PRETENSION OPTIMIZATION OF CABLE-SUPPORTED SPATIAL FRAMES

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ABSTRACT

Two methods are presented for optimization of prestressing order of cable-supported frames. The cable forces at the final state are first optimized, and the construction process is inversely traced. The objective function of the problem for optimizing prestressing order is the sum or the maximum value of the forces introduced by the jacks. The temporary supports are modeled similarly to the cables. The globally optimal order of prestressing cables and removing temporary supports is found by the dynamic programming approach. An approximate method is also presented based on a heuristic measure for selecting a cable or a temporary support at each step while tracing the inverse construction process only once. The results by two methods are compared in the example.

1. INTRODUCTION

Increasing number of cable-supported frames have been recently built for the purpose of material saving, and or simply from aesthetic point of view. By using the cables, overall stiffness is increased against various loading conditions. Cable forces can easily be adjusted to recover overall stiffness. Recently, there have been several studies for determination of cable forces at the final state (e.g. [1]).

It is important for cable-supported structures that the cost for construction strongly depends on the pretensioning order. The construction period, safety during construction, and the structural performance at the final state can also be improved by seeking an appropriate pretensioning order. Although it is very difficult to determine the cost prior to the construction, it can be estimated by a simple model based on the number of temporary supports needed or the sum of the forces applied by the jacks. Kaneko *et al.* [2] optimized the construction process of trusses using a genetic algorithm.

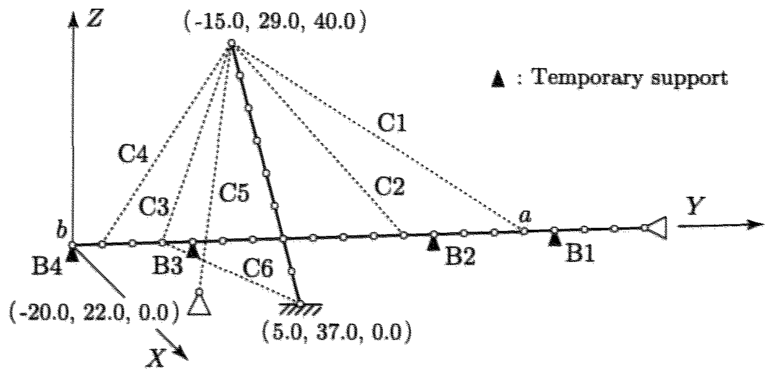

Fig. 1: A cable-supported frame.

Arase *et al.* [3] presented a method for determination of cable forces of a tower-type structure under the given prestressing order. Chen [4] optimized the cable forces so as to minimize the amount of the forces actually introduced to the cables under the specified prestressing order. Ohsaki *et al.* [5] presented a method for finding an approximately optimal order of prestressing cables of a plane frame. To the authors' knowledge, however, there have been no general and practically applicable method for optimizing order of pretensioning cables as well as removing temporary supports for a space frame.

In this paper, the globally optimal order of prestressing cables and removing temporary supports is found by using a dynamic programming approach. Approximate solutions are also obtained by using heuristic measures through the inverse construction analysis.

2. DESCRIPTION OF THE MODEL AND BASIC ASSUMPTIONS

Consider a cable-supported frame as shown in Fig. 1. The optimal cable forces at the final state is first found, and the construction process is inversely traced from the final state. Therefore, the order of removal of cables and insertion of temporary supports, which is called *construction order* for brevity, is to be optimized through the inverse construction analysis based on the following assumptions:

1. Cross-sectional and material properties of the frame members and cables are given.
2. Locations of cables and temporary supports are specified.
3. The extensional stiffness of a cable does not depend on the tensioning force.
4. Constraints on stresses and displacements are considered under the vertical loads including live load and self-weight; i.e. horizontal loads are not considered.
5. Cables and temporary supports are modeled similarly by truss members.
6. Additional loads during construction are not considered.

3. OPTIMIZATION OF CABLE FORCES AT THE FINAL STATE

Let $\boldsymbol{P}^0$ denote the static nodal loads. The stiffness matrix of the frame without cables is denoted by $\boldsymbol{K}$. Let $\boldsymbol{N}^c = \{N_i^c\}$ denote the vector of cable forces. Similar forms are used for expressing the components of vectors. Deformation during the construction process may be large and assumption of small deformation may not be applied. At the final state with appropriate cable forces, however, deformation against $\boldsymbol{P}^0$ can be assumed to be small. Let $\boldsymbol{P}^c$ denote the vector of nodal loads to the frame that are equivalent to $\boldsymbol{N}^c$. The vector $\boldsymbol{u}$ of nodal displacements is obtained from

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{P}^0 + \boldsymbol{P}^c \tag{1}$$

Let superscripts $(\ )^L$ and $(\ )^U$ denote lower- and upper-bound values. The design variables are $\boldsymbol{N}^c$, and an optimization problem is formulated as follows for minimizing a performance index $P(\boldsymbol{N}^c)$ defined by the cable forces [5]:

$$\text{Minimize} \quad P(\boldsymbol{N}^c) \tag{2}$$
$$\text{subject to} \quad u_i^L \leq u_i \leq u_i^U, \quad (i = 1, 2, \cdots, n) \tag{3}$$
$$\sigma_i^L \leq \sigma_i \leq \sigma_i^U, \quad (i = 1, 2, \cdots, m) \tag{4}$$
$$N_i^L \leq N_i^c \leq N_i^U, \quad (i = 1, 2, \cdots, m^c) \tag{5}$$

where m and m^c are the numbers of frame members and cables, respectively, and n is the number of freedom of displacements.

The following four performance indices are considered in the examples:

Index 1 Sum of the cable forces: $P_1 = \sum_{i=1}^{m^c} N_i^c$

Index 2 Maximum cable force: $P_2 = \max_i N_i^c$.

Index 3 Difference between the maximum and minimum cable forces: $P_3 = \max_i N_i^c - \min_i N_i^c$.

Index 4 Deviation of N_i^c from the target value $\bar{N}_i^c$: $P_4 = \sum_{i=1}^{m^c} (N_i^c - \bar{N}_i^c)^2$.

The optimal cable forces are found using a gradient-based optimization algorithm. In order to reduce the error due to the assumption of infinitesimal deformation, the direction of N_i^c is updated using the nodal locations after optimization, and N_i^c is re-optimized. Optimization is to be carried out several times, if necessary, before the nodal locations converge.

4. OPTIMIZATION OF CONSTRUCTION ORDER

The optimal construction order is to be found based on the following two objective functions:

- Π_s: Sum of jack forces actually used for cables and temporary supports.
- Π_m: Maximum value of jack forces.

The jack force needed for tensioning a cable is equal to the force of the cable itself. Let u_0 denote the vertical displacement of a node against unit vertical load at the node. The distance to be raised by inserting a temporary support at the node is denoted by Δu. Then the jack force for the temporary support is evaluated by $\Delta u / u_0$.

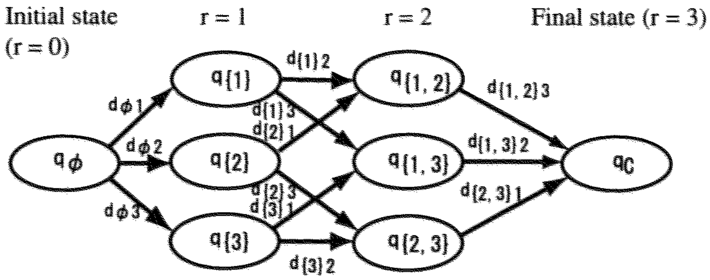

Fig. 2: State transition for $m^c = 3$.

4.1. Geometrically nonlinear analysis

In the process of inverse construction analysis, we should take into account the effect of geometrical nonlinearity. A cable is considered as a truss member, and removal of a cable is modeled as a process of decreasing its cross-sectional area to 0. The unstressed length of a cable is calculated from N_i^c, length at the final state, the elastic modulus and the cross-sectional area so that N_i^c is equal to its optimal values obtained in the previous section. The temporary support is also modeled as truss member with sufficiently large cross-sectional area.

Let $\boldsymbol{F}$ denote the vector of internal nodal forces of the frame members. The equilibrium equation for $\boldsymbol{P}_0$, $\boldsymbol{F}$ and $\boldsymbol{N}^c$ is given in terms of the equilibrium matrix $\boldsymbol{B}$ and $\boldsymbol{B}^c$ as

$$\boldsymbol{B}\boldsymbol{F} + \boldsymbol{B}^c\boldsymbol{N}^c = \boldsymbol{P}^0 \tag{6}$$

where $\boldsymbol{N}^c$ includes the axial forces of the truss members representing the temporary supports.

The truss members representing cables and temporary supports are successively removed or inserted in the inverse construction analysis. The equilibrium states are traced by using the well-known Newton-Raphson type incremental iterative method. Note that the geometrical stiffness is not considered for frames, because the linear stiffness of the frame is sufficiently large.

4.2. Optimization by dynamic programming approach

The construction order can be defined by a permutation of the indices for cables and temporary supports. In this sub-section, the temporary supports are not considered for brevity. The simplest method for finding the globally optimal construction order is to enumerate all the possible permutations. In this case, however, the number of steps of nonlinear analysis is equal to $m^c!m^c$, and the computational cost becomes very large if n, m^c and the number of temporary supports are very large. Therefore, the dynamic programming approach is used.

Fig. 2 illustrates the state transition for $m^c = 3$. Let I denote the list of cable numbers that have not been removed. q_I denotes the minimum cost for reaching the state defined by I from the initial state. The cost for removing cable j from the state defined by I is denoted by d_{Ij}. In Fig. 2, e.g., the optimal path to the final state is obtained from $\min(q_{\{1,2\}} + d_{\{1,2\}3}, q_{\{2,3\}} + d_{\{2,3\}1}, q_{\{1,3\}} + d_{\{1,3\}2})$.

Consider an intermediate state S where several cables exist. According to the principle of optimality of dynamic programming, the construction order after S should be the optimal order to the final state assuming S as the initial state. The construction order before S should also be optimal among the paths from the initial state to S. Based on this principle, the number of steps H of the nonlinear analysis is reduced to $\sum_{k=0}^{m^c-1} {}_{m^c}C_k(m^c - k)$; e.g., H is about 2.46×10^5 for $m^c = 15$ while H is about 1.96×10^{13} if the enumeration method is used.

4.3. An approximate method based on heuristic measures.

The computational cost is still very large if the dynamic programming approach is applied to a structure with many cables and frame members. Therefore, we propose an approximate method using heuristic measures for selecting a cable or a temporary support while tracing the inverse construction process only once. Let $\boldsymbol{R} = \{R_i\}$ denote the vector of jack forces needed for removal of cables or insertion of temporary supports. J denotes the set of indices of the remaining cables and non-existent temporary supports.

The increment of R_j due to removal of ith cable or insertion of ith temporary support is denoted by ΔR_j^i. The following measures are considered for selecting a cable to be removed or a temporary support to be inserted, where α_i and β_i are weight coefficients:

Case 1 Minimize the maximum jack force at the next step:

$$\min_{i \in J} \max_{j \in J, j \neq i} [\beta_j (R_j + \Delta R_j^i)] \tag{7}$$

Case 2 Consider the jack force actually used at the current step in addition to the performance measure of Case 1:

$$\min_{i \in J} \left[\max_{j \in J, j \neq i} \{\beta_j (R_j + \Delta R_j^i) + \alpha_i \beta_i R_i\} \right] \tag{8}$$

Case 3 Maximize the sum of reduction of the jack forces:

$$\max_{i \in J} \left[\sum_{j \in J, j \neq i} \beta_j \Delta R_j^i \right] \tag{9}$$

Note that ΔR_j^i should be replaced by R_j if $R_j < \Delta R_j^i$.

Case 4 Divide the performance measure of Case 3 by the jack force actually used at the current step:

$$\max_{i \in J} \left[\sum_{j \in J, j \neq i} \beta_j \Delta R_j^i \right] \Big/ \beta_j R_i \tag{10}$$

Note that we skip the cable or the temporary support with $R_i = 0$.

Since estimation of state variables after the construction step should be accurate, a re-analysis method [7, 8] is used for estimating ΔR_j^i. The increment of the variables during a construction step is denoted with Δ. The equilibrium equation after the step is written as

$$\boldsymbol{B}(\boldsymbol{F} + \Delta \boldsymbol{F}) + (\boldsymbol{B}^c + \Delta \boldsymbol{B}^c)(\boldsymbol{N}^c + \Delta \boldsymbol{N}^c) = \boldsymbol{0} \tag{11}$$

which is rewritten by using (6) and $\boldsymbol{B}\Delta\boldsymbol{F} = \boldsymbol{K}\Delta\boldsymbol{u}$, $\boldsymbol{B}^c\Delta\boldsymbol{N}^c = (\boldsymbol{K}^c + \boldsymbol{K}_G^c)\Delta\boldsymbol{u}$, $\Delta\boldsymbol{B}^c\Delta\boldsymbol{N}^c = \Delta\boldsymbol{K}^c\Delta\boldsymbol{u}$ as

$$(\boldsymbol{K} + \boldsymbol{K}^c + \boldsymbol{K}_G^c + \Delta\boldsymbol{K}^c)\Delta\boldsymbol{u} = \Delta\boldsymbol{P}^c \tag{12}$$

where $\Delta\boldsymbol{P}^c$ corresponds to the direct increment of the nodal loads due to removal of a cable or insertion of a temporary support. $\Delta\boldsymbol{u}$ is obtained by using the reanalysis method and ΔR_j^i is calculated. Since the stiffness matrix is factorized only once at each step of estimation, accurate increments are obtained within moderately small computational cost.

5. OPTIMIZATION RESULTS

Optimal cable forces at the final state and the optimal construction order have been found for a cable-supported roof-type frame as shown in Fig. 1, where a beam of 95 m is supported by six cables connected to the mast or the ground and the displacements in the x-direction of the nodes along the beam are constrained. The locations of the selected modes are as shown in Fig. 1. The static loads for the beam is 40.0 kN/m. The cross-sectional properties are as shown in Table 1, where A_i is the cross-sectional area, I_{1i} and I_{2i} are the second moment of areas, E_i is the elastic modulus, and G_i is the shear modulus.

The cable forces at the final state are optimized considering four indices, where $N_i^L = 100$ kN, $N_i^U = 2500$ kN, $\sigma_i^U = -\sigma_i^U = 0.2$ kN/mm^2, $u_i^U = -u_i^L = 200$ mm except $u_i^L = -100$ mm for the vertical displacement of the beam. Optimization has been carried out by DOT Ver. 5.0 [6] and the method of the sequential quadratic programming has been used. Table 2 shows the optimal values of cable forces and performance indices. It has been confirmed that each index has the minimum value if it is taken as the objective function.

The construction order has been optimized from the optimal final state for Index 1. The optimal and the worst-case construction orders obtained by the dynamic programming approach are as shown in Table 3 for the case of minimizing Π_s which is the sum of

Table 1: Cross-sectional and material properties.

	A_i (mm^2)	I_{1i} (mm^4)	I_{2i} (mm^4)	E_i (kN/mm^2)	G_i (kN/mm^2)
beam	1.600×10^5	2.763×10^{10}	6.633×10^9	210.00	81.00
mast	3.657×10^5	1.721×10^{11}	1.721×10^{11}	210.00	81.00
cable	5.84×10^3	0.0	0.0	160.00	0.0

Table 2: Optimal cable forces (kN), performance indices P_1, P_2, P_3 (kN) and P_4 ($\times10^6$ (kN)^2) at the final state.

	N_1^c	N_2^c	N_3^c	N_4^c	N_5^c	N_6^c	P_1	P_2	P_3	P_4
Index 1	1107.4	2500.0	1658.1	511.9	499.9	272.8	6550.2	2500.0	2227.2	3.731
Index 2	1689.9	1689.9	1307.6	911.7	490.9	1689.9	7779.9	1689.9	1199.0	1.317
Index 3	1703.1	1703.2	1441.1	1053.6	534.6	1701.2	8136.7	1703.2	1168.7	1.279
Index 4	1716.5	1711.0	1141.6	1102.4	500.3	1297.8	7469.5	1716.5	1216.3	1.039

Table 3: Construction orders and sum of jack forces.

	Π_s	Construction order
Optimal	4514.8 (kN)	(B3) - B1 - (C6, B2) - C1 - (B4) - C2 - (C4, C5) - C3
Worst-case	14222.7 (kN)	C5 - C6 - C3 - C2 - C1 - B1 - C4 - B3 - B2 - B4
Case 1	80143 (kN)	B2 - C2 - C1 - B1 - (C5, C6) - B3 - C3 - B4 - (C4)
Case 2	5548.8 (kN)	B2 - (B3, B4) - C6 - C5 - C4 - B1 - C1 - C2 - C3
Case 3	4706.4 (kN)	B1 - C5 - (C6, B2, B3, B4) - C1 - C2 - (C4) - C3
Case 4	5269.4 (kN)	B1 - C5 - (C6, B2, B3, B4) - C2 - C1 - (C4) - C3

Table 4: A variation of cable forces (kN), reaction of temporary supports (kN) and displacements (mm).

(a): Optimal construction order

	N_1^c	N_2^c	N_3^c	N_4^c	N_5^c	N_6^c	F_1^b	F_2^b	F_3^b	F_4^b	δz_a	δz_b
Final	1072.4	2491.4	1665.8	514.5	462.8	311.3	0.0	0.0	0.0	0.0	-111.6	210.2
S(B1)	834.1	2307.1	1521.6	478.4	345.7	0.0	592.1	0.0	0.0	0.0	5.0	133.6
S(C1)	0.0	2873.3	1405.7	486.5	149.3	0.0	805.1	0.0	0.0	0.0	8.3	3.1
S(C2)	0.0	0.0	215.3	0.0	0.0	0.0	1017.1	1242.8	1030.6	418.8	-1.9	0.0
S(C3)	0.0	0.0	0.0	0.0	0.0	0.0	1010.7	1261.6	1176.5	449.9	-1.8	0.0

(b): Case 3

	N_1^c	N_2^c	N_3^c	N_4^c	N_5^c	N_6^c	F_1^b	F_2^b	F_3^b	F_4^b	δz_a	δz_b
Final	1072.4	2491.4	1665.8	514.5	462.8	311.3	0.0	0.0	0.0	0.0	-111.6	210.2
S(B1)	834.1	2307.1	1521.6	478.4	345.7	0.0	592.1	0.0	0.0	0.0	5.0	133.6
S(C5)	729.5	2272.1	1530.0	476.3	0.0	0.0	670.0	0.0	0.0	0.0	3.6	146.0
S(C1)	0.0	2823.6	1416.3	485.0	0.0	0.0	829.0	0.0	0.0	0.0	7.5	16.3
S(C2)	0.0	0.0	215.3	0.0	0.0	0.0	1017.1	1242.8	1030.6	418.8	-1.9	0.0
S(C3)	0.00	0.00	0.0	0.00	0.00	0.00	1010.7	1261.6	1176.5	449.9	-1.8	0.0

the jack forces. In Table 3, 'Bi' and ''Ci' indicate the numbers of temporary support and cable, respectively, to be inserted and removed. The numbers in brackets indicate that the cable is already slack or the node is already above the temporary support, and no jack force is needed at the corresponding step. It is observed from Table 3 that Π_s can be reduced by inserting the temporary supports at the early stage of the inverse construction process. Early removal of the cables connected to the nodes near the supports also leads to reduction of Π_s. Note that the total jack forces including those for the temporary supports is 4514.8 kN which is about 69% of the sum of the cable forces at the final state.

The approximate solutions based on heuristic measures are also listed in Table 3. It should be noted from Table 3 that an almost optimal solution that minimizes Π_s has been obtained in Case 3. The variation of the cable forces, reactions F_i^b at the temporary supports, and the vertical displacements δz_a and δz_b of nodes a and b, as defined in Fig. 1, are listed in Table 4. In Table 4, 'S(Bi)' or 'S(Ci)' indicate the state immediately after ith temporary support and cable, respectively, are inserted and removed. Note that the largest deformation is observed at the final state, and the deformation during construction is sufficiently small.

6. CONCLUSIONS

Two methods have been presented for optimization of prestressing order of cable-supported frames by inversely tracing the construction process from the final state with optimal cable forces. The cables and temporary supports are modeled as truss members, and the cross-sectional areas of the truss members are adjusted to represent the removal and insertion of cables and temporary supports, respectively, during the inverse construction analysis.

The globally optimal prestressing order is found by the dynamic programming approach. An approximate method is also presented based on a heuristic measure for selecting the cable to be removed or the temporary support to be inserted at each step while tracing the inverse construction process only once. It has been shown in the example of a cable-supported roof-type frame that the order of removal of cables and insertion of temporary supports can be successfully optimized by the dynamic programming approach. A nearly optimal construction order that minimizes the sum of jack forces actually used can be obtained by using the approximate method within a fraction of computational cost that is needed for the dynamic programming approach.

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