# PARAMETRIC SENSITIVITY OF OPTIMAL ARCH-TYPE FRAMES SUBJECTED TO SPATIALLY VARYING SEISMIC MOTIONS

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## ABSTRACT

An optimization method is presented for long-span arch-type frames for specified seismic responses considering spatial variation of ground motions. Sensitivity coefficients of the optimum objective value with respect to the parameters defining spatial variation of the seismic motions are computed based on post-optimal analysis. It is shown that the second-order parametric sensitivity coefficients are easily obtained if the first order coefficients vanish due to symmetry and antisymmetry of the structure and ground motions.

## 1. INTRODUCTION

In the design process of shells and spatial structures, it is important to consider the effects of spatial variation of seismic motions which are classified into (a) wave passage effect due to delay of the wave being transmitted to the supports, (b) incoherency effect due to reflections and refractions of the wave in the heterogeneous medium, and (c) the local effect or site response effect due to difference of the soil conditions near the supports. Der Kiureghian and Neuenhofer [1] presented a response spectrum approach based on coherency functions between the ground motions at the supports.

Hao and Duan [2] showed that incoherency of the support motions leads to torsional responses of frame structures. Zembaty [3] carried out a parametric study to discuss the effect of incoherency parameters. No attempt, however, has been made for developing an optimal design method considering effects of spatial variation of seismic motions.

Since the parameters for the spatial variation cannot be clearly defined, it is useful if sensitivity coefficients of optimal solution with respect to those parameters are obtained. Dependence of optimal solutions on parameters defining the optimization problem may be evaluated by the parametric programming approach or post-optimal analysis [4]. Application of the parametric programming to structural optimization problems is called optimum design sensitivity analysis [5].

In this paper, an optimization method by Ohsaki et al. [6] is extended to carry out post-optimal analysis of long-span arch-type structures considering spatial variation of ground motions. It is shown that the second-order parametric sensitivity coefficient is easily obtained if the first-order parametric sensitivity vanishes.

## 2. RESPONSE TO SPATIALLY VARYING GROUND MOTIONS

Consider a long-span structure discretized by the finite element method. The total Degrees-Of-Freedom (DOF) of the internal nodes and supports are divided into Unconstrained DOF (UDOF) and Support DOF (SDOF). Let $\boldsymbol{x}$ and $\boldsymbol{u}$ denote the vectors of absolute displacements corresponding to UDOF and SDOF, respectively. The stiffness matrix, mass matrix and damping matrix, denoted by $\boldsymbol{K}, \boldsymbol{M}$ and $\boldsymbol{C}$, respectively, are divided into the components corresponding to UDOF and SDOF as indicated by the subscripts $x$ and $u$, respectively. The equation of motion can then be written as

$$\begin{bmatrix} \boldsymbol{M}_x & \boldsymbol{M}_{xu} \\ \boldsymbol{M}_{xu}^T & \boldsymbol{M}_u \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}} \\ \ddot{\boldsymbol{u}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_x & \boldsymbol{C}_{xu} \\ \boldsymbol{C}_{xu}^T & \boldsymbol{C}_u \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{u}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_x & \boldsymbol{K}_{xu} \\ \boldsymbol{K}_{xu}^T & \boldsymbol{K}_u \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{f} \end{Bmatrix} \tag{1}$$

where $\boldsymbol{f}$ is the vector of reaction forces.

Let $\boldsymbol{x}^s$ and $\boldsymbol{x}^d$ denote the pseudo-static and dynamic components of $\boldsymbol{x}$, where $\boldsymbol{x}^d$ is divided into components of eigenmodes $\boldsymbol{\phi}_i$ $(i = 1, 2, \dots, n)$ of fixed SDOF. The $i$th natural circular frequency and damping coefficient are denoted by $\omega_i$ and $\zeta_i$, respectively. A vector $\boldsymbol{r}_k$ is defined as

$$\boldsymbol{r}_k = -\boldsymbol{K}_x^{-1} \boldsymbol{K}_{xu} \boldsymbol{i}_k \tag{2}$$

where $\boldsymbol{i}_k$ is a vector whose $k$th component is 1 and the remaining components are 0.

Let $s_{ki}$ denote the modal response to the input $u_k$ at the $k$th SDOF. Suppose the representative response $z(t)$ such as a strain of an element is defined as

$$z(t) = \sum_{k=1}^{s} \sum_{i=1}^{n} b_{ki} s_{ki}(t) + \sum_{k=1}^{s} a_k u_k(t) \tag{3}$$

$$a_k = \boldsymbol{q}_1^T \boldsymbol{r}_k + \boldsymbol{q}_2^T \boldsymbol{i}_k, \quad (k = 1, \dots, s) \tag{4}$$

$$b_{ki} = \boldsymbol{q}_1^T \boldsymbol{\phi}_i \beta_{ki}, \quad (k = 1, \dots, s;\ i = 1, \dots, n) \tag{5}$$

where $s$ is the number of SDOFs, and $\beta_{ki}$ is the participation factor of the $i$th mode to the input at the $k$th SDOF. Note that Der Kiureghian and Neuenhofer [1] defined $z(t)$ only in terms of $\boldsymbol{x}$. In this paper, the second term in (3) has been added to include in $z(t)$ the response defined by $\boldsymbol{x}$ and $\boldsymbol{u}$, e.g. the strain of a member connecting to a support.

Consider a response to the horizontal component of a ground motion. A coherency function is used for representing the wave passage effect and the incoherency effect. The coherency between the ground accelerations $\ddot{u}_k$ and $\ddot{u}_l$ at $k$th and $l$th SDOFs is given as [7].

$$\gamma_{kl}(\mathrm{i}\omega) = \exp\left[ -\left( \frac{\alpha \omega d_{kl}}{v_{\mathrm{s}}} \right)^2 \right] \exp\left( \mathrm{i} \frac{\eta \omega d_{kl}^{\mathrm{L}}}{v_{\mathrm{app}}} \right) \tag{6}$$

where $\alpha$ is the incoherency factor, $d_{kl}$ is the horizontal distance between $k$th and $l$th SDOF, $d_{kl}^{\mathrm{L}}$ is the projected distance of $d_{kl}$ to the horizontal plane, $v_{\mathrm{s}}$ is the velocity of shear wave, and $v_{\mathrm{app}}$ is the apparent velocity of the shear wave. Note that $\eta$ is an auxiliary parameter for indicating incorporation of the wave passage effect with $\eta = 1$, and that effect is not considered if $\eta = 0$.

Assuming that $u_k$ is a zero-mean jointly stationary process, and that the peak factors for the input quantities and the responses are same, the mean maximum response $E[\max|z(t)|]$ of $z(t)$ is written as

$$\mathrm{E}[\max|z(t)|] \approx \left[\sum_{k=1}^{s}\sum_{l=1}^{s} a_k a_l \rho_{u_k u_l} u_k^{\max} u_l^{\max} + 2\sum_{k=1}^{s}\sum_{l=1}^{s}\sum_{j=1}^{n} a_k b_{lj} \rho_{u_k s_{lj}} u_k^{\max} S_{Dl}(\omega_j, \zeta_j) + \sum_{k=1}^{s}\sum_{l=1}^{s}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} S_{Dk}(\omega_i, \zeta_i) S_{Dl}(\omega_j, \zeta_j)\right]^{\frac{1}{2}} \tag{7}$$

where $u_k^{\max}$ and $S_{Dk}(\omega, \zeta)$ are the specified maximum displacement and the displacement response spectrum of the $k$th SDOF, and $\sigma_{u_k}$, $\sigma_{s_{ki}}$, $\rho_{u_k u_l}$, $\rho_{u_k s_{lj}}$ and $\rho_{s_{ki} s_{lj}}$ may be referred to Refs. 1 and 6; e.g. $\sigma_{s_{ki}}$ is given as

$$\sigma_{s_{ki}} = \left[\int_{-\infty}^{\infty} \frac{1}{(\omega_j^2 - \omega^2)^2 + 4\zeta_j^2 \omega_j^2 \omega^2} G_{\ddot{u}_k \ddot{u}_k}(\omega) \mathrm{d}\omega\right]^{\frac{1}{2}} \tag{8}$$

where the power spectrum $G_{\ddot{u}_k \ddot{u}_k}(\omega)$ is defined in terms of the response spectrum as

$$G_{\ddot{u}_k \ddot{u}_k}(\omega) = \frac{\omega^{c_k+2}}{\omega^{c_k} + \omega_{(\mathrm{f})k}^{c_k}} \left(\frac{2\zeta\omega}{\pi} + \frac{4}{\pi\tau}\right) \left[\frac{S_{Dk}(\omega, \zeta)}{c_s(\omega)}\right]^2 \tag{9}$$

where $\tau$ is the duration of the motion and $c_s(\omega)$ is the peak factor of the response to white noise. The parameter $c_k$ is equal to 3 in the following example, and $\omega_{(\mathrm{f})k}$ is calculated to represent finite power for the pseudo-static input.

The difference in the local amplification can be modeled by simply assigning different values of $S_{Dk}(\omega, \zeta)$ for the SDOFs. This simple model is used in the examples to evaluate the effect of the pseudo static components on the total response quantities.

## 3. OPTIMIZATION PROBLEM AND DESIGN SENSITIVITY ANALYSIS

Consider a structure such as a frame or a truss discretized by one-dimensional finite elements. The number of elements is denoted by $m$. Let $L_i$ and $A_i$ denote the length and cross-sectional area of the $i$th element. The design variable is the vector $\boldsymbol{A} = \{A_i\}$ assuming other cross-sectional properties such as the second moment of area are functions of $A_i$.

Let $\varepsilon_i^{\mathrm{v}}(\boldsymbol{A})$ denote the representative mean maximum strain of the $i$th member computed from (7), where $a_k$ and $b_{lk}$ should be defined appropriately. The strain due to self-weight and the static loads is denoted by $\varepsilon_i^{\mathrm{W}}(\boldsymbol{A})$. The optimization problem for minimizing the total structural volume is formulated as

$$\text{Minimize} \quad V(\boldsymbol{A}) = \sum_{i=1}^{m} A_i L_i \tag{10}$$

$$\text{subject to} \quad \varepsilon_i(\boldsymbol{A}) = \varepsilon_i^{\mathrm{V}}(\boldsymbol{A}) + |\varepsilon_i^{\mathrm{W}}(\boldsymbol{A})| \leq \bar{\varepsilon}^{\mathrm{U}}, (i = 1, \dots, m) \tag{11}$$

$$A_i \geq \bar{A}_i \tag{12}$$

where $\bar{A}_i$ is the lower bound for $A_i$.

The problem defined above is to be solved by a gradient based method. Therefore the Design Sensitivity Coefficients (DSCs) of the objective and constraint functions need to be computed. Since the DSCs of static strains are obtained from a well-established method, only the formulations for the DSC of $\varepsilon_i^{\mathrm{V}}(\boldsymbol{A})$ are briefly summarized.

Let $A^*$ denote a representative design variable which is a component of $\boldsymbol{A}$. Eq. (2) is first rewritten as

$$\boldsymbol{K}_x \boldsymbol{r}_k = -\boldsymbol{K}_{xu} \boldsymbol{i}_k \tag{13}$$

and is differentiated with respect to $A^*$ as

$$\boldsymbol{K}_x \boldsymbol{r}'_k = -\boldsymbol{K}'_x \boldsymbol{r}_k - \boldsymbol{K}'_{xu} \boldsymbol{i}_k \tag{14}$$

where a prime indicates partial differentiation with respect to $A^*$. Therefore, computation of the inverse matrix or its DSCs is not needed. By using (4),

$$a'_k = \boldsymbol{q}_1^{T\prime} \boldsymbol{r}_k + \boldsymbol{q}_1^T \boldsymbol{r}'_k + \boldsymbol{q}_2^{T\prime} \boldsymbol{i}_k \tag{15}$$

is derives. The DSC of $b_{ki}$ is obtained by differentiating (5) as

$$b'_{ki} = \boldsymbol{q}_1^{T\prime} \boldsymbol{\phi}_i \beta_{ki} + \boldsymbol{q}_1^T \boldsymbol{\phi}'_i \beta_{ki} + \boldsymbol{q}_1^T \boldsymbol{\phi}_i \beta'_{ki} \tag{16}$$

Note that $G_{\ddot{u}_k \ddot{u}_k}(\omega)$, $G_{u_k \ddot{u}_l}(\mathrm{i}\omega)$, $G_{u_k u_l}(\mathrm{i}\omega)$, and $\sigma_{u_k}$ do not depend on $A^*$. From (8),

$$\begin{aligned}
\sigma'_{s_{lj}} &= \int_{-\infty}^{\infty} \left( \frac{1}{(\omega_j^2 - \omega^2)^2 + 4\zeta_j^2 \omega_j^2 \omega^2} \right)' G_{\ddot{u}_l \ddot{u}_l}(\omega) \mathrm{d}\omega \\
&= -(\omega_j^2)' \int_{-\infty}^{\infty} \left( \frac{2(\omega_j^2 - \omega^2) + 4\zeta_j^2 \omega^2}{\{(\omega_j^2 - \omega^2)^2 + 4\zeta_j^2 \omega_j^2 \omega^2\}^2} \right) G_{\ddot{u}_l \ddot{u}_l}(\omega) \mathrm{d}\omega
\end{aligned} \tag{17}$$

is derived. Other terms are differentiated similarly. Since partial differentiation is not included in the integrand, integration should be carried out only once in the process of response analysis. Therefore, the increase of number of design variables does not lead to rapid increase of the computational cost.

## 4. POST-OPTIMAL ANALYSIS

Since the solution of an optimization problem depends on the parameters which appear in the objective and/or constraint functions but are kept constant during the optimization process, it is practically important to investigate the sensitivity of the optimal solution and/or the optimal objective value with respect to such parameters.

Consider an optimization problem as

$$\text{Find} \qquad \hat{C}(\boldsymbol{p}) = \underset{\boldsymbol{x}}{\text{Min}}\, C(\boldsymbol{x}, \boldsymbol{p}) \tag{18}$$

$$\text{subject to} \quad G_i(\boldsymbol{x}, \boldsymbol{p}) \leq 0, \quad (i = 1, 2, \ldots, r) \tag{19}$$

where $\boldsymbol{x}$ and $\boldsymbol{p} = \{p_j\}$ are the vectors of variables and parameters, and $\hat{C}(\boldsymbol{p})$ is the optimal objective value that is conceived as a function of the parameter vector $\boldsymbol{p}$.

Suppose that the Lagrange multipliers for the constraints have been found as the result of optimization. If the multipliers are not automatically computed, those are obtained by solving a system of linear equations [8]. Sensitivity of the optimal objective value with respect to $p_j$ is found from [4, 5]

$$\frac{\partial \hat{C}}{\partial p_j} = \frac{\partial C}{\partial p_j} + \sum_{i=1}^{r} \lambda_i \frac{\partial G_i}{\partial p_j} \tag{20}$$

where $\lambda_i$ is the Lagrange multiplier for the $i$th inequality constraint. Note that the sensitivity coefficients of the optimal variables with respect to the parameter are not needed for computing those of the optimal objective value.

By differentiating (20) with respect to $p_k$, the second order parametric sensitivity of the optimal objective value is written as [4, 5]

$$\begin{aligned} \frac{\partial^2 \hat{C}}{\partial p_j \partial p_k} = {} & \frac{\partial^2 C}{\partial p_j \partial p_k} + \sum_{q=1}^{m} \frac{\partial^2 C}{\partial p_j \partial x_q} \frac{\partial \hat{x}_q}{\partial p_k} + \sum_{i=1}^{r} \sum_{q=1}^{m} \lambda_i \frac{\partial^2 G_i}{\partial p_j \partial x_q} \frac{\partial \hat{x}_q}{\partial p_k} \\ & + \sum_{i=1}^{r} \left[ \frac{\partial G_i}{\partial p_j} \frac{\partial \hat{\lambda}_i}{\partial p_k} + \lambda_i \frac{\partial^2 G_i}{\partial p_j \partial p_k} \right] \end{aligned} \tag{21}$$

where $m$ is the number of variables and a hat indicates the optimal value that is a function of $\boldsymbol{p}$. Note that $\partial \hat{x}_q / \partial p_k$ and $\partial \hat{\lambda}_i / \partial p_k$ are needed for computing $\partial^2 \hat{C} / \partial p_j \partial p_k$. Since the Hessian of the constraint function with respect to the variables is needed for finding $\partial \hat{x}_q / \partial p_k$ and $\partial \hat{\lambda}_i / \partial p_k$, it is not practically acceptable to compute those values for a complicated optimization problem as considered in this paper.

Consider a case where all the variables of the optimal solution are even functions of a parameter $p_j$. In this case, the first order parametric sensitivity coefficients $\partial \hat{x}_i / \partial p_j$ and $\partial \hat{\lambda}_i / \partial p_j$ vanish at $p_j = 0$ and (21) is reduced to

$$\frac{\partial^2 \hat{C}}{\partial p_j \partial p_k} = \frac{\partial^2 C}{\partial p_j \partial p_k} + \sum_{i=1}^{r} \lambda_j \frac{\partial^2 G_j}{\partial p_j \partial p_k} \tag{22}$$

Although the parametric sensitivities of the variables are not obtained, the parametric sensitivities of the optimal objective values will lead to useful information in the process of designing structures. Note that the first order sensitivity usually dominates over the second order, and the latter need not be computed. For a symmetric structure, however, due to

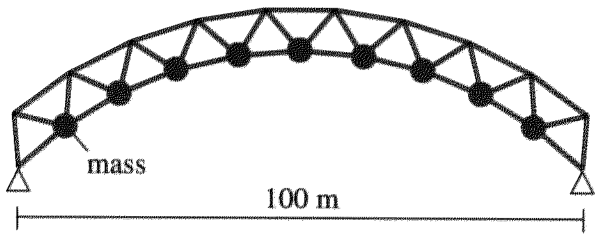


Fig. 1: A 39-bar arch-type plane frame.

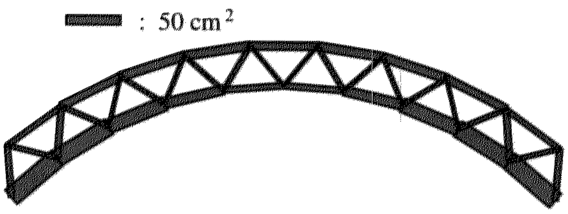


Fig. 2: Optimum cross-sectional areas without spatial variation of seismic motion.

symmetry or antisymmetry conditions of the structure and ground motions, the first order sensitivity coefficients vanish and the second order coefficients give useful information on characteristics of optimal solutions.

## 5. EXAMPLES

The response spectrum by Newmark and Hall [9] is used in the examples. Let $C_A, C_V$ and $C_D$ denote the maximum values of acceleration, velocity and displacement of the ground motion. In their definition, however, the maximum response displacement at $\omega = 0$ does not agree with $C_D$. Therefore $S_D$ for the small range of $\omega$ is given as

$$S_D(\omega, \zeta) = C_D(1.0 + \kappa\omega^2) \tag{23}$$

where $\kappa$ is a parameter defined by $\zeta$. A scaling parameter $\mu_k$ is used for incorporating the local amplification effect, where $u_k^{\max}$ is also scaled as $u_k^{\max} = \mu_k C_D$.

Consider a rigidly-jointed 39-bar arch-type plane frame as shown in Fig. 1. The span length is 100.0 m and the lower nodes are located along a circle where the open angle is 50 deg. The upper nodes are also on a circle, and the difference between the radii of two circles with the same center is 7.5 m. Note that the upper chords, lower chords, and diagonals have same lengths, respectively. The upper-bound strain $\bar{\varepsilon}^{\mathrm{U}}$ is 0.001, and the mass density of the member is $7.86 \times 10^{-3}$ kg/cm$^3$. A nonstructural mass of $2.0 \times 10^4$ kg is located at each lower node. The vertical load corresponding to the weights of the members and the nonstructural masses is applied at each node. The parameters are $v_{\mathrm{s}} = 400.0$ m/s, $v_{\mathrm{app}} = 2000.0$ m/s, $\zeta_i = 0.02$ for all the modes, $C_A = 201.0$ cm/s$^2$, $C_V = 25.0$ cm/s, $C_D = 18.75$ cm, $\tau = 25.0$ sec. In this case, the value of $\omega_{(\mathrm{f})k}$ is 0.258.

The frame is assumed to be made of sandwich beams with $I_i = h^2 A_i$, where $I_i$ is the second moment of area, and $h = 50.0$ cm is the distance between two flanges which is considered as constant. The representative strain of each member is the maximum value of the edge strain at the member ends. The package DOT Ver. 5.0 [10] has been used for optimization, where the sequential quadratic programming is used. The 16-point Gaussian quadrature is used for integration. Computation has been carried out on a personal computer with AMD Athron 1.0 G Hz.

Fig. 2 shows the optimal cross-sectional areas without the effects of spatial variation; i.e. $\alpha = \eta = 0$. Note that the width of each member in Fig. 2 is proportional to the cross-sectional area. It may be observed from Fig. 2 that the lower chords near the supports

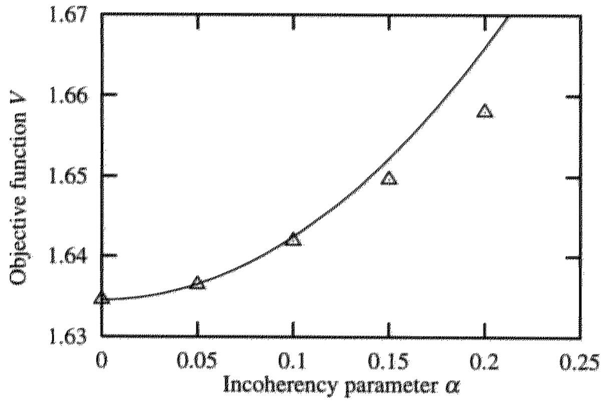


Fig. 3: Optimal objective values for $\alpha = 0.0, 0.01, 0.1, 0.15, 0.2$ and second order approximation at $\alpha = 0.0$.

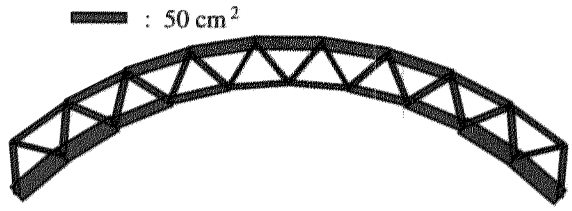


Fig. 4: Optimum cross-sectional areas considering incoherency effect ($\alpha = 1.0$).

and the upper chords around the center have large values of cross-sectional areas. The optimal objective value $\hat{V}$ is equal to $1.63454\,\mathrm{m}^3$. The triangles in Fig. 3 are the values of $\hat{V}$ for $\alpha = 0.0, 0.05, 0.1, 0.15, 0.2$. The solid curve shows the second order approximation at $\alpha = 0.0$, where $\partial^2\hat{V}/\partial\alpha^2 = 1.57076\,\mathrm{m}^3$. Note that $\hat{V}$ is an even function of $\alpha$; i.e. $\partial\hat{V}/\partial\alpha = 0$ at $\alpha = 0$, which is obvious from the quadratic term of $\alpha$ in (6). It may be observed from Fig. 3 that the optimal objective values are successfully approximated as a quadratic function of $\alpha$. The value of $\alpha d_{kl}/v_s$ corresponding to $\alpha = 0.2$ is 0.05 which is not large enough for a possible incoherency. The value of the second order sensitivity, however, gives us the estimate for the effect of incoherency on the optimal objective value.

The optimum design for $\alpha = 1.0$ is as shown in Fig. 4 which has larger cross-sectional areas in the upper chords around the center than those for $\alpha = 0$, because the difference in the movements of two supports causes pseudo static deformation that leads to bending deformation around the center. The optimal objective value for $\alpha = 1.0$ is $1.71497\,\mathrm{m}^3$. The value of $\partial\hat{V}/\partial\alpha$ at $\alpha = 0.5$ is $0.485797\,\mathrm{m}^3$ which agrees in a good accuracy with the result by the central difference method. CPU time for optimization is 9.884 sec, whereas that for post-optimal analysis is 0.020 sec. Therefore, optimal objective value for a range of parameter is estimated within only a fraction of time for finding an optimal solution.

The optimal cross-sectional areas considering the wave passage effect only, i.e. $\alpha = 0, \eta = 1$ are almost same as those in Fig. 4, where $\hat{V} = 1.69317\,\mathrm{m}^3$. Note that variation of $\eta$ indirectly corresponds to change of $v_{\mathrm{app}}$ as observed in (6). The parametric sensitivity coefficient with respect to $\eta$ at $\eta = 1.0$ is $5.14221 \times 10^{-2}\,\mathrm{m}^3$ which agrees in good accuracy with the result by the central difference method.

Finally, optimum designs are found by considering the difference in the amplification of the local soils. Let $\mu_1$ and $\mu_2$ denote the scaling factors of the maximum ground displacements and the response spectra in the horizontal displacements of the two supports. The distribution of optimal cross-sectional areas for $\mu_1 = 1.2, \mu_2 = 0.8$ is similar to that in Fig. 4, and $\hat{V} = 1.77694\,\mathrm{m}^3$. Note that $\hat{V}$ is an even function of $\mu_1 + \mu_2$, and an odd function of $\mu_1 - \mu_2$. The sensitivity coefficients with respect to $\mu_1$ and $\mu_2$ are same and equal to 0.493577 $m^3$, whereas the coefficients by the central difference method with $\Delta\mu_1 = \Delta\mu_2 = 0.05$ is 0.5107 $m^3$.

## 6. CONCLUSIONS

A method has been presented for optimum design of structures for specified response to spatially varying ground motions. The response is evaluated by the response spectrum approach. Dependence of the optimum objective value on the parameters defining the spatial variation of the ground motion is explicitly evaluated by using the technique of parametric programming or post-optimal analysis.

The parametric sensitivity coefficients obtained by the proposed method have been verified by the example of an arch-type frame subjected to horizontal ground motions. It has been shown that the second order parametric sensitivity coefficients are easily found if the first order coefficients vanish. Note that the first order coefficients are usually dominant over the second order, and the latter need not be computed. For a symmetric structure, however, the first order coefficients vanish due to symmetry and antisymmetry of the structure and ground motions.

It has been shown that all of the wave passage effect, incoherency effect, and local amplification effects lead to increase of the optimal total structural volume that is the objective function of the optimization problem. This way, the effect of the spatial variation of the seismic motions may be evaluated through the structural volume that is required for constructing the structure so as to satisfy the constraints on the response quantities.

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