{d}) = -\frac{\mathbf{\Phi}_m^\top\mathbf{B}\mathbf{d}}{V'_{,m}}\varepsilon - \frac{1}{V'_{,m}}\sqrt{\mathbf{d}^\top\tilde{\mathbf{G}}\mathbf{d}}\;|\varepsilon| \tag{25}$$

Hence, the anti-optimization problem to minimize λ^{c} is formulated as

$$\mathrm{AOP1}: \;\; \text{minimize} \;\;\; \lambda^{\mathrm{c}}(\mathbf{d}) \tag{26}$$
$$\text{subject to} \;\; \mathbf{d}^\top\mathbf{d} = 1 \tag{27}$$

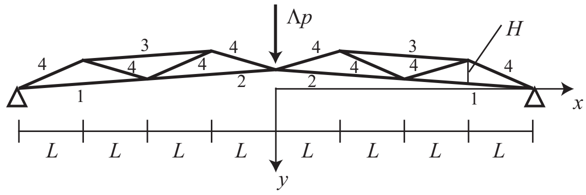


Figure 1: An arch-type truss.

Next, we consider the imperfection in the direction of the active coordinates; i.e., the imperfection pattern vector $\mathbf{d}$ is defined as the linear combination of the critical mode vectors $\mathbf{\Phi}_i$ $(i = 1, \ldots, m)$ as

$$\mathbf{d} = \sum_{i=1}^{m} c_i \mathbf{\Phi}_i \tag{28}$$

where c_i is the coefficients. Assume the following orthogonality condition:

$$\mathbf{\Phi}_i^\top \mathbf{B} \mathbf{\Phi}_m = 0, \ \ (i = 1, \ldots, m-1) \tag{29}$$

The validity of this assumption is confirmed in the example of an arch-type truss.

Let $\mathbf{D}^B$ denote the matrix, which has $\mathbf{\Phi}_i$ $(i = 1, \ldots, m-1)$ in the ith column, and $\mathbf{c}$ is divided into the components of bifurcation modes $\mathbf{c}^B = (c_1, \ldots, c_{m-1})^\top$ and the limit point mode $c^L = c_m$.

Define $\mathbf{G}^*$ and e_m as

$$\mathbf{G}^* = \mathbf{D}^{B\top} \tilde{\mathbf{G}}^\top \mathbf{D}^B \tag{30}$$

$$e_m = \mathbf{\Phi}_m^\top \mathbf{B} \mathbf{\Phi}_m \tag{31}$$

Using (28)–(31), (25) is reduced to

$$\lambda^{\mathrm{c}}(\mathbf{c}) = -\frac{e_m c^L}{V'_{,m}} \varepsilon - \frac{1}{V'_{,m}} \sqrt{\mathbf{c}^{B\top} \mathbf{G}^* \mathbf{c}^B} \ |\varepsilon| \tag{32}$$

Hence, λ^{c} defined by (17) is regarded as a function of $\mathbf{c}$, and the anti-optimization problem to minimize λ^{c} is formulated as

$$\mathrm{AOP2}: \ \mathrm{minimize} \quad \lambda^{\mathrm{c}}(\mathbf{c}) \tag{33}$$

$$\mathrm{subject\ to} \ \ \mathbf{c}^\top \mathbf{c} = 1 \tag{34}$$

Since the right-hand-side of (32) is divided into the two terms corresponding to c^L and $\mathbf{c}^B$, respectively, we first maximize $\mathbf{c}^{B\top} \mathbf{G}^* \mathbf{c}^B$ under constraint of

$$\mathbf{c}^{B\top} \mathbf{c}^B = a, \ \ (0 \leq a \leq 1) \tag{35}$$

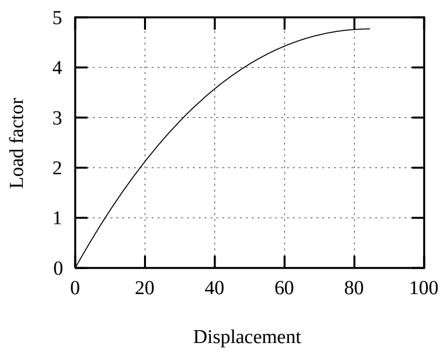
Let μ denote the maximum eigenvalue of $\mathbf{G}^*$, which is a real symmetric matrix. The minimum value of the second term is given as $-(1/V'_{,m})\sqrt{a\mu}\ |\varepsilon|$. Hence, a is obtained by minimizing $\lambda^{\mathrm{c}}(a)$ defined by

$$\lambda^{\mathrm{c}}(a) = -\frac{e_m}{V'_{,m}} \sqrt{1-a}\ \varepsilon - \frac{1}{V'_{,m}} \sqrt{a\mu}\ |\varepsilon| \tag{36}$$

By differentiating (36) with respect to a, we obtain $a = \mu / \sqrt{e_m^2 + \mu}$ for the worst imperfection, and the anti-optimal solution $\mathbf{c}$ of AOP2 is obtained as

$$c^L = \frac{e_m^2}{e_m^2 + \mu}, \ \ \mathbf{c}^B = \sqrt{\frac{\mu}{e_m^2 + \mu}}\ \mathbf{g} \tag{37}$$

where $\mathbf{g}$ is the eigenmode corresponding to the maximum eigenvalue of $\mathbf{G}^*$, where $\mathbf{g}$ is normalized by $\mathbf{g}^\top \mathbf{g} = 1$.



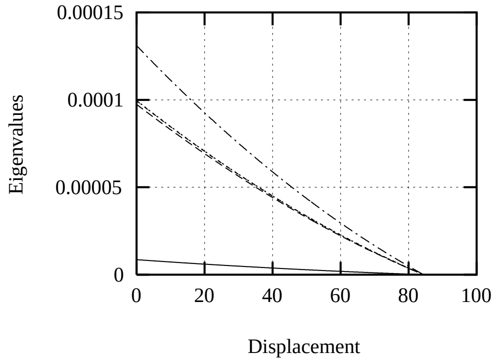
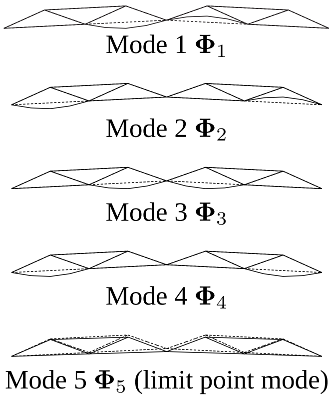


Figure 2: Relation between the load factor and the vertical displacement of the center node.

Figure 3: Relation between the eigenvalues and the vertical displacement of the center node.



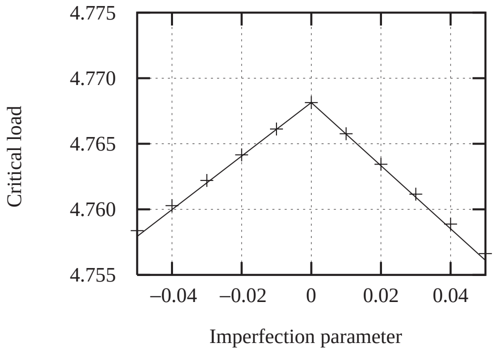


Figure 4: Four member buckling modes $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_4$ and limit point mode $\mathbf{\Phi}_5$

Figure 5: Critical loads of imperfect structures corresponding to the sum of five modes.

5 IMPERFECTION SENSITIVITY OF AN ARCH-TYPE TRUSS

Imperfection sensitivity analysis is carried out for a pin jointed arch-type truss as shown in Fig. 1, where $L = 1000$, $H = 400$, and the height of the center node is 200. A proportional load Λp is applied in the y-direction at the center node, where $p = 0.001$, and the stiffness is scaled so that the elastic modulus is equal to 1. In the following, the units of length and force are omitted.

The members are divided into four groups as shown in Fig. 1. Let A_i and I_i denote the cross-sectional area and the moment of inertia of the members in group i, which are related by the constant γ_i as

$$I_i = \gamma_i^2 A_i \tag{38}$$

where $(A_1, A_2, A_3, A_4) = (100, 100, 1000, 300)$, $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (18.15, 18.05, 30.0, 100.0)$. Each member is divided into four elements, and Green's strain is used for the definition of the strain.

5.1 Imperfection sensitivity

The relation between the load factor Λ and the vertical displacement v of the center node is as shown in Fig. 2, where a limit point is reached at $\Lambda = \Lambda^{c0} = 4.7681$. The relations between the five eigenvalues and v are plotted in Fig. 3.

The five critical modes $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_5$ are as shown in Fig. 4, where $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_4$ correspond to member buckling, and $\mathbf{\Phi}_5$ is the limit point mode. The 3rd-order differential coefficients of V are computed as

$$\begin{aligned} &V_{,115} = V_{,335} = -2.0842 \times 10^{-7}, \;\; V_{,225} = V_{,445} = -2.1415 \times 10^{-7}, \\ &V_{,345} = -2.2262 \times 10^{-9}, \;\; V_{,555} = -1.8214 \times 10^{-8} \end{aligned} \tag{39}$$

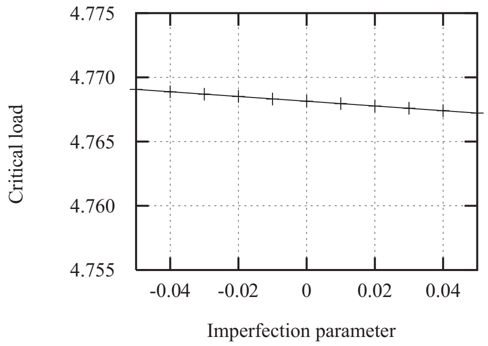


Figure 6: Critical loads of imperfect structures corresponding to the limit point mode.

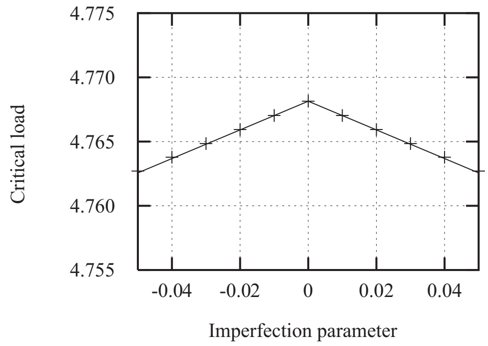


Figure 7: Critical loads of imperfect structures corresponding to $\mathbf{\Phi}_3$.

where the terms neglected in (13) (15) have been confirmed to be very small. Other coefficients are

$$\begin{aligned} &\dot{V}_{,1} = \dot{V}_{,2} = -9.7979 \times 10^{-5}, \;\; \dot{V}_{,3} = \dot{V}_{,4} = -9.7963 \times 10^{-5}, \\ &\dot{V}_{,5} = -4.7352 \times 10^{-5}, \;\; V'_{,5} = -7.0748 \times 10^{-11} \end{aligned} \tag{40}$$

The relation between the imperfection parameter in the direction of $\sum_{i=1}^{5} \mathbf{\Phi}_i$ and the critical load is computed as

$$\Lambda^{\mathrm{c}} = \Lambda^{\mathrm{c}0} - 1.8393 \times 10^{-2}\varepsilon - 0.22176|\varepsilon| \tag{41}$$

which is plotted in Fig. 5, where '+' denotes the limit point load computed by path-tracing analysis. Note that each mode is normalized by (2), and the maximum nodal imperfection for $\varepsilon = 1$ is about 0.5. Hence, for a small range of imperfection, the imperfection sensitivity can be approximated in a good accuracy by the piecewise linear law (41).

For the imperfection in the direction of the limit point mode, the second term in (41) does not exist, and the relation between Λ^{c} and ε is linear as

$$\Lambda^{\mathrm{c}} = \Lambda^{\mathrm{c}0} - 1.8393 \times 10^{-2}\varepsilon \tag{42}$$

which is plotted in Fig. 6. In this example, the imperfection in the limit point mode is less sensitive than that in the bifurcation mode.

The imperfection sensitivity in the direction of $\mathbf{\Phi}_3$ is symmetric as

$$\Lambda^{\mathrm{c}} = \Lambda^{\mathrm{c}0} - 0.11249|\varepsilon| \tag{43}$$

which is plotted as Fig. 7. Therefore, the small asymmetry in $\mathbf{\Phi}_3$ does not lead to visible asymmetry in the imperfection sensitivity.

In the above examples, small imperfections that are not practically acceptable have been considered. Consider next a moderately large nodal imperfection of about 2.0, which is 1/100 of the height 200.0. Since the vertical deflection of the center node in the limit point mode normalized by (2) is 0.25745, the range $-8 \leq \varepsilon \leq 8$ is considered. The critical loads are plotted in Fig. 8, which exhibits good accuracy between the linear law and the critical point computed by the path-tracing analysis.

For the members, the initial deformation is considered to be 2.0, which is about 1/1000 of the member length. Since the maximum absolute value of the components of modes 1–4 is about 0.5, the range $-4 \leq \varepsilon \leq 4$ is considered. The critical loads of imperfect structures are plotted in Fig. 9. Note that the approximation is not very good as the imperfection in the direction of the limit point mode.

Next, path-tracing analysis is carried out considering random imperfections. Nodal imperfections in x- and y-directions are generated by normal distribution of probability $N(0, \delta^2)$. The same distribution is used for the imperfection Δ_i at the center node of the ith member, where the imperfection of $1/\sqrt{2}\Delta_i$ is given for the nodes located at 1/4 from each member end. The results for $\delta = \Delta_i = 1$ are as shown in '+' in Fig. 10(a).

5.2 *Worst imperfection*

The coefficient $\mathbf{c}$ of the worst imperfection is obtained from AOP2 as

$$(c_1, c_2, c_3, c_4, c_5) = (0.0, -0.93362, 0.0, 0.32002, 0.16108) \tag{44}$$

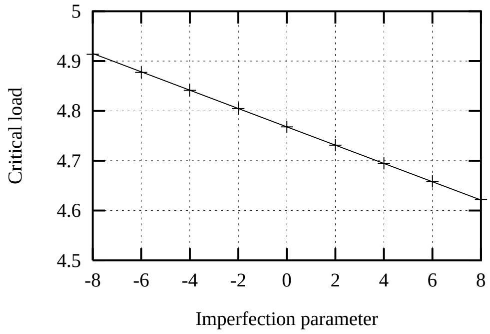


Figure 8: Critical loads of imperfect structures corresponding to the limit point mode (large imperfection).

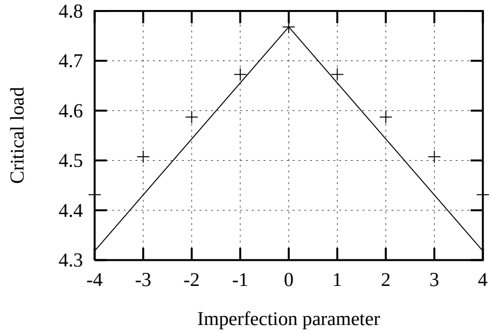
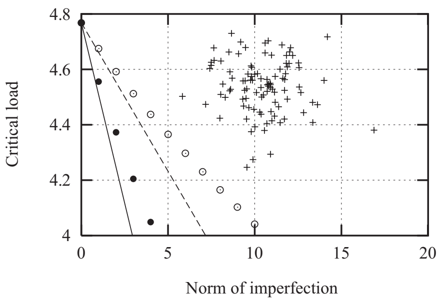
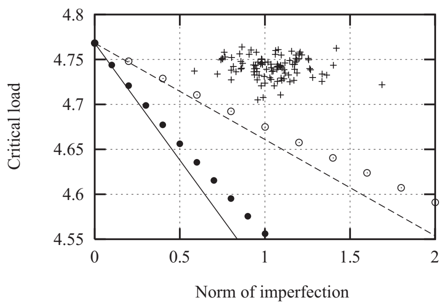


Figure 9: Critical loads of imperfect structures corresponding to $\mathbf{\Phi}_3$ (large imperfection).

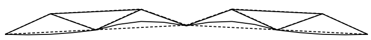


(a) small imperfection



(a) large imperfection

Figure 10: Relation between imperfection norm and critical load for random imperfection



(a) solution of AOP2

(a) solution of AOP1

Figure 11: Worst imperfection modes.

Since orthogonality conditions are not satisfied for $\mathbf{\Phi}_i$ $(i = 1, \dots, 5)$, the norm of $\mathbf{d} = \sum_{i=1}^{5} c_i \mathbf{\Phi}_i$ is 1.0478, which is not equal to 1. On the other hand, the absolute value of $\mathbf{\Phi}_i^\top \mathbf{B} \mathbf{\Phi}_m$ $(i = 1, \dots, m-1)$ is less than that of 1/100000 of $\mathbf{\Phi}_m^\top \mathbf{B} \mathbf{\Phi}_m$ $(i = 1, \dots, m-1)$; hence, the assumption (31) holds.

The mode $\mathbf{d}$ is as shown in Fig. 11(a). The optimal (worst) objective value of AOP2 is -0.11231, which is divided by the norm 1.0478 to obtain the sensitivity coefficient -0.10719. The linear law in the direction of the worst imperfection is plotted in the broken line in Fig. 10(a), where '$\circ$' is the result of path-tracing analysis.

On the other hand, the worst imperfection obtained by AOP1 is as which in Fig. 11(b), and the sensitivity coefficient is -0.25822. The result of the linear approximation is as shown in the solid line in Fig. 10(a), where '$\bullet$' is the result of path-tracing analysis. The similar results for $\delta = 0.1$ are plotted in Fig. 10(b). It can be confirmed from these results that the worst imperfection serves as the lower bound of the critical load of the imperfect structures. The coefficients of the worst imperfection mode for $\mathbf{\Phi}_i$ $(i = 1, \dots, 5)$ are 0.0, 0.41847, 0.0, 0.10663, -0.071310. Since the sum of the absolute values of the coefficients is less than 1, it is seen that the component of passive coordinates are included in the worst imperfection mode. Therefore, considering only active coordinates (linear combination of buckling modes) is not enough to estimating the worst imperfection mode.

6 CONCLUSIONS

The conclusions drawn from this study are as follows:

1. Imperfection sensitivity of hilltop branching point, which has many individually symmetric bifurcation points at a limit point, obeys a piecewise linear law; hence, the sensitivity is less severe than that of a bifurcation point. Therefore, the existence of member buckling at the limit point is not dangerous in view of imperfection sensitivity. The 'simultaneous mode design' for this case is not that pessimistic as was

cautioned the ‘erosion of optimization by compound bifurcation.’

2. The worst mode of nodal imperfection can be obtained by solving an anti-optimization problem to minimize the critical load for specified norm of imperfection. An explicit form can be obtained if the imperfection is limited within a linear combination of the buckling modes.

3. The simultaneous member buckling at a limit point can be modeled as a hilltop branching point with many individually symmetric bifurcation points. The accuracies of the imperfection sensitivity laws and the worst imperfection have been confirmed by the example of an arch-type truss.

4. It has been shown in the example of an arch-type truss that the imperfection sensitivity of the worst imperfection is about twice of that for the worst imperfection in the direction of a linear combination of the buckling modes. Therefore, it is not appropriate to use only buckling modes for investigating the worst-case scenario.

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